PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC, **123**:976–980, 2011 August

Call to Adopt a Nominal Set of Astrophysical Parameters and Constants to Improve the Accuracy of Fundamental Physical Properties of Stars

PETR HARMANEC

Astronomical Institute of the Charles University, Faculty of Mathematics and Physics, V Holešovičkách 2, CZ-180 00 Praha 8, Czech Republic; hec@sirrah.troja.mff.cuni.cz

AND

ANDREJ PRŠA

Villanova University, Department of Astronomy & Astrophysics, 800 Lancaster Avenue, Villanova, PA 19085

Received 2011 March 19; accepted 2011 June 3; published 2011 July 29

ABSTRACT. The increasing precision of astronomical observations of stars and stellar systems is gradually getting to a level where the use of slightly different values of the solar mass, radius, and luminosity, as well as different values of fundamental physical constants, can lead to measurable systematic differences in the determination of basic physical properties. An equivalent issue with an inconsistent value of the speed of light was resolved by adopting a nominal value that is constant and has no error associated with it. Analogously, we suggest that the systematic error in stellar parameters may be eliminated by (1) replacing the solar radius $R_\odot$ and luminosity $L_\odot$ by the nominal values that are *by definition* exact and expressed in SI units: 1 $\mathcal{R}_\odot^N = 6.95508 \times 10^8$ m and 1 $\mathcal{L}_\odot^N = 3.846 \times 10^{26}$ W; (2) computing stellar masses in terms of $M_\odot$ by noting that the measurement error of the product $GM_\odot$ is 5 orders of magnitude smaller than the error in G; (3) computing stellar masses and temperatures in SI units by using the derived values $\mathcal{M}_\odot^{2010} = 1.988547 \times 10^{30}$ kg and $\mathcal{T}_\odot^{2010} = 5779.57$ K; and (4) clearly stating the reference for the values of the fundamental physical constants used. We discuss the need and demonstrate the advantages of such a paradigm shift.

1. INTRODUCTION AND MOTIVATION

Notable improvements in the precision of astronomical observations start to challenge our current models and understanding of the physical processes in stars and stellar systems. Until recently, the accuracy of fundamental stellar parameters derived from observations was limited by the stochastic uncertainty of observations, but with new space-borne instruments such as the Molonglo Observatory Synthesis Telescope (MOST; Ruciński et al. 2003), *COROT* (Auvergne et al. 2009), and *Kepler* (Borucki et al. 2010), systematic effects due to model inadequacies are starting to play an important role.

One of the increasingly important deficiencies is the use of the solar mass $M_\odot$, solar radius $R_\odot$, and solar luminosity $L_\odot$ as units in which the fundamental stellar properties, deduced via fitting the observations by appropriate models, are usually expressed. Such stellar properties inevitably depend on the particular values of the fundamental solar characteristics adopted by different researchers in their studies. This inconsistency is rooted deeply in the literature, because the effect of discrepant parameters was often well within the systematic and stochastic uncertainties of observations. However, in certain fields, such as binary star research, the associated artifacts become increasingly important, especially with the accumulation of precise observations of systems with longer orbital periods. They actually remain a fundamental issue across all fields, because eclipsing binaries (EBs) are the most accurate and commonly used calibrators for the masses and radii of *single* stars (Torres et al. 2010). A typical accuracy of stellar parameters for well-studied EBs is claimed to be better than ~2% (Andersen 1991). The modern methods for spectroscopic analysis, such as cross-correlation (Simkin 1974; Da Costa et al. 1977; Tonry & Davis 1979), broadening functions (Ruciński 1992, 1998), and disentangling (Simon & Sturm 1994; Hadrava 1995, 1997; Ilijić et al. 2004), as well as improved light-curve synthesis codes (Wilson & Devinney 1971; Wilson 1979, 1994, 2008; Prša & Zwitter 2005, 2006) can push these limits even further. The systematic error due to inconsistently used solar parameters hence propagates to the derived M-R-L-T calibrations and has a notably adverse effect on the computed absolute scales and distances. The situation, however, can be significantly improved if we are willing to part with the *current* values of canonical solar parameters and replace them with the nominal values.

This issue is not novel in astronomy and a precedent has been set with the exact value of the speed of light. In 1975, Resolution 2 of the 15th Conférence Générale des Poids et Mesures (CGPM) proposed the value of the speed of light in vacuum to be $c = 299,792,458\ \mathrm{m\,s^{-1}}$. The value was chosen considering the excellent agreement to $\delta c/c \sim 4 \times 10^{-9}$ among different measurement methods across all wavelengths. In 1983, an argument was made that the *unchanging* speed of light needs to be maintained, notably for astronomy and geodesy, and this was formalized by Resolution 1 of the 17th CGPM, which defines the speed of light to be the value recommended in 1975. Consequently, 1 m was *redefined* to be the distance traveled by light in vacuum in $1/299,792,458$ s. This way the nominal value of 1 m depends on a fundamental natural constant rather than the other way around, as was historically the case.[1]

The redefinition of the bolometric magnitude followed at the 23rd General Assembly of the International Astronomical Union, held in Kyoto in 1997 August, with adopting a resolution[2] that specifies that the zero point of the bolometric magnitude scale will no longer be defined through the bolometric luminosity of the Sun, but rather by defining that $M_{\rm bol} = 0.0$ mag corresponds to the bolometric luminosity $L_{\rm bol} = 3.055 \times 10^{28}$ W. This introduces an *absolute scale* of bolometric magnitudes, $M_{\rm bol} = 71.2125$ mag–$2.5 \log L$, where the bolometric luminosity L is given in watts. The convenience of comparing L with the solar value remains, since a new definition complies with the most-often-quoted value of the solar bolometric magnitude: $L_\odot = 3.846 \times 10^{26}$ W $\Rightarrow M_{\rm bol,\odot} = 4.75$ mag.

Another important deficiency is the use of inconsistent (often outdated) values of the fundamental physical constants without providing a reference to the source of the value used. The derived constants cannot be made exact, since they *observationally* depend on fundamental SI units. The universal constant of gravitation G, for example, is defined as the proportionality constant in Newton's law and is one of the most difficult constants to measure to a high accuracy (Gillies 1997). The currently recommended 2010 value of the constant by the Committee on Data for Science and Technology (CODATA)[3] is $G = (6.67384 \pm 0.00080) \times 10^{-11}\ {\rm m^3\,kg^{-1}\,s^{-2}}$. Since it is derived, this value is subject to change in the future as more precise measurements become available. The solution to this problem is to commit to the use of constants set forth by the IAU or CODATA and meticulously provide a reference to the used value.

In this article we quantify the systematic effect of inconsistent values of various parameters and propose to adopt the nominal values for the mass, radius, and luminosity of the Sun. We further propose to unify the astrophysical constants across modeling tools and make a strong effort to keep the values of the derived constants up to date with the IAU, CGPM, and CODATA resolutions. An excellent example is a recent review on accurate stellar masses and radii by Torres et al. (2010), who quote values of all the relevant constants used.

[1] See http://www.bipm.org/en/CGPM/db/17/1.

[2] Suggested by Dr. Roger Cayrel from Paris.

[3] See http://physics.nist.gov/cuu/Constants/index.html.

2. QUANTIFICATION OF THE EFFECT

The recently published Kepler Eclipsing Binary Catalog (Prša et al. 2011; Slawson et al. 2011) contains over 2200 EBs in the 105 deg^2 field of view. The period distribution due to Kepler's uninterrupted baseline does not suffer from any significant selection effects toward longer periods (∼10^2 days) and a significant number of sources with $P_{\rm orb} \geq 50$ d are emerging from the sample. Let us consider as an example a binary system with two 1 $M_\odot$ components in a circular orbit with the orbital period of 200 days. The separation between the components can be readily computed from Kepler's third law. If we adopt the preceding quoted value of G and compute the separation based on the 1 $M_\odot$ values listed in Table 1, we will arrive at the following values (in 10^9 m): 126.1624 ± 0.0050, 126.2026 ± 0.0050, 126.1586 ± 0.0050, and 126.1476 ± 0.0050. This accounts for the relative error of 4.0×10^{-5}, or an absolute error of 5.0×10^6 m. A single event in Kepler long-cadence data (30 minute exposure) can be timed to ∼6 minutes; a $P_{\rm orb} = 200$ day binary will have 18 events (nine primary and nine secondary eclipses) over the 5 yr mission lifetime, which will reduce the timing error to ∼1.4 minutes or, in relative terms, to 5×10^{-6}. This is an order of magnitude smaller than the effect of using inconsistent solar-mass values and will be the cause of significant systematics. If short-cadence data (1 minute exposures) are available for the given target, the timing is improved by another order of magnitude, and the systematics will overpower the stochastic error.

The same is true for single stars as well. Consider a giant star with a radius of 30 $R_\odot$ and an equatorial rotational velocity of 5 ${\rm km\,s^{-1}}$. The period of rotation of such a star would then be 303.801 days for the IAU 1976 value of the solar radius but 303.473 days for the recent Brown & Christensen-Dalsgaard (1998) value (see Table 2). Such a difference is readily detectable after a few rotational periods covered by relevant observations.

Also worth considering is the dependence of the effective temperature on the value of stellar radius through $L_\odot = 4\pi R_\odot^2 \sigma T_{\rm eff}^4$, where σ is the Stefan-Boltzmann constant. Table 2 lists several values of the solar radius, either recommended in

TABLE 1

VALUES OF 1 SOLAR MASS IN THE LAST 35 YEARS

$M_\odot$ (10^{30} kg)	Source[a]
1.9891	1 (1976 IAU constants)
1.991 ± 0.002	2 (Handbook of Chem. Phys.)
1.98892 ± 0.00025	3 (Astron. Almanac 1994)
1.9884 ± 0.0002	1 (Astron. Almanac 2011)

[a] (1) http://asa.usno.navy.mil/SecK/Constants.html; (2) Weast & Astie 1980; (3) Astronomical Almanac for the year 1994, U.S. Government Printing Office, Washington, and Her Majesty's Stationery Office, London 1993.

TABLE 2

VALUE OF MEAN SOLAR RADIUS $R_\odot$ ADOPTED FROM VARIOUS SOURCES AND THE CORRESPONDING SOLAR EFFECTIVE TEMPERATURE

$R_\odot$ (km)	$T_{\rm eff}$ (K)	Source[a]
696200.0	5776.702 ± 0.005	A
695990.0	5777.573 ± 0.005	B
695508.0	5779.575 ± 0.005	C
695835.3	5778.215 ± 0.005	D
695833.1	5778.224 ± 0.005	E
695770.0	5778.486 ± 0.005	F
695658.0	5778.952 ± 0.005	G

NOTE.—The (almost negligible) errors in values of the effective temperature are given solely by the error in the determination of the Stefan-Boltzmann constant σ.

[a] (A) IAU system of constants 1976; (B) Allen third edition: Allen 1976; (C) Allen fourth edition: Cox 2000, adopted from Brown & Christensen-Dalsgaard 1998; (D) This article: a sinusoidal fit to all Cote d'Azur data 1975–1998; (E) This article: a sinusoidal fit to good Cote d'Azur data 1975–1998; (F) Tripathy & Antia 1999; (G) Haberreiter et al. 2008.

various compilations or derived from recent measurements. For each value we give the corresponding effective temperature calculated for the solar luminosity value of $L_\odot = 3.846 \times 10^{26}$ W and $\sigma = 5.670373(21) \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. The systematic differences in $T_{\rm eff}$ values are larger than the propagated errors.

3. THE SOLAR UNITS

It is customary in stellar research to express the basic physical properties of the stars, such as their luminosity, mass, or radius, in solar units. While convenient, this is somewhat unfortunate, for the following reasons:

1. The values of the solar luminosity, mass, and radius are subject to continuous improvement due to the increasingly precise observational techniques.
2. Solar luminosity and radius vary measurably with the solar cycle and perhaps also on other, even shorter, timescales (see, e.g., Selhorst et al. 2004). Due to effects such as mass loss via stellar wind or mass gain due to infalling material, the change of the solar mass may become measurable on longer timescales.

At the same time, some important quantities, such as the projected velocity of the stellar rotation, are measured in absolute units (km $\mathrm{s^{-1}}$ in this particular case) that depend directly on the adopted values of the fundamental solar properties.

We suggest a simple solution to the problem of inconsistent use of various different values of the solar units—one that follows the precedent for the speed of light and bolometric magnitude. Let us deprecate the use of the current values of $R_\odot$ and $L_\odot$ with all their uncertainties and time variations, and replace them with the *exact, nominal* units $\mathcal{R}_\odot^N$ and $\mathcal{L}_\odot^N$, where their numerical values are chosen to be close to the recently derived (or adopted) values: the solar radius from Brown & Christensen-Dalsgaard (1998), also recommended in the fourth edition of Allen's Astrophysical Quantities (Cox 2000), and the solar luminosity to comply with the aforementioned IAU resolution on bolometric magnitude.

Once the values of $R_\odot$ and $L_\odot$ are made nominal, the relationship between the nominal values $\mathcal{R}_\odot^N$ and $\mathcal{L}_\odot^N$ and the effective temperature $\mathcal{T}_\odot^{2010}$ can be determined readily from

$$\mathcal{L}_\odot^N = 4\pi \mathcal{R}_\odot^{N^2} \sigma (\mathcal{T}_\odot^{2010})^4,$$
$$\text{where } \sigma = 5.670373(21) \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}.$$

Note that the small error in the determination of the Stefan-Boltzmann constant σ propagates to negligibly small errors in the determination of the effective temperature.

Unfortunately, the same cannot be proposed for the solar mass: while the universal gravitational constant $G = (6.67384 \pm 0.00080) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ is one of the least precisely determined fundamental constants in nature, the product $GM_\odot = 1.32712442099(10) \times 10^{20}\ \mathrm{m^3\,s^{-2}}$ is determined much more precisely (Petit et al. 2010). All physical parameters that can be expressed in terms of $GM_\odot$ can thus be determined much more accurately than the ones depending on $M_\odot$ alone. Table 3 lists several important ones, most notably, stellar masses in terms of solar mass and the absolute scale of the system in SI units. In particular, the mass of a binary star component j $(j = 1, 2)$ can be expressed as $GM_j = [K_{3-j}P(K_1 + K_2)^2]/(2\pi \sin^3 i)$, where K_1 and K_2 are radial velocity semiamplitudes in km $\mathrm{s^{-1}}$, and i is the inclination. If we divide this expression by $GM_\odot$, we obtain $M_j/M_\odot$ in terms of $GM_\odot$, which is much more precise than computing it with the more uncertain value of G. However, both solar and stellar masses in SI units will still be limited by the measurement error in G. If solar mass were to be made nominal, the precision in $M_j/\mathcal{M}_\odot^{2010}$ would be degraded by 5 orders of magnitude. To make the distinction clear, we use the designation N to represent nominal parameters, and we use 2010 to denote parameters that are derived from the latest values of $GM_\odot$ and σ.

The proposed nominal and derived parameters to be used in computations are

$$1\ \mathcal{R}_\odot^N = 6.95508 \times 10^8\ \mathrm{m}$$
$$1\ \mathcal{L}_\odot^N = 3.846 \times 10^{26}\ \mathrm{W}$$
$$1\ GM_\odot^{2010} = 1.32712442099(10) \times 10^{20}\ \mathrm{m^3\,s^{-2}}$$
$$1\ \mathcal{M}_\odot^{2010} = 1.988547 \times 10^{30}\ \mathrm{kg}$$
$$1\ \mathcal{T}_\odot^{2010} = 5779.57\ \mathrm{K}.$$

4. SELECT EXAMPLES

There are a number of frequently used formulae where numerical constants are affected by the adopted values of the solar mass, radius, and luminosity. We list some of them for the

TABLE 3

Select Examples of Commonly Used Equations Utilizing Nominal Solar Values Proposed by This Article and Constants Recommended by IAU

Select example	Equation	Comments
Kepler's third law	$A\ (\mathrm{m}) = 2,927,699,260.629(74)P^{\frac{2}{3}}(M_1+M_2)^{\frac{1}{3}}$ $A\ (\mathrm{AU}) = 0.01957046077547(49)P^{\frac{2}{3}}(M_1+M_2)^{\frac{1}{3}}$ $A[\mathcal{R}^N_\odot] = 4.20944009361(11)P^{\frac{2}{3}}(M_1+M_2)^{\frac{1}{3}}$	A: semimajor axis in m, AU, and $\mathcal{R}^N_\odot$; M_1 and M_2: point masses in $M^{2010}_\odot$; P: period in mean solar days.
Surface gravity	$\log g\ (\mathrm{cgs}) = 4.438307381330(33) + \log M - 2\log R$	M: stellar mass in $M^{2010}_\odot$; R: stellar radius in $\mathcal{R}^N_\odot$. With the use of the nominal values of the solar mass and radius, the error only depends on the error of the $GM_\odot$ product constant and the conversion formula becomes very accurate, highly exceeding the current precision of our knowledge of stellar masses and radii.
Stellar bolometric magnitude:	$M_{\mathrm{bol}} = 42.3689588(40) - 5\log R - 10\log T_{\mathrm{eff}}$	R: stellar radius in $\mathcal{R}^N_\odot$; T_{eff}: effective temperature in K. The error in the conversion constant is due to the uncertainty in the Stefan-Boltzmann constant σ.
Parallactic radius	$R = 107.5457584245(22)\theta p^{-1}$	R: radius in $\mathcal{R}^N_\odot$; θ: angular stellar diameter in arcseconds; p: stellar parallax in arcseconds. The constant is computed from the recommended value of 1 AU and 1 pc by IAU 2009 (149,597,870,700(3) m and $3.085677581503(62) \times 10^{16}$ m, respectively).
Angular orbit diameter:	$a = (215.0915168490(43))^{-1}Ap$	a: angular orbit diameter in arcseconds; A: semimajor axis in AU; p: stellar parallax in arcseconds. The same preceding constants were used.
Equatorial rotational velocity	$V = 50.57877RP^{-1}_{\mathrm{rot}}$	V: equatorial rotational velocity in $\mathrm{km\,s^{-1}}$; R: stellar radius in $\mathcal{R}^N_\odot$; P_{rot}: rotational period in mean solar days. Note that the constant in the preceding equation has no uncertainty, due to our use on the nominal value of the solar radius.
Mass functions	$M_1\sin^3 i = 1.036149050206(78) \times 10^{-7}K_2(K_1+K_2)^2P(1-e^2)^{1.5}$ $M_2\sin^3 i = 1.036149050206(78) \times 10^{-7}K_1(K_1+K_2)^2P(1-e^2)^{1.5}$ $f_1(M) = 1.036149050206(78) \times 10^{-7}K_1^3P(1-e^2)^{1.5}$ $f_2(M) = 1.036149050206(78) \times 10^{-7}K_2^3P(1-e^2)^{1.5}$	M_1: stellar mass in $M^{2010}_\odot$; i: inclination in deg; $K_{1,2}$: RV semiamplitudes in $\mathrm{km\,s^{-1}}$; P: orbital period in mean solar days; e: eccentricity. The uncertainty stems from the uncertainty in the G $M_\odot$ product.
Projected orbital sizes	$a_1\sin i = 0.019771142K_1P(1-e^2)^{0.5}$ $a_2\sin i = 0.019771142K_2P(1-e^2)^{0.5}$ $A\sin i = 0.019771142(K_1+K_2)P(1-e^2)^{0.5}$	$a_{1,2}$: distance of the orbiting component from the center of mass, in $\mathcal{R}^N_\odot$; $A = a_1 + a_2$: semimajor axis in $\mathcal{R}^N_\odot$; i: inclination in degrees; $K_{1,2}$: RV semiamplitudes in $\mathrm{km\,s^{-1}}$; P: orbital period in days; e: orbital eccentricity. Note that there is no error in the transformation constant.
Solar effective temperature	$\mathcal{T}^N_{\mathrm{eff}} = 5779.5747(54)$	$\mathcal{T}^N_{\mathrm{eff.}}$ is the effective temperature of the Sun for the nominal value of the solar radius

suggested nominal values in Table 3. In all examples, the propagated errors of the fundamental physical constants used are given in parentheses and denote the uncertainty of the last two significant digits. The expressions for the semimajor axis A from the third Kepler law are given for the value of A in kilometers, AU, and $\mathcal{R}^N_\odot$.

5. CONCLUSIONS

We presented the case for obsoleting the use of $R_\odot$, $L_\odot$, and $M_\odot$ as units and replacing two of them with their nominal counterparts $\mathcal{R}^N_\odot$ and $\mathcal{L}^N_\odot$ that are *exact*. In all determinations of stellar masses expressed in $M_\odot$, the product $GM^{2010}_\odot$, known to a very high precision, should be used. For the conversion of the thus derived stellar masses to SI units, $\mathcal{M}^{2010}_\odot$ can be used. Once the precision of the universal gravitational constant G is improved, it will be beneficial to deprecate the actual value of the solar mass and replace it by an exact, nominal value $\mathcal{M}^N_\odot$.

All the recommendations presented in this article would reduce the systematics that stem from using discrepant values of fundamental properties of the Sun. This is a consequence of the true variations of these values due to intrinsic effects such as the magnetic solar cycle, as well as of a steady improvement in their determination via more precise observations. We further implore the community to use fundamental physical constants recommended by the IAU and/or CODATA and call for meticulous referencing of the used sources.

We appreciate several useful discussions on the subject, which we had with M. Brož, P. Mayer, D. Vokrouhlický, and P. Zasche. The research of P. H. was supported by grant P209/10/0715 of the Czech Science Foundation and also from the Research Program MSM0021620860 "Physical Study of Objects and Processes in the Solar System and in Astrophysics of the Ministry of Education of the Czech Republic." A. P. acknowledges the NASA/SETI subcontract 08-SC-1041. We acknowledge the use of the electronic database from CDS Strasbourg and electronic bibliography maintained by the NASA/ADS system.

REFERENCES

Allen, C. W. 1976, Astrophysical Quantities, ed. C. W. Allen (3rd ed.; London: Athlone)
Andersen, J. 1991, A&A Rev., 3, 91
Auvergne, M., Bodin, P., & Boisnard, L., et al. 2009, A&A, 506, 411
Borucki, W. J., Koch, D., & Basri, G., et al. 2010, Science, 327, 977
Brown, T. M., & Christensen-Dalsgaard, J. 1998, ApJ, 500, L 195
Cox, A. N. 2000, Allen's Astrophysical Quantities, ed. A. N. Cox (4th ed.; New York: AIP Press and Springer)
Da Costa, G. S., Freeman, K. C., Kalnajs, A. J., Rodgers, A. W., & Stapinski, T. E. 1977, AJ, 82, 810
Gillies, G. T. 1997, Rep. Prog. Phys., 60, 151
Haberreiter, M., Schmutz, W., & Kosovichev, A. G. 2008, ApJ, 675, L 53
Hadrava, P. 1995, A&AS, 114, 393
———. 1997, A&AS, 122, 581
Ilijíc, S., Hensberge, H., Pavlovski, K., & Freyhammer, L. M. 2004, ASP Conf. Ser., Vol. 318, Spectroscopically and Spatially Resolving the Components of the Close Binary Stars, ed. R. W. Hilditch, H. Hensberge, & K. Pavlovski (San Francisco: ASP), 111
Petit, G., & Luzum, B., eds. 2010, IERS Conventions (IERS Tech. Note 36; Frankfurt: Verlag des Bundesamts für Kartographie und Geodäsie)
Prša, A., Batalha, N. M., & Slawson, R. W., et al. 2011, AJ, 141
Prša, A., & Zwitter, T. 2005, ApJ, 628, 426
———. 2006, Ap&SS, 304, 347
Rucínski, S., Carroll, K., Kuschnig, R., Matthews, J., & Stibrany, P. 2003, Adv. Space Res., 31, 371
Rucínski, S. M. 1992, AJ, 104, 1968
———. 1998, Turkish J. Physics, 1, 1
Selhorst, C. L., Silva, A. V. R., & Costa, J. E. R. 2004, A&A, 420, 1117
Simkin, S. M. 1974, A&A, 31, 129
Simon, K. P., & Sturm, E. 1994, A&A, 281, 286
Slawson, R. W., Prsa, A., & Batalha, N. M., et al. 2011, AJ, in press
Tonry, J., & Davis, M. 1979, AJ, 84, 1511
Torres, G., Andersen, J., & Giménez, A. 2010, A&A Rev., 18, 67
Tripathy, S. C., & Antia, H. M. 1999, Sol. Phys., 186, 1
Weast, R., & Astie, M. 1980, CRC Handbook of Chemistry and Physics (61st ed.; Boca Raton: CRC Press)
Wilson, R. E. 1979, ApJ, 234, 1054
———. 1994, PASP, 106, 921
———. 2008, ApJ, 672, 575
Wilson, R. E., & Devinney, E. J. 1971, ApJ, 166, 605