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ξ Tauri...

J. Nemravová

[1] Institute of Astronomy, Charles University, Prague, V Holešovičkách 2, 18000 Prague 8, Czech Republic,
e-mail:
[2]

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8. N-body model of ξ Tauri with mutual interactions

Given the quadruple nature of ξ Tauri and its relatively close packing, it is necessary to proceed with an advanced N-body model which would account for mutual gravitational interactions of *all* four components. To this point, we shall now describe our numerical integrator, a definition of a suitable χ^2 metric and overall results of our fitting procedure. Hereinafter, we beg to switch to a numerical notation for individual ξ Tau components — namely 1 ... Aa, 2 ... Ab, 3 ... B, 4 ... C — in order to simplify the equations below.

8.1. Numerical integrator and χ^2 metric

We use a standard Bulirsch–Stöer N-body numerical integrator from the SWIFT package (Levison & Duncan 1994). Our method is quite general — we can model classical Keplerian orbits, of course, but also non-Keplerian ones (involving N-body interactions). We treat all the stars as point masses only thought; we have no higher-order gravitational terms and no tides in our model.

As explained below, this is a significant improvement of our previous application in Brož et al. (2010), because we can now account not only for the light-time effect but for complete TTV variations of the eclipsing binary, arising from both direct and indirect gravitational perturbations. At the same time, we do not use the simplification of Brož et al. (2010) and we consider all the components separately, because the equivalent gravitational moment:

$$J_2 \simeq \frac{1}{2}\left(\frac{a}{r}\right)^2 \frac{m_1 m_2}{(m_1+m_2)^2} \doteq 2\times 10^{-3} \tag{1}$$

of the inner eclipsing binary is large at the distance of the 3rd body.

We are forced to use five different coordinate systems: (i) 1-centric (usually, for a specification of initial conditions); (ii) barycentric (for the numerical integration itself); (iii) 1+2 photocentric (for a comparison with interferometric observations of the 3rd body); (iv) 1+2+3 photocentric (dtto for the 4th body); (v) Jacobian (for computations of hierarchical orbital elements).

Initial conditions at a selected epoch T_0 can be specified either in Cartesian coordinates — with x, y in the sky plane and z in the radial direction — or in *osculating* orbital elements. This very choice has a substantial role, because the outcome of the fitting procedure will be generally different. The orbital elements can be considered less correlated quantities than 1-centric Cartesian coordinates.

We try to account for as much observational data as we can using the following joint metric:

$$\chi^2 = \chi^2_{\rm rv} + \chi^2_{\rm ttv} + \chi^2_{\rm ecl} + \chi^2_{\rm sky} + \chi^2_{\rm vis} + \chi^2_{\rm clo}\,, \tag{2}$$

$$\chi^2_{\rm rv} = \sum_{j=1}^{3}\sum_{i=1}^{N_{{\rm rv}\,j}} \frac{\left(v'_{{\rm zb}\,j} + \gamma - v_{{\rm rad}\,ji}\right)^2}{\sigma^2_{{\rm rv}\,ji}}\,, \tag{3}$$

$$\chi^2_{\rm ttv} = \sum_{i=1}^{N_{\rm ttv}} \frac{\left(t'_{1+2} - t_{1+2\,i}\right)^2}{\sigma^2_{{\rm ttv}\,i}}\,, \tag{4}$$

$$\chi^2_{\rm ecl} = \sum_{i=1}^{N_{\rm ecl}} \frac{\left(\epsilon'_{1+2} - \epsilon_{1+2\,i}\right)^2}{\sigma^2_{{\rm ecl}\,i}}\,, \tag{5}$$

$$(\Delta x_{ji}, \Delta y_{ji}) = \mathbf{R}\left(-\phi_{\rm ellipse} - \frac{\pi}{2}\right) \times \begin{pmatrix} x'_{{\rm p}j} - x_{{\rm p}ji} \\ y'_{{\rm p}j} - y_{{\rm p}ji} \end{pmatrix}, \tag{6}$$

$$\chi^2_{\rm sky} = \sum_{j=3}^{4}\sum_{i=1}^{N_{{\rm sky}\,j}} \frac{(\Delta x_{ji})^2}{\sigma^2_{{\rm sky\,major}\,ji}} + \frac{(\Delta y_{ji})^2}{\sigma^2_{{\rm sky\,minor}\,ji}}\,, \tag{7}$$

$$V'(u,v) = \frac{1}{L_{\rm tot}} \sum_{j=1}^{4} L_{ij}\, 2\frac{J_1\left(\pi\theta_j\sqrt{u^2+v^2}\right)}{\pi\theta_j\sqrt{u^2+v^2}}\, {\rm e}^{-2\pi {\rm i}(u x'_{{\rm a}j} + v y'_{{\rm a}j})}\,, \tag{8}$$

$$L_{ij}(T_{{\rm eff}\,j}, R_j) \doteq \int_{\lambda_i-\Delta\lambda_i/2}^{\lambda_i+\Delta\lambda_i/2} 4\pi R_j^2\, \pi B_\lambda(T_{{\rm eff}\,j})\,{\rm d}\lambda\,, \tag{9}$$

$$\chi^2_{\rm vis} = \sum_{i=1}^{N_{\rm vis}} \frac{\left(|V'(u_i,v_i)|^2 - V_i^2\right)^2}{\sigma^2_{{\rm vis}\,i}}\,, \tag{10}$$

$$T'_3 = V'(u_1,v_1)V'(u_2,v_2)V'(-(u_1+u_2), -(v_1+v_2))\,, \tag{11}$$

$$\chi^2_{\rm clo} = \sum_{i=1}^{N_{\rm clo}} \frac{\left(\arg T'_3 - \arg T_{3i}\right)^2}{\sigma^2_{{\rm clo\,phase}\,i}} + \frac{\left(|T'_3| - |T_{3i}|\right)^2}{\sigma^2_{{\rm clo\,modul}\,i}}\,, \tag{12}$$

where the notation is briefly described in Table 1. The dashed quantities are the model values linearly interpolated to the exact

Table 1. Notation used for various coordinates, velocities, other quantities and uncertainties. which we use in our N-body model.

$x_{\rm h}, y_{\rm h}, z_{\rm h}$	1-centric ("heliocentric") coordinates
$v_{x{\rm h}}, v_{y{\rm h}}, v_{z{\rm h}}$	1-centric velocities
$x_{\rm p3}, y_{\rm p3}$	1+2 photocentric sky-plane coordinates (3rd body)
$x_{\rm p4}, y_{\rm p4}$	1+2+3 photocentric coordinates (4th body)
$x_{\rm a}, y_{\rm a}$	1-centric coordinates in an angular measure
$v_{z{\rm b}}$	the barycentric radial velocity
γ	the systemic velocity
$v_{\rm rad}$	the observed radial velocity
t_{1+2}	the minimum of the eclipsing binary
ϵ_{1+2}	the eclipse duration
V	complex visibility, squared visibility is $\lvert V\rvert^2$
T_3	complex triple product, closure phase is $\arg T_3$
u, v	projected baselines (expressed in cycles, $\frac{B}{\lambda}$)
$\theta = \frac{2R}{d}$	angular diameter
d	distance
$L, L_{\rm tot}$	component luminosity and the total one
$T_{\rm eff}$	effective temperature
R	stellar radius (uniform-disk)
λ, $\Delta\lambda$	effective wavelength and bandwidth
$B_\lambda(T)$	the Planck function
$\sigma_{\rm rv}$	uncertainty of the radial velocity
$\sigma_{\rm ttv}$	uncertainty of the minimum timing
$\sigma_{\rm ecl}$	uncertainty of the eclipse duration
$\sigma_{\rm sky\,major,\,minor}$	uncertainty of the astrometric position, angular sizes of the uncertainty ellipse
$\phi_{\rm ellipse}$	position angle of the ellipse
$\mathbf{R}(\ldots)$	the corresponding rotation matrix
$\sigma_{\rm vis}$	uncertainty of the squared visibility
$\sigma_{\rm clo\,phase,\,modul}$	uncertainty of the closure phase and $\lvert T_3\rvert$

times t_i of observations. The index j corresponds to the individual components of ξ Tau and i to the observational data.

Regarding the observational data, we have radial-velocity measurements for the three components (1, 2, 3), $N_{\rm rv} = 843$, minima timings for the eclipsing binary (1+2), $N_{\rm ttv} = 35$, and interferometry (for components 3 and 4), $N_{\rm sky} = 49$, i.e. a subset of measurements from NPOI and WDS, for which it was possible to convert fringe visibilities (averaged over one night) to distance–angle values.[1] The individual uncertainties of the observations used in this section were modified as follows: $\sigma_{\rm rv} \geq 2\,{\rm km\,s^{-1}}$ due to calibration uncertainties, $\sigma_{\rm ttv} \geq 0.001\,{\rm d} = 1.5\,{\rm min}$, because the quasiperiodic oscillations visible on MOST light curves shift minima timings in a random fashion, and $\sigma_{\rm sky} = 3\,{\rm mas}$ (as in Tokovinin et al. 2015) or 5 mas if not reported in WDS.

We assumed the (nominal) distance $d = 64.1\,{\rm pc}$ for ξ Tau. The radii for eclipses detection were $R_1 = 1.752\,R_\odot$, $R_2 = 1.542\,R_\odot$, in accord with photometric inversion. Note that a to-be-expected correlation between R_1, R_2, eclipse depth, eclipse duration and third light contribution is removed to some extent thanks to spectroscopic observations (cf. Table **??**).

The synthetic minimum distance Δ' between components 1 and 2 in the sky plane was computed analytically as the distance of the piece-wise straight line $(x_{\rm h2}, y_{\rm h2})$ from the origin in 1-centric coordinates, as given by the numerical integration. The condition for an eclipse is then simply $\Delta' \leq R_1 + R_2$ and the corresponding time t'_{1+2} is linearly interpolated from neighbouring points. The eclipse duration is then given by a simple geometry, $\epsilon'_{1+2} = 2\sqrt{(R_1+R_2)^2 - \Delta'^2}/\bar{v}$, where $\bar{v}$ denotes the average velocity between the points. We thus straightforwardly account for disappearing eclipses and their durations, but we do not model (possible) eclipse depth variations at this stage.

[1] To be crystal clear, in this N-body model we do *not* fit the observed spectra by synthetic ones, individual light curve points, or interferometric fringes. We use higher-level observational data instead which were reduced and derived in previous sections.

Table 2. Minima timings t_{1+2} and eclipse durations ϵ_{1+2} determined from MOST light curves — corrected for quasiperiodic oscillations by means of Eq. (13) — and corresponding uncertainties $\sigma_{\rm ttv}$, $\sigma_{\rm ecl}$.

t_{1+2} JD	$\sigma_{\rm ttv}$ day	ϵ_{1+2} day	$\sigma_{\rm ecl}$ day
2456224.724205	0.0010	0.2656	0.0069
2456228.301662	0.0012	0.2611	0.0035
2456231.868584	0.0010	0.2678	0.0069
2456235.445218	0.0010	0.2573	0.0035

In order to remove minor systematics in minima timings and eclipse durations, we attempted to suppress quasiperiodic oscillations visible in MOST light curves by subtracting a function of the following form:

$$f(t) = C_0 + C_1(t-T_1) - [A_0 + A_1(t-T_1)]\sin\left[\frac{2\pi(t-T_1)}{P_0 + P_1(t-T_1)}\right]; \quad (13)$$

its coefficients $(C_0, C_1, T_1, A_0, A_1, P_0, P_1)$ were always determined by a *local* fit in the surroundings of the given minimum. The resulting data are reported in Table 2.

The relative luminosities for photocentre computations were set $L_1 = 0.1362$, $L_2 = 0.1067$, and $L_3 = 0.7571$ — again, in accord with photometric observations.

There are also mass constraints arising from from the spectroscopic classification of ξ Tau components (AV, AV, BV, FV). We can easily enforce reasonable limits for the component masses with the following artificial term:

$$\chi^2_{\rm mass} = \sum_{j=1}^{4}\left[\left(m_j - \frac{m_{j\rm min} + m_{j\rm max}}{2}\right)\cdot\frac{2}{m_{j\rm max} - m_{j\rm min}}\right]^{100}, \quad (14)$$

where we used m_1 and $m_2 \in (0.9, 3.0)\,M_\odot$, $m_3 \in (3.5, 3.9)\,M_\odot$, $m_4 \in (0.9, 2.0)\,M_\odot$ as the limits.

Optional weights $w_{\rm sky} = 10$, $w_{\rm ecl} = 10$, $w_{\rm ttv} = 1$, $w_{\rm rv} = 1$ of the χ^2 terms in Eq. (2) can be also used to prevent systematic deviations of the model from astrometric measurements of the 4th body or eclipse durations of the 1+2 pair which are (alas) not as numerous as other data sets.

The integrator and its *internal* time step was controlled by the parameter $\epsilon_{\rm BS} = 10^{-8}$ (unitless) which ensures a sufficient accuracy. The time span was 1,000 d forward and 11,000 d backward, the output time step $\Delta t = 0.5\,{\rm d}$ for initial runs. We verified that this sampling is sufficient even for the trajectory with the largest curvature and all necessary interpolations to the times of observations. For final optimisations we decreased the value further down to $\Delta t = 0.1\,{\rm d}$ to suppress interpolation errors.

We use a standard simplex algorithm (Press et al. 1997) to search for (local) minima of χ^2. We have 23 of potentially free parameters — masses m_j, coordinates $x_{{\rm h}j}, y_{{\rm h}j}, z_{{\rm h}j}$, velocities $v_{x{\rm h}j}, v_{y{\rm h}j}, v_{z{\rm h}j}$ in 1-centric frame — or, alternatively, masses m_j and three sets of orbital elements $a_j, e_j, I_j, \Omega_j, \omega_j, M_j$ in Jacobian coordinates, and the systemic velocity γ. The convergence tolerance for χ^2 was set $\epsilon_{\rm tol} = 10^{-6}$, the maximum number of iterations 10,000 or as low as 300 for extended surveys of the parameter space. We verified that this low number is sufficient to quickly find local minima or to exclude their existence.

The initial epoch $T_0 = 2456224.724705$ corresponds to the first very precise minimum on MOST light curve. We can thus (almost) fix $x_{\mathrm{h2}} \doteq y_{\mathrm{h2}} \doteq 0$. At the same time, it is possible to (approximately) fix positions $x_{\mathrm{p3}}, y_{\mathrm{p3}}$ and $x_{\mathrm{p4}}, y_{\mathrm{p4}}$, derived by interferometry for an epoch close to T_0.