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An improved model of the seasonal Yarkovsky force for regolith–covered asteroid fragments

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Abstract. We consider an exact solution of the seasonal Yarkovsky effect, the mean–motion frequency mode of the recoil force due to reradiated sunlight, on a spherical asteroid fragment. The body is assumed to have a low–conductivity ("regolith–like") surface layer covering a more conductive core. The penetration depth of the seasonal thermal wave in the low–conductivity surface material is typically larger than its geometrical thickness, a fact that has not been taken into account consistently by the previous simplified solutions. Our approach removes this drawback and the results indicate (i) a significantly increased seasonal mobility of the dekameter–sized (and larger) fragments with an insulating surface layer and (ii) a significantly decreased seasonal mobility of the meter–sized fragments with this structure. These results may affect the accuracy of simulations of the meteorite and NEA delivery to the Earth.

Key words: Celestial mechanics – Minor planets, asteroids – Meteoroids

1. Introduction

Thermal phenomena on asteroid fragments have attracted considerable attention over the last few years since their dynamical aspects, known as the Yarkovsky effects, were found to be important in solving some interesting problems in meteoritics and the delivery of NEAs to the Earth vicinity (e.g. Farinella et al. 1998, Hartmann et al. 1999, Vokrouhlický & Farinella 1998, Farinella & Vokrouhlický 1999). A detailed understanding of the real astronomical role of the Yarkovsky effects requires (i) a precise knowledge of the thermal and other parameters affecting their efficiency (e.g. surface thermal conductivity of the asteroid fragments, details of their collisional dynamics etc.) and (ii) a precise modelling of the thermal effects themselves. While the first problem basically needs observational inputs (such as thermal observations of the asteroid surfaces during fly–by space missions), the second requires theoretical efforts. In this note we extend the precision of the thermal modelling for a particular, but important, class of asteroid fragments.

Probably the most uncertain modelling of the thermal effects concerns bodies which have a two–layer structure: a high–conductive core covered with a thin layer of a low–conductive material ("regolith"). Though there is no direct evidence of a regolith cover for the small, meter–sized asteroid fragments we must consider this case as one of the possibilities (e.g. Farinella et al. 1998). Planetary moons such as Phobos typically do have regolith surface layers and future fly–by missions will investigate this possibility for small near–Earth asteroids. Theoretical considerations also support a possibility of electrostatically sticked dust particles on cosmic bodies (e.g. Lee 1996).

The physical reason why the previous thermal models might not be appropriate in the case of the regolith–covered bodies is different for the seasonal and the diurnal components of the Yarkovsky effect.

In the seasonal effect case, the previous models (e.g. Rubincam 1998; Farinella et al. 1998; Vokrouhlický 1999) are inappropriate due to the assumption of homogeneity of the thermal parameters throughout the body. The penetration depth of the seasonal thermal wave in the low–conductive surface regolith is of the order $l_s^1 \approx 15$ cm. This value is to be compared with the estimated geometrical thickness of the surface layer (possibly ≤ 1 cm). Because the corresponding penetration depth of the seasonal thermal wave in the high–conductive core of the body is much larger – e.g. $l_s^2 \approx 2$ m for a bare–rock body – the seasonal temperature changes affect a much larger volume of the body than just the surface layer of l_s^1 thickness (as expected from a simple thermal theory for a homogeneous body). Consequently, the previously obtained estimates of the seasonal Yarkovsky effect mobility for such two–layer

Send offprint requests to: D. Vokrouhlický asteroid fragments may be significantly incorrect and in need of revision.

Modelling the diurnal variant of the Yarkovsky effect for bodies with very low surface conductivity presents a different problem. Taking into account the lunar regolith thermal data (e.g. Langseth et al. 1973) we obtain an estimate of $38.3/\sqrt{\omega}\,\mu$m for the penetration depth of the diurnal thermal wave (ω is the rotation frequency of the body in rad/s). Assuming a linear dependence of the rotation period with the characteristic size of the body (e.g. Farinella et al. 1998), we may further express the estimated diurnal penetration depth as a function of the fragment's size: $34.2\sqrt{R}\ \mu$m (R in meters). For meter–sized bodies of interest in meteoritics this depth is submillimetric. Since on this scale the surface is likely to be dusty or particulate, that is a complex mixture of micrometric grains and voids of the same size, the previous models based on the homogeneity assumption may lead to wrong results. However, the solution of this problem is much more complicated than in the previously mentioned case of the seasonal effect.

In this note, we consider the seasonal effect only and we remove the homogeneity assumptions of the previous models by taking into account the two–layer structure of the body.

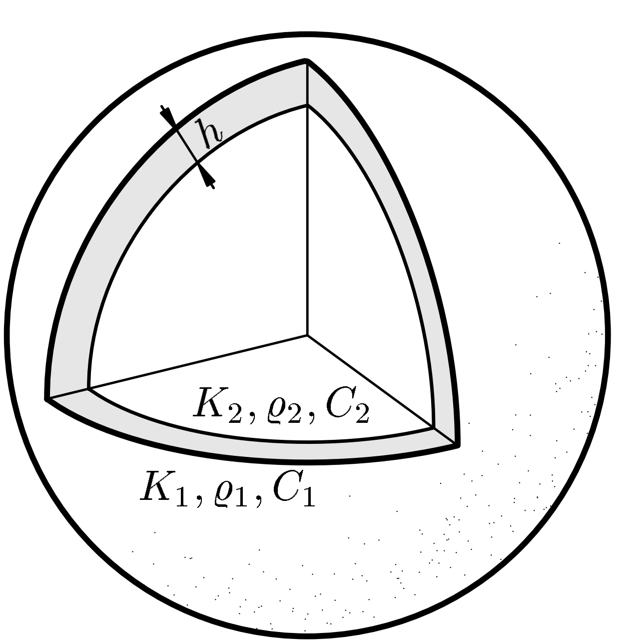

Fig. 1. Interior structure of the two–layer body discussed in the text.

2. Theory

Hereafter, we use the mathematical approach and, to a large extent, also the notation introduced in the series of papers by Vokrouhlický (1998a,b; 1999). The reader is referred details to these references for details.

We consider a linearized solution of the temperature variations in a spherical and rotating body that revolves around a radiating source (the Sun in our case) on a circular orbit. The body of radius R is assumed to have a thin and homogeneous surface layer of thickness h, characterized by thermal conductivity K_1, thermal capacity C_1 and density ρ_1. The corresponding parameters of the core are K_2, C_2 and ρ_2. As noted above, we assume that the thermal conductivities K_1 and K_2 are different; typically $\xi_2 \equiv K_1/K_2 \simeq 1/50$. For further use we also introduce the ratio of the thermal depths in the two parts of the body as $\xi_1 \equiv \sqrt{K_1\rho_2 C_2/K_2\rho_1 C_1} \simeq 1/15$. Figure 1 shows the geometrical structure of the bodies discussed in this note.

Since we shall use the linearized heat conduction theory, the temperature T throughout the body is assumed to be oscillate around a constant mean value $T_{\rm av}$: $T = T_{\rm av} + \Delta T$. The constant average part is defined by the formal conservation of the absorbed and reradiated energy without taking into account the heat conduction effects. The heat conduction problem then means to solve the temperature variation ΔT at any time t everywhere in the body. The volume elements in the body are to be parametrized by the spherical coordinates $(r;\theta,\phi)$.

A suitable scaling of the variables simplifies the mathematical formulation of the problem. The radial coordinate r is scaled by the thermal penetration depth $l_s^1 = \sqrt{K_1/\rho_1 C_1 n}$ of the surface layer: $r \to r' = r/l_s^1$ (scaled quantities will be denoted by prime; n denotes the mean motion of the fragment's revolution around the Sun). The temperature ΔT will be scaled by the subsolar surface temperature $T_\star$, defined from $\epsilon\sigma T_\star^4 = \alpha\mathcal{E}_\star$ (ϵ is the thermal emissivity and α the optical absorptivity of the surface, σ the Stefan–Boltzmann constant and $\mathcal{E}_\star$ is the solar radiation flux at the fragment's distance from the Sun). Finally, the time t will be replaced by a complex variable $\zeta = \exp(i\lambda)$, where $\lambda = n(t - t_0)$ is the mean longitude along the orbit. The time origin t_0 is chosen so that the solar colatitude θ_0 in the body's reference system (measured from the spin axis) is such that $\cos\theta_0 = \sin\gamma\sin\lambda$ (γ is the fragment's rotation axis obliquity).

The linearized heat conduction problem then satisfies the Fourier equation (e.g. Vokrouhlický 1998a; 1999)

$$
\begin{aligned}
i\zeta\frac{\partial}{\partial\zeta}\ \Delta T'(r';\theta,\phi;\zeta) &= \\
&= \frac{1}{r'^2}\left\{\frac{\partial}{\partial r'}\left(r'^2\frac{\partial}{\partial r'}\right) + \Lambda\left(\theta,\phi\right)\right\}\Delta T'(r';\theta,\phi;\zeta)\ ,
\end{aligned}
\tag{1}
$$

with the operator $\Lambda(\theta,\phi)$ given by

$$
\Lambda\left(\theta,\phi\right) = \frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\right]\ . \tag{2}
$$

The general solution of Eq. (1) is constrained by three boundary conditions. The energy flux conservation at the fragment's surface ($r' = R'$) reads

$$\sqrt{2}\Delta T'(R') + \Theta \left[\frac{\partial \Delta T'(R')}{\partial r'}\right] = \Delta\mathcal{E}' \ , \tag{3}$$

with the incident radiation flux $\mathcal{E}$ normalized by the "subsolar value" $\mathcal{E}_\star$ ($\mathcal{E}' = \mathcal{E}/\mathcal{E}_\star$) and $\Theta = \sqrt{K_1\rho_1 C_1}\sqrt{n}/\epsilon\sigma T_\star^3$ defined as the seasonal thermal parameter of the surface low–conductive layer. A dependence of the boundary condition (3) on the particular surface element (θ, ϕ) and time (ζ) is assumed. The right–hand side term in Eq. (3) ($\Delta\mathcal{E}' = \mathcal{E}' - \frac{1}{4}$) can be developed in the spherical harmonics series

$$\Delta\mathcal{E}' = \sum_{n\geq 1}\sum_{k=-n}^{n} \varepsilon_{nk}(\zeta)\, Y_{nk}(\theta,\phi) \ , \tag{4}$$

from which only the dipole zonal part will be relevant below. Vokrouhlický (1999) showed that the corresponding amplitude $\varepsilon_{10}(\zeta)$ reads

$$\varepsilon_{10}(\zeta) = \frac{1}{2i}\sqrt{\frac{\pi}{3}}\,\sin\gamma\,\left(\zeta - \zeta^{-1}\right) \ . \tag{5}$$

A new type of constraint is due to the boundary between the surface low–conductive layer and the core (i.e. at $r' = R' - h'$). Both the temperature and the heat flux are to be continuous on this surface, thus satisfying

$$\Delta T'(R'-h')_+ = \Delta T'(R'-h')_- \ , \tag{6}$$

$$\xi_2\left[\frac{\partial \Delta T'(R'-h')}{\partial r'}\right]_+ = \left[\frac{\partial \Delta T'(R'-h')}{\partial r'}\right]_- \ . \tag{7}$$

The indexes $\pm$ indicate that the corresponding quantity has to be computed as a limit $r' \to R' - h'$ from the surface layer (+) and the core (−) region.

Thanks to the spherical geometry the temperature inside the body may be developed as a spherical harmonics series

$$\Delta T'(r';\theta,\phi;\zeta) = \sum_{n\geq 1}\sum_{k=-n}^{n} t'_{nk}(r';\zeta)\, Y_{nk}(\theta,\phi) \ , \tag{8}$$

with the coefficients $t'_{nk}(r';\zeta)$ depending on the scaled radial distance r' from the center of the body and the time ζ. Only the dipole part $t'_{10}(r';\zeta)$ is relevant to compute of the seasonal Yarkovsky acceleration $\mathbf{a}$ (e.g. Vokrouhlický 1999), since

$$\mathbf{a}(\zeta) = -\frac{4}{3}\sqrt{\frac{2}{3\pi}}\,\alpha\Phi\, t'_{10}(R';\zeta)\,\mathbf{s} \ . \tag{9}$$

Here, $\mathbf{s}$ is the unit vector in the direction of the fragment's spin axis and $\Phi = \pi R^2 \mathcal{E}_\star/mc$ is the usual radiation pressure factor (m is the mass of the body and c the light velocity).

After a great deal of algebra we obtain

$$t'_{10}(R';\zeta) = \sqrt{\frac{\pi}{6}}\,\sin\gamma\,\frac{E_{R'}\sin(\lambda + \delta_{R'})}{1+\chi} \ , \tag{10}$$

where

$$E_{R'}\exp(i\delta_{R'}) = \frac{\beta_4 - \beta_3}{\beta_1\beta_4 - \beta_2\beta_3} \ . \tag{11}$$

The auxiliary complex β–factors are given by

$$\beta_1 = 1 + \frac{\chi}{1+\chi}\psi_1(z_1) \ , \tag{12}$$

$$\beta_2 = 1 - \frac{3\chi}{1+\chi} + \frac{\chi}{1+\chi}\psi_2(z_1) \ , \tag{13}$$

$$\beta_3 = \left[\xi_2 - 1 + \xi_2\psi_1(z_2) - \psi_1(z_3)\right]\,\psi_3(z_2) \ , \tag{14}$$

$$\beta_4 = \left[-2\xi_2 - 1 + \xi_2\psi_2(z_2) - \psi_1(z_3)\right]\,\psi_3(z_1) \ , \tag{15}$$

with

$$\psi_1(z) = \frac{z}{j_1(z)}\frac{dj_1(z)}{dz} - 1 \ , \tag{16}$$

$$\psi_2(z) = \frac{z}{n_1(z)}\frac{dn_1(z)}{dz} + 2 \ , \tag{17}$$

$$\psi_3(z) = \frac{j_1(z)}{n_1(z)} \ . \tag{18}$$

Here, $j_1(z)$ and $n_1(z)$ are the spherical Bessel functions of the first and the second kind. The complex arguments of the β–functions read: $z_1 = \sqrt{-i}\,R'$, $z_2 = \sqrt{-i}\,(R' - h')$ and $z_3 = \sqrt{-i}\,\xi_1(R' - h')$. Finally, the real parameter χ is defined by: $\chi = \Theta/\sqrt{2}R'$.

Using the corresponding Gauss equation we may then estimate the secular rate of the semimajor axis evolution as:

$$\frac{da}{dt} = \frac{4\alpha}{9}\,\frac{\Phi}{n}\,\frac{E_{R'}\sin\delta_{R'}}{1+\chi}\,\sin^2\gamma \ . \tag{19}$$

This result is formally identical to Eq. (26) in Vokrouhlický (1999) that has been derived for a homogeneous spherical fragment. However, in our case the amplitude $E_{R'}$ and the seasonal thermal phase lag $\delta_{R'}$ are given by the much more complex expressions (11) – (18) above (see Appendix A for a numerical method that enables the evaluation of the $E_{R'}\,\sin\delta_{R'}$ factor for $R' \gg 1$). Obviously, taking the limit of a homogeneous body (i.e. $K_1 = K_2$, $\rho_1 = \rho_2$ and $C_1 = C_2$) the semimajor axis secular drift (19) reduces to the simpler result (26) in Vokrouhlický (1999). The same applies for the limit $h \to R$, when the surface layer formally extends over the whole body, and the limit $h \to 0$ when the surface layer disappears.

3. Examples and conclusions

To assess the importance of the previously discussed effects related to the two–layer structure of asteroid fragments, we have performed two tests. First, we have esti-

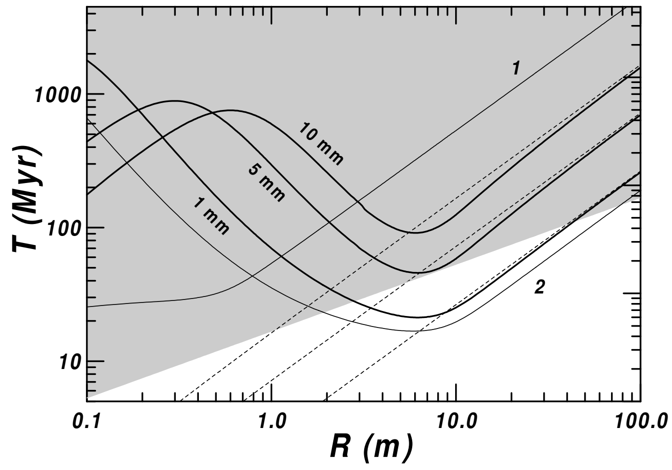

Fig. 2. The average time (in Myr) necessary to move an asteroid fragment by 0.05 AU in the semimajor axis vs. its radius R (in meters) by the seasonal variant of the Yarkovsky effect. The complete solution of this paper for three thicknesses of the low–conductive regolith layer (thick solid lines; h = 1 mm, h = 5 mm, h = 10 mm) is compared with the simplified solution for a homogeneous body (thin solid lines; $K_1 = 0.0015$ W/m/K corresponding to the surface regolith – curve 1, $K_2 = 1$ W/m/K corresponding to the highly conductive core – curve 2). The asymptotic behavior, based on the solution of the 1-D heat diffusion problem (see Appendix B) is shown by the dashed lines. Timescales longer than the estimated lifetime of the fragments are denoted by the shaded upper region.

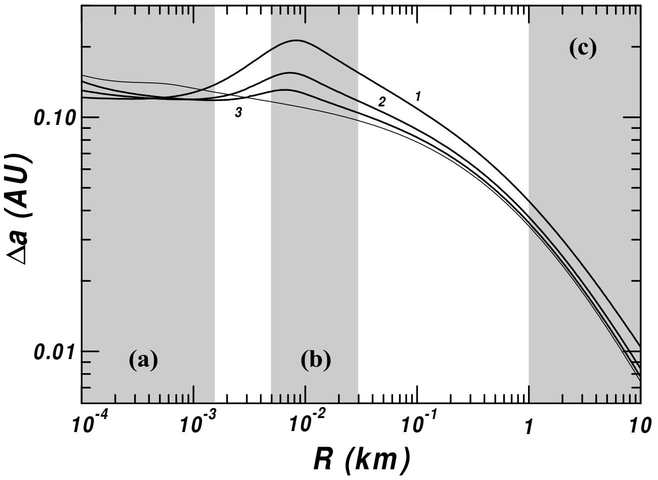

Fig. 3. Average semimajor axis drift (in AU) within the lifetime of an asteroid fragment vs. its radius R (in kilometers). Both diurnal and seasonal effects included. The assumed collisional lifetime of $16.8\sqrt{R}$ Myr is taken from Farinella & Vokrouhlický (1999). The complete solution of the seasonal effect (this paper) is shown for three thicknesses of the low–conductive regolith layer (thick solid lines; h = 1 mm curve 1, h = 5 mm curve 2, h = 10 mm curve 3) and is compared with the simplified solution for a homogeneous body whose thermal parameters correspond to the surface layer (thin solid line). The diurnal variant of the Yarkovsky effect is the same in all cases. The region (b) of the figure, where the most important differences are found, correspond to Tunguska–like Earth impactors with typical sizes of 5 – 30 meters.

mated the timescale required to move a main–belt body by 0.05 AU due to the seasonal Yarkovsky effect, assuming a body initially at 2.25 AU (for a similar discussion see Hartmann et al. 1999). The statistical method of Farinella & Vokrouhlický (1999) was used to include the effect of random changes in the spin axis orientation due to collisions with the background population of main–belt bodies. The results, shown in Figure 2, indicate that the previous, simplified approach significantly overestimates the seasonal effect mobility of meter–sized, regolith–covered fragments, while at the same time it significantly underestimates the seasonal effect mobility of the dekameter–sized (and larger) bodies with the same structure. Both results are easy to understand. We recall that the assumed geometrical thickness of the regolith–layer (up to 10 mm in Figure 2) is smaller than the penetration depth l_s^1 of the seasonal thermal wave corresponding to the thermal parameters of the regolith (about 15 cm). The surface temperature variations thus penetrate well into the highly conductive core of the body. Since the typical penetration depth l_s^2 of the seasonal thermal wave in the core is of the order of 1 – 3 meters, the efficient heat conduction through the entire bulk of the meter–sized body results in a significant decrease of its semimajor axis mobility. In other words, the effect of the highly conductive core helps the temperature to get closer to an average value and thus decreases the efficiency of the seasonal Yarkovsky effect. On the other hand, the high conductivity of the sub–surface layer helps in increasing the "average" thermal parameter (indicating roughly how much the thermal reemission lags behind the absorption of sunlight) in the case of dekameter–sized and larger bodies. This effect then leads to a faster seasonal Yarkovsky mobility in the semimajor axis, as indicated in Figure 2 (a shorter time is necessary to move the body by 0.05 AU).

Figure 3 shows the average semimajor axis mobility of asteroid fragments within their estimated collisional lifetime (compare with Figure 3 in Farinella & Vokrouhlický 1999). The more efficient mobility of dekameter–sized fragments covered with a low–conductive surface layer is again confirmed [see the bump on the thick curves in the strip (b)]. Bodies of this typical size have both a high diurnal effect mobility and, on top of this, a significantly increased seasonal effect mobility (as discussed above).

Let us now briefly comment on the possible astronomical importance of the enhanced ("large bodies") or inhibited ("small bodies") seasonal Yarkovsky effect mobility resulting from the improved model presented in this paper.

First, an increased mobility in the semimajor axis of dekameter–size bodies (Figure 3), provided they are endowed with a regolith cover, could significantly enhance the flux of these bodies to the main–belt resonances. Their preferential overabundance in the resonances should then result in a corresponding effect among the Earth–crossing objects. Rabinowitz's (1993, 1994) analysis indicates a possibility for this effect, and Hartmann et al. (1999) and Vokrouhlický & Farinella (1999a) suggest the Yarkovsky mobility of main–belt dekameter–sized fragments as the most plausible explanation. In a future quantitative simulation of these effects one should consider in detail all aspects that might affect the semimajor axis mobility of bodies with critical sizes (5 – 20 meters), including the implications of this paper.

Second, the quantitative simulations of the cosmic ray exposure ages of different types of meteorites by Vokrouhlický & Farinella (1999b) might depend on the mobility of the dekameter–sized fragments. The current results indicate a significant importance of secondary fragmentations for the delivery of the parent bodies of meteorites (an idea that has been pioneered by Wetherill 1985). Since the dekameter–sized objects are the most likely immediate precursors of the meter–sized meteoroids, it is very important to asses their abundance in the main belt. In particular, one must determine whether the dekameter–sized bodies succeed in escaping from the main belt before being disrupted or whether they typically suffer a catastrophic collision before reaching the resonances (yielding thus a secondary population of meter–sized meteoroids). Assume, for instance, a dekameter–sized, regolith–covered body released at a 0.15 AU distance from a resonance in the inner region of the asteroid belt – this is roughly the case of asteroid Vesta. Using the results in Figure 3 we may conclude that the simplified approach predicts a typical drift of about 0.1 AU within the estimated lifetime of a dekameter–sized body, which is not sufficient to reach the resonances for many fragments released from Vesta. However, when the more precise theory of this paper is used, the enhanced mobility may protect the dekameter–sized bodies from being disrupted on their way to the resonances, inhibiting thus to some extend the importance of the secondary fragmentations.

A precise analysis of the orbital dynamics of the near–Earth asteroid (1566) Icarus represents a special problem where the results of this paper might provide an element of the solution. According to Sitarski (1991), Icarus' orbit determination might reveal an unexplained secular decrease of the semimajor axis of about 7.5×10^{-4} AU/Myr. Given the fact that Icarus, which is about ≈ 900 m in diameter, could have a regolith cover, the conclusions from this paper are that one should be cautious in modelling the seasonal Yarkovsky effect on this body (apart from the fact that Icarus' orbit is very eccentric, which also poses a special problem for the evaluation of the seasonal variant of the Yarkovsky effect). Indeed, a first attempt to model the semimajor axis mobility of Icarus' orbit (Vokrouhlický, in preparation) indicates that the classical Yarkovsky effect models fail to predict the "observed" value as reported by Sitarski (1991). Obviously, there might be other possibilities to explain this discrepancy, that is (i) Sitarski's value is overestimated [see, e.g., the analysis of Yeomans (1994), who was not able to confirm Sitarski's finding], (ii) Icarus' size is smaller. As additional possibility we may mention that the seasonal effect influence has been underestimated and a fully numerical model that would simulate consistently the abrupt change in the thermal conductivity of the surface layer (similar to the linearized analysis in this paper) is required. Although this possibility seems plausible to us, we must await a detailed re–analysis of Icarus' orbit. In any case, Icarus' dynamics might offer a unique test of the Yarkovsky effect and thus deserves a careful attention.

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Appendix A: Evaluation of the term (11)

In this appendix we comment on the numerical evaluation of the complex function $E_{R'} \exp(i\delta_{R'})$ in Eq. (11), which is fundamental for estimating the semimajor axis drift (19). Curiously, this problem is rather tricky for reasons that will be explained below.

First, a more suitable representation of (11) reads

$$E_{R'}\exp(i\delta_{R'}) = \frac{1}{\beta_1 + \beta_3 \frac{\beta_1-\beta_2}{\beta_4-\beta_3}} \; . \qquad (20)$$

where the ratio of the β–functions in the second term of the denominator is of particular concern. Using simple algebraic identities for the spherical Bessel functions one proves that

$$\beta_1 - \beta_2 = \frac{\chi}{1+\chi}\,\frac{z_1}{\psi_3(z_1)}\,\frac{d\psi_3(z_1)}{dz} \; . \qquad (21)$$

More involved algebra then leads to

$$\beta_4 - \beta_3 \;=\; -z_1\,\frac{d\psi_3(z_1)}{dz}\left[\xi_2 + \frac{h}{R}\,\alpha_1\left(z_1; R, \xi_1, \xi_2\right) + \left(\frac{h}{R}\right)^2 \alpha_2\left(z_1; R, \xi_1, \xi_2\right) + \ldots\right] , \qquad (22)$$

where α_1, α_2 etc. are some complex functions. Notice that both the numerator $(\beta_1 - \beta_2)$ and the denominator $(\beta_4 - \beta_3)$ factors are proportional to the first derivative of the ψ_3 function defined in (18). One easily proves that this term decays to zero extremely fast when the norm $|z_1|$ of its argument increases. In quantitative terms we estimate

$$\frac{d\psi_3(z_1)}{dz} \propto \exp\left(-\sqrt{2}R'\right) \; . \qquad (23)$$

Since a typical value of the penetration depth of the seasonal thermal wave in the regolith layer is $l_s^1 \approx 15$ cm and the radius R of the bodies of interest may be of the order of 100 meters (say), we obtain $|d\psi_3(z_1)/dz| \approx 10^{-400}$! While both the numerator and the denominator factors are of this order, their ratio is a finite number. Obviously, without factorizing the $d\psi_3(z_1)/dz$ term, precise evaluation of the complex number $E_{R'}\exp(i\delta_{R'})$ in the left–hand side of (20) would not be possible. In the previous examples have computed the (h/R)–development in (22) up to the third power by using a Taylor expansion of the β_3 and β_4 functions.

Appendix B: Plane–parallel solution

As it was explained in the previous Appendix A, evaluating the seasonal semimajor axis drift (19) requires a special attention for large bodies ($R' \gg 1$). In this limit, the plane–parallel ("1–D") approximation of the heat diffusion problem might become appropriate. This approach has been developed by Rubincam (1995) and, in the notations of this paper, by Vokrouhlický (1998b). The corresponding non–linearized theory can be found in Vokrouhlický & Farinella (1998).

We have generalized the analysis in Vokrouhlický (1998b) to account for (i) the seasonal variant of the Yarkovsky effect, and (ii) the composite, two–layer bodies studied in this paper. The final formula for the semimajor axis drift due to the seasonal variant of the Yarkovsky effect is identical with (19), provided the $E_{R'}\exp(i\delta_{R'})$ function reads

$$\frac{E_{R'}\exp(i\delta_{R'})}{1+\chi} = -\frac{1}{1 - \Theta\sqrt{-\frac{i}{2}\frac{1+\xi_{12}\exp\left(2\sqrt{-i}h'\right)}{1-\xi_{12}\exp\left(2\sqrt{-i}h'\right)}}} \; , \qquad (24)$$

keeping the same notation as above and

$$\xi_{12} = \frac{\xi_1 - \xi_2}{\xi_1 + \xi_2} \qquad (25)$$

[obviously, the dependence on R' disappeared from Eq. (24)]. Interestingly, formula (24) may be evaluated without any numerical problems and can check the quality of our approximation developed in the previous Appendix A. Results based on (24) are shown in Figure 2 by the dashed curves. When $R' \gg 1$, we note a fairly good match with the complete solution (solid line), while at small sizes $R' \leq 1/\xi_1$ the plane–parallel approximation obviously fails to predict correct results.

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