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Yarkovsky–driven leakage of Koronis family members
I. The case of 2953 Vysheslavia

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ABSTRACT

The orbit of the asteroid 2953 Vysheslavia is presently locked in a tiny chaotic zone very close to the 5 : 2 mean motion jovian resonance. Its dynamical lifetime is estimated to be of the order of only about 10 Myr. Since Vysheslavia is a member of the Koronis family, such a short dynamical lifetime opens a variety of interesting questions concerning its origin and evolution. Milani and Farinella (1995, *Icarus* **115**, 209) considered a number of plausible scenarios and suggested that most probably Vysheslavia is an outcome of a recent secondary fragmentation event in the family. Here we propose that Vysheslavia might have been placed on its peculiar orbit by a slow inward–drift of the semimajor axis due to the Yarkovsky effect. Numerical simulations confirm that such evolutionary process can take 100 – 500 Myr, a period comparable, but still shorter than the probable age of the family (on the order of a Gyr), depending on the thermal properties of Vysheslavia's surface, orientation of its spin axis and its size. We have also integrated orbits of the asteroids 7340 (1991 UA_2) and 1993 FR_{58}, located very close, but outside of the chaotic zone that triggers the Vysheslavia's orbit instability, and we show that orbits of these asteroids may also slowly evolve toward the chaotic zone. Such an erosion of the asteroid families, caused by a slow leakage to the nearby powerful resonances, could be fairly common in the main asteroid belt.

Subject headings: asteroids, Yarkovsky effect, chaotic motion, dynamical lifetime

1. Introduction

In a broader sense, the aim of this paper is to contribute to the understanding of evolution and fate of the asteroid families. Starting from the birth by a large collisional event in the main asteroid belt, a typical family suffers different kinds of "erosion" processes. The most obvious causes of such an

[1]In the course of preparing this paper, Paolo Farinella passed away on March 25, 2000. He was at the peak of his scientific productivity, and was a "driving force" in shaping the scenario presented in this manuscript. This concept was only one of the many innovative ideas of Paolo, whose scientific style combined creativity with deep physical insight and mathematical rigor. Through publishing this paper, we honour Paolo's memory.

effacement effect are the collisional grinding (e.g., Marzari *et al.* 1995, 1999) and the chaotic diffusion (see Milani and Farinella 1994 who studied the case of Veritas family; also Milani et al. 1997, Nesvorný and Morbidelli 1998, Knežević 1999), but there are also others. An interesting mechanism, recently proposed by Farinella and Vokrouhlický (1999), is the semimajor axis diffusion by the Yarkovsky effect, spreading out otherwise closely packed family members. This non–gravitational phenomenon, due to the recoil force of the thermal radiation of an asteroid (e.g. Rubincam 1995, 1998; Farinella *et al.* 1998), alters secularly the semimajor axes of small asteroids up to a size of few kilometers. Farinella and Vokrouhlický (1999) observed that the small members of the Astrid family exhibit a larger scatter of the semimajor axes than the big members. Since the Yarkovsky effect is size dependent (larger mobility for small bodies), they concluded that the Astrid data support the idea of the Yarkovsky spreading of the families. In this paper, we shall consider another case where the Yarkovsky mobility of small family members might result in observable consequences.

The key argument for the present analysis derives from a particularly "lucky" position of the asteroid 2953 Vysheslavia, a small member of the Koronis family (we estimate its size to about 15 kilometers from the absolute magnitude and a typical geometric albedo of 0.2 observed in the family). Milani and Farinella (1995) noticed that Vysheslavia is located in a tiny chaotic zone (about 10^{-3} AU wide) very close to the border of the strong 5 : 2 mean motion resonance with Jupiter, and that this peculiar location makes the orbit of Vysheslavia rather unstable with respect to falling into the 5 : 2 resonance. Given the very short lifetime of bodies residing inside this resonance (Gladman *et al.* 1997), and the probability of a Vysheslavia–like orbit transfer to this resonance (Milani and Farinella 1995), the expected dynamical lifetime of this body is estimated to be of the order of about 10 Myr. This is an extremely short timescale, since Vysheslavia is a member of the Koronis family, whose age is, according to the available evidence, presumably much longer, of the order of $1 - 2$ Gyr (Chapman *et al.* 1996, Greenberg *et al.* 1996, Giblin *et al.* 1998). Milani and Farinella (1995), and later Knežević *et al.* (1997), pointed out this apparent contradiction and suggested several possible solutions, being however unable to decisively discriminate between them. In fact, all of the plausible possibilities seem to be related to the "family–aging processes" mentioned above, while it appears that we can safely rule out the possibility that Vysheslavia is an interloper genetically unrelated to the Koronis family. Both the spectroscopic analysis indicating that Vysheslavia is an ordinary S–type asteroid, like other members of the family (Bus 1999), and the interloper analysis of the Koronis family (Migliorini *et al.* 1995) predicting very few interlopers of the size of Vysheslavia, make this possibility very unlikely.

According to Milani and Farinella (1995) the present location and lifetime of Vysheslavia can be best explained by assuming that it is an outcome of a secondary fragmentation of a large (possibly originally the largest) member of the Koronis family that occurred several tens of Myr ago. Such a catastrophic event might have placed Vysheslavia onto its present orbit, but, given the estimated sizes of Vysheslavia and of its hypothetical parent body (50–100 km), there is only about ≤ 5 % probability that such a collision occurred in the last 100 Myr. Moreover, if this was the case, Vysheslavia's spectrum should be less altered by the space weathering processes and should slightly differ from the typical spectrum of the older Koronis members; related to the assumed recent collisional origin of Vysheslavia is also its possible state of non–principal axis rotation that might produce a complex photometric curve (Harris 1994). Vysheslavia has been observed during two nights only very recently by L. Šarounová (2000; private communication). Apart from small amplitude of the lightcurve (only 0.2 magnitude), which may mean that Vysheslavia is a moderately elongated object, no conclusions can be drawn yet. On the other hand, Vysheslavia might have been placed onto its present orbit by a nondisruptive collision against the asteroid itself, or by a close encounter with Ceres, but Milani and Farinella again found these scenarios to be highly unlikely (encounters with Ceres, for example, typically result in a total shift of about 10^{-3} AU in the semimajor axis over the age of the Solar System, so that there is less than 1 % probability that a close enough encounter occurred in the last 100 Myr).

In this paper we investigate the possible role of the third family–aging process mentioned above – the semimajor axes spreading by the Yarkovsky effect – in relation to the Vysheslavia case (note that some preliminary results of this analysis have already been presented in Vokrouhlický *et al.* 1999). In particular, we would like to understand whether Vysheslavia, being originally located further away from the 5 : 2 resonance, could have been placed onto its present orbit by a Yarkovsky drift in the semimajor axis. A typical timescale for such a process would be of main concern here. Moreover, to better describe the mechanism of delivery and capture by the tiny chaotic zone in which Vysheslavia is locked, we present also the integrations of the orbits of two other Koronis family asteroids, 7340 (1991 UA_2) and 1993 FR_{58}, that are located in the vicinity of this region (see Knežević *et al.* 1997). We show that for these two bodies, once they enter into the chaotic zone, the subsequent orbital evolution cannot be predicted in a deterministic way; their orbits may remain locked in the region for a long time (of the order of tens of millions of years), or they may quickly cross it, drifting towards the 5 : 2 resonance with Jupiter. We estimate the probability of both outcomes from a number of numerical simulations (each of them starting from slightly different initial conditions), and we propose that the evolution we observed for the two small asteroids might have occurred in the past to Vysheslavia itself.

2. Numerical simulations

We have implemented both variants of the Yarkovsky effect (diurnal and seasonal) in a number of different numerical integrators. For a precise formulation of the corresponding accelerations, and the way in which we have incorporated them into the codes, see Brož (1999) or the paper by Brož *et al.* (2000). Long–term evolution studies usually profit from (or even require) a fast integrator based on a mapping method. For that reason, we have implemented the Yarkovsky effect in the `swift_rmvs3` integrator (e.g. Levison and Duncan 1994; in what follows, we shall use designation `swift_rmvs3` or `swift` even for our modified version of this integrator). It should be noted that due to weak dissipation through the Yarkovsky effect the symplecticity of the integrator is violated (as it is, in fact, anyway because of handling the close encounters with planets). We may mention papers by Malhotra (1994), Codeiro *et al.* (1997) or Mikkola (1998), where the authors considered a similar possibility to include nonconservative phenomena in the originally symplectic integrators (our method corresponds to that of Codeiro *et al.* (1997) and Mikkola (1998)). We have carefully tested that the integrator reproduces analytical results (when available) of the semimajor axis secular evolution. As a check of the results with the modified `swift_rmvs3` integrator, we also performed some simulations using other integrators, in particular (i) the `Bulirsh-Stoer` integrator (Press *et al.* 1994), and (ii) the `ORBIT9` integrator kindly provided by A. Milani. The latter has been used by Milani and Farinella (1995) and Knežević *et al.* (1997) in the previous integrations of Vysheslavia.

Since Vysheslavia is located at the outer edge of the 5 : 2 jovian mean motion resonance, we typically include in our integrations the outer planets only (except from Pluto). Their initial state vectors and masses were taken from the JPL DE403 ephemerides, while for the integrated asteroids (2953 Vysheslavia, 7340 (1991 UA_2) and 1993 FR_{58}) the initial osculating elements were taken from Bowell's catalogue (Bowell *et al.* 1994). All the initial data correspond to the epoch of JD2451400.5. Whenever only four outer planets were used in our simulations, the initial data have been corrected for the indirect effect of the inner planets (the so called "barycentric correction"; see Milani and Knežević 1992, 1994). We typically used a timestep of 36 days, but in several cases where a precision of the integration was of particular concern we used also a shorter step.

To check the robustness of conclusions based on the simulations that include the giant planets only, we have performed two separate series of tests. First, by propagating the orbits back in time, we verified that the state vectors of the integrated asteroids match the initial conditions used by Milani and Farinella (1995) and Knežević *et al.* (1997) if referred to the epochs given by these authors. Next, we have also included perturbations of the inner Solar System planets in our integrations. As expected, their influence is small at Vysheslavia's distance, and the main conclusions derived from the integrations with giant planets only do not change. The single major difference of the integrations with and without the inner planets pertains to the shorter lifetime of bodies that eventually terminated in the 5 : 2 resonance in the former case. This observation concurs with the conclusion of Gladman *et al.* (1997) that the 5 : 2 residing bodies are removed principally by close encounters with the inner planets.

Realizing that the "anomalous" fluctuation of the Vysheslavia's semimajor axis due to a close encounter with sufficiently massive perturber could have placed it onto its peculiar orbit (see Sec. 1), we have performed another series of tests that included gravitational effects of asteroids Ceres and Pallas. The results of these tests seem to confirm the negative conclusion by Milani and Farinella (1995), notably that the scenario assuming Vysheslavia's orbit to be strongly affected by Ceres is unlikely.

In order to check the proposed scenario we performed simulations with and without Yarkovsky effect; the latter essentially coincide with the previous work of Milani and Farinella (1995) and Knežević *et al.* (1997). Let us emphasize that by doing so we actually do not compare two equally plausible approaches. Rather we thus compare a more complete, and thus more realistic model (with the Yarkovsky effect included), with a less complete and less realistic model (purely gravitational model, without the Yarkovsky effect).

There are two major unknown parameters related to the Yarkovsky effect modelling: (i) the thermal parameters (the thermal conductivity) of the Vysheslavia surface, and (ii) its rotation state (orientation of the spin axis and rotation period). Being aware of this caveat we tried to span the realistic values of these parameters. In particular, we performed simulations with different spin axis orientations, using two different values of the surface conductivity (one at the low end of the plausible range, and the other at the high end). We draw our conclusions only after considering all the results obtained with the full range of values for these quantities. In any case, we believe that taking into account the Yarkovsky effect, even with the unconstrained parameters (and spanning their physically reasonable values), is more realistic than not taking the Yarkovsky effect at all.

In order to properly model the Yarkovsky effect, we use the formulation developed by Vokrouhlický *et al.* (2000) and Brož *et al.* (2000). Since both variants – the diurnal and the seasonal (see, e.g., Farinella *et al.* 1998; Vokrouhlický 1999) – of the Yarkovsky effect have been included in our code, we can span the entire range of possible values of the surface thermal conductivity. We remind that the Yarkovsky acceleration in the low conductivity regime is dominated by the diurnal variant, while the high conductivity regime is dominated by the seasonal variant (see also Table 4). The ISO observations reported by Müller and Lagerros (1999) suggest that a comparably higher weight should be given to the case of low surface conductivity (since nearly all asteroids observed by ISO do indicate a very low value for this parameter). Such a conclusion is supported by the further observations (such as that of Phobos) indicating that surfaces of small bodies in the solar system might have a regolith (insulating) cover and/or exhibit a high degree of porosity (induced probably by a numerous microimpacts). We can thus assume $K = 0.0015$ W/m/K to be the most probable value of the Vysheslavia surface conductivity (this is the lunar regolith value,

Table 1: Orbital parameters of the close clones (CC). The increments refer to the nominal orbital elements of the asteroid.

Code	Δa ($\times 10^{-7}$ AU)	Δe ($\times 10^{-7}$)
CC_0^0	0	0
$CC_{\pm 1}^0$	± 1	0
$CC_{\pm 2}^0$	± 2	0
$CC_{\pm 3}^0$	± 3	0
$CC_0^{\pm 1}$	0	± 5
$CC_0^{\pm 2}$	0	± 10
$CC_1^{\pm 1}$	1	± 5
$CC_{-1}^{\pm 1}$	-1	± 5

Rubincam 1995, that also roughly corresponds to the measurements reported by Müller and Lagerros 1999). Nevertheless, we performed a series of test integrations with considerably higher surface conductivity $K = 1$ W/m/K, that corresponds to a solid rock surface (Yomogida and Matsui 1983). In passing we mention that the linearized formulation of the Yarkovsky effect may not be appropriate for very high eccentricity orbits (see, e.g., Vokrouhlický and Farinella 1998, 1999; Spitale and Greenberg 2000). This drawback is, however, not very important for our conclusions that are based on a transport of asteroids to the 5:2 resonance on low–eccentricity orbits (typically $e \leq 0.2$ in our simulations).

The strength of the Yarkovsky effect depends sensitively on the orientation of the spin axis and the rotation period of the body (Rubincam 1995, 1998; Farinella *et al.* 1998). Unfortunately, we presently do not have any explicit information about these parameters neither for Vysheslavia nor for 7340 (1991 UA_2) and 1993 FR_{58}. We have thus performed a series of simulations with different assumptions about the orientation of their spin axes. As for the rotation period, we assumed it to be[2] 5 hours as a typical value observed in the family (Binzel *et al.* 1989). Surface geometric albedo was taken to be equal to 0.2 and the thermal emissivity equal to 0.9. From these values we have derived Vysheslavia's radius of approximately 7.5 km. Similarly, we got 2.8 km for the 7340 (1991 UA_2) radius and 3 km for the radius of 1993 FR_{58}. Given these values, we may estimate a collisional lifetime of these asteroids to be at least 1 Gyr (even 1.4 Gyr for Vysheslavia). Our longest integrations with the Yarkovsky effect accounted for span 0.5 Gyr, that is, significantly less than the collisional lifetime. Yet, we shall demonstrate that the Yarkovsky effect has the capability to shift quite significantly the semimajor axes of these objects.

It has been mentioned above that Vysheslavia's orbit is presently locked in a chaotic zone. In principle, any individual orbit in such a zone quickly loses its deterministic meaning, with a typical timescale for this to occur characterized by the Lyapounov time. Previous numerical experiments (e.g., Milani *et al.* 1997) demonstrated that a typical Lyapounov time of the strongly chaotic orbits in the main asteroid belt is of

[2] At the late stage of preparation of this paper we have learnt about photometric observations made by L. Šarounová (2000; personal communication). She observed Vysheslavia on Jan 15/16/17, 2000 at the St. Veran Observatory in France. A preliminary reduction of the light curve indicates rotation period of 6.29 ± 0.05 hr, roughly consistent with our assumption. The amplitude of the lightcurve is only about 0.2 magnitude which may indicate only moderately elongated shape of Vysheslavia. Unfortunately, this single observation cannot reveal any information about the Vysheslavia spin axis orientation.

Table 2: Orbital parameters of the fictitious neighbors (FN). The increments refer to the nominal orbital elements of the asteroid.

Code	Δa ($\times 10^{-4}$ AU)	Δe ($\times 10^{-4}$)
FN_0^0	0	0
$FN_{\pm 1}^0$	± 5	0
$FN_{\pm 2}^0$	± 10	0
$FN_{\pm 3}^0$	± 15	0
$FN_0^{\pm 1}$	0	± 5
$FN_0^{\pm 2}$	0	± 10
$FN_1^{\pm 1}$	5	± 5
$FN_{-1}^{\pm 1}$	-5	± 5

the order of $10^3 - 10^4$ years. The Lyapounov time for the Vysheslavia orbit is ≈ 27 kyr (see, e.g., `AstDyS` web page maintained in Pisa by A. Milani), indicating that the degree of chaoticity in the Vysheslavia's vicinity is somewhat smaller than in the most chaotic zones of the main belt (as in the 5 : 2 resonance). However, the indicated Lyapounov time of Vysheslavia is still very short with respect to our integrations that typically cover a time span 10^4 up to 2.5×10^4 longer. Most of the simulations presented in this paper have thus a statistical meaning only. To explore the stochastic characteristics of the chaotic zone in which Vysheslavia is located, we used the well-known technique of the "fictitious neighbors" (e.g., Milani and Farinella 1995). However, we introduced these fictitious neighbors not only for Vysheslavia itself, but also for the objects on stable orbits, like the two above mentioned asteroids 7340 (1991 UA_2) and 1993 FR_{58}; by integrating their orbits with the Yarkovsky effect accounted for, we expected to be able to observe and follow the evolutionary paths of currently stable bodies drifting towards the chaotic zone.

Two levels of "zooming" were used in these simulations. First, to trace statistically the fate of a given orbit, we introduced very close neighbors, which we refer to as the "close clones". These are supposed to be so tightly clustered around a given orbit that an avegaring over their evolutionary states would express the corresponding statistical uncertainty of the orbit itself. Notice that the close clones cover about a 3σ uncertainty area (in both semimajor axis and eccentricity) resulting from the current orbit determination (see, e.g., `AstDyS` page at http://newton.dm.unipi.it/asteroid/). For the sake of minimizing the computational expenses, however, we defined the close clones by slightly modifying the initial semimajor axis and eccentricity only. Table 1 gives the variations of the close clones' initial semimajor axes and eccentricities with respect to the nominal values for the real asteroid itself. Second, we also introduced somewhat less close neighbors – hereafter referred to as the "fictitious neighbors" – whose orbits are supposed to map the Vysheslavia zone on a broader scale. In particular, their semimajor axes spanned the entire chaotic zone (about 0.001 AU wide). Increments of their initial semimajor axes and eccentricities with respect to these of the real asteroids are given in the Table 2. In the Vysheslavia's case, the most distant FN_{+3}^0 body is located just outside the chaotic zone, thus representing both (i) a suitable delimiter of the chaotic zone, and (ii) a suitable probe to understand the role of the Yarkovsky perturbations in contrast to the gravitational effects only.

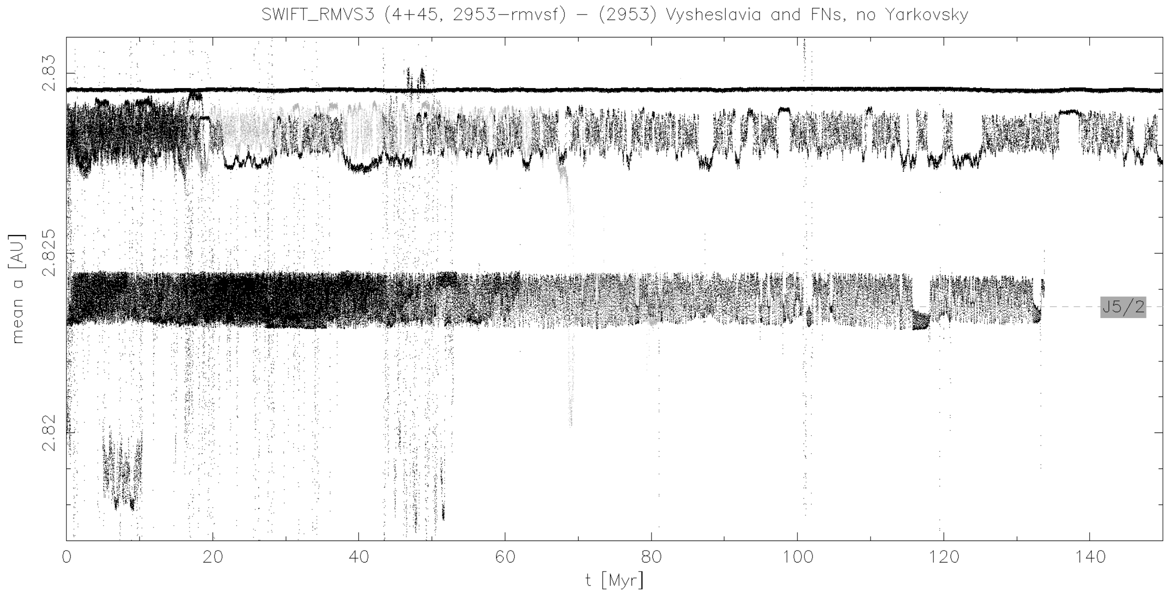

Fig. 1.— Mean semimajor axis (in AU) vs. time (in Myr) for Vysheslavia (grey curve) and its 14 fictitious neighbors (FN). Perturbations due to outer planets are included, while the Yarkovsky effect is not considered in this simulation. Notice the chaotic zone characterized by about 0.001 AU random fluctuations of the semimajor axis. Vysheslavia and FNs originally placed in this zone fall in the 5 : 2 resonance (centered at about 2.823 AU – see the dashed line) typically within 10 Myr.

2.1. Integrations without the Yarkovsky effect

Before considering the influence of the Yarkovsky effects on the orbits of the Koronis family asteroids in question, we performed integrations where these effects have not been included. The purpose of the integrations was twofold: (i) to check our software and integration procedure by comparison with previous results (notably Milani and Farinella 1995, and Knežević *et al.* 1997), and (ii) to contrast these results with those including the Yarkovsky effect. As far as the first item is concerned, we primarily wanted to verify reliability of the results obtained with the `swift` integrator. This is a very important issue, since we need to perform a very long integrations (several hundred Myr) with the Yarkovsky effects included. As mentioned above, the `swift` integrator profits in terms of the speed from its mapping structure, but `swift` is also known to be less precise on a short–term than the slower standard integrators (especially because its lack of reproducing correctly the short–periodic part of the perturbation; e.g. Michel and Valsecchi 1996). Since, on the other hand, Vysheslavia is located in a chaotic zone, we anyway cannot infer results of a firmly deterministic value, but we still need to be sure that the results obtained with the `swift` integrator are at least statistically correct (i.e., about the same fraction of orbits reaching the same terminal state within comparable timespan). The comparison of results obtained with the standard, highly precise integrators over shorter timespans (typically tens of Myr), and those obtained with the `swift` integrator thus serve to assess credibility of the long–term results obtained by means of the `swift` integrator itself.

Figure 1 shows the mean semimajor axes of Vysheslavia and its 14 fictitious neighbors (FN; see Table 2) as a function of time obtained by means of the `swift_rmvs3` code. We notice the chaotic zone between 2.8275 AU and 2.829 AU in semimajor axis that has been found by Milani and Farinella (1995).

Vysheslavia and its FNs originally residing in this zone (that is, all except FN^0_{+3}) fall into the 5 : 2 resonance after a median time of 12.1 Myr, a value which matches well the results of Milani and Farinella (1995). Table 3 lists the median time of fall to the 5 : 2 resonance in all Vysheslavia integrations. Despite of the short median dynamical lifetime in the chaotic zone, some of the neighbors may stay captured there for a remarkably long time ($\geq$ 200 Myr for the FN^{+1}_{+1}; see Figure 1). The FN^0_{+3} body, originally put just above the chaotic zone, seems to have a very stable orbit without any noticeable variations of the mean semimajor axis. Notice also a "symmetric" chaotic zone on the opposite side of the 5 : 2 resonance, bracketed by 2.818 AU and 2.820 AU in the semimajor axis, where a very similar chaotic behaviour occurs as in the zone where the present Vysheslavia's orbit resides. One of our objects (FN^{+1}_0) has been temporarily captured in that zone before finally falling into the 5 : 2 resonance.

We have repeated the above simulation using the `Bulirsh-Stoer` integrator with the same initial conditions and dynamical model, and covering the time span of 50 Myr. We found qualitatively similar results with the median time to fall into the resonance of 6.6 Myr. The difference of the median times to fall in the 5 : 2 resonance found in the two integrations is well within the uncertainty due to the small statistical sample of objects. Out of the integrated 15 FNs only about $4 - 5$ define effectively the median time and this is too few. Some of the FNs lie already too close to the bottom part of the chaotic zone, so that they fall in the 5 : 2 resonance in a very short time (both in the simulation by `swift_rmvs3` and `Bulirsh-Stoer` integrator). Others lie near the upper border of the chaotic zone and they persist in our simulation for a long time. What thus matters is the same order of magnitude of the two results. Due to their chaotic nature, the individual orbits of Vysheslavia and its FNs were different from those in the previous simulation (except, again, for the FN^0_{+3} body that has a stable orbit). Also, some of FNs were temporarily captured into the symmetric chaotic zone centered at about 2.819 AU.

Much better agreement of the two integrators (in terms of the median to fall in the 5 : 2 resonance) is found for the close clones (see the Table 3). Here all 15 particles contribute to definition of the median and this is already good statistical sample. In particular, the median time to fall into the 5 : 2 resonance was found to be 14.1 Myr with the `swift` integrator and 12.9 Myr with the `Bulirsh-Stoer` integrator. Comparison with the corresponding medians for FNs indicates, that the chaotic mixing of orbits in the whole zone is very efficient, so that the statistical properties of the orbits very close to that of Vysheslavia match the corresponding statistics taken over orbits departing from a much broader region.

Finally, we may mention that the mean lifetime of particles that dropped *in* the 5 : 2 resonance was on the order of 10 Myr in all the integrations. We shall comment more on this issue in the Sec. 2.3.1 where the inclusion of the perturbations by inner planets will be discussed.

We have also integrated, using both `swift` and `Bulirsh-Stoer` integrators, the orbits of 7340 (1991 UA_2) and 1993 FR_{58} and of their FNs. Results of these integrations agreed fairly well. The asteroid 7340 (1991 UA_2) lies close to another tiny chaotic zone between 2.8329 AU and 2.8335 AU. Some of its FNs that were placed in this chaotic zone remained captured in it for the entire time span of the integration (200 Myr). The asteroid 1993 FR_{58} itself is placed just at the upper boundary of this chaotic zone and might get temporarily captured in it without being ejected to the 5 : 2 resonance. Other FNs that lie outside this zone have remarkably stable orbits. Notice that Knežević *et al.* (1997) considering a fictitious neighbor still closer to the 5 : 2 resonance detected the chaotic behavior (placing a body in the same location we found the same behavior), but, by taking a bigger displacement of the semimajor axis for the 1993 FR_{58} FNs, they overlooked (skipped) the tiny chaotic zone indicated above.

Generally, however, we can conclude that the results we obtained using both the classical `Bulirsh-Stoer`

Table 3: Vysheslavia's median lifetime (in Myr) against the fall to the 5 : 2 resonance computed from the integration of the fictitious neighbors (FNs) or close clones (CCs) in different simulations. Three cases considered: (i) the gravitational interaction with planets only without the Yarkovsky perturbation (NY), (ii) the Yarkovsky perturbation included and obliquity of all particles set to 135^0 [Y($\gamma = 135^0$)], and (iii) the Yarkovsky perturbation included and obliquity of the particles isotropic [Y(γ iso.)]. Two types of the integrators (column Int) are also compared: (i) `swift_rmvs3` (and its corresponding extension to include the Yarkovsky perturbation), and (ii) `Bulirsh-Stoer` (BS) integrator. The case of CCs is more relevant for the particular orbit of Vysheslavia. A good correspondence to the FNs results indicates a large degree of chaotic mixing at the Vysheslavia's location inside the chaotic zone.

	Int	NY	Y($\gamma = 135^0$)	Y(γ iso.)
CCs	`swift`	14.1	10.6	13.9
	`BS`	12.9	10.7	10.3
FNs	`swift`	12.1	2.7	9.6
	`BS`	6.6	7.7	14.9

integrator and the `swift` match very well the previous findings of Milani and Farinella (1995) and Knežević *et al.* (1997).

2.2. Integrations with the Yarkovsky effect

As mentioned above, we do not have a precise knowledge of a couple of necessary parameters that tune the Yarkovsky effect. As far as the surface thermal conductivity is concerned we therefore always performed two, otherwise identical integrations, with a low and a high value of this parameter. The chosen values approximately bracket the interval of physically admissible values, with the lower conductivity appearing more likely to represent the reality, according to the ISO measurements. We, however, did not notice any really important difference between the simulations with the low and the high value of the surface thermal conductivity of the bodies, apart from a longer timescale in the latter case. For sake of illustration we list in Table 4 the estimated maximum secular drift of the semimajor axis (we assume zero obliquity for the diurnal variant and ninety degrees obliquity for the seasonal variant of the Yarkovsky effect). We recall that the diurnal variant scales roughly as $\propto \cos\gamma$ and the seasonal variant roughly as $\propto \sin^2\gamma$ (where γ is the obliquity).

Next, we miss information about the orientation of the spin axes of Vysheslavia, 7340 (1991 UA_2) and 1993 FR_{58}. Hence, we again performed two series of simulations: (i) assuming the initial obliquity of the spin axes of all bodies to be 135°, and (ii) assuming an isotropical distribution of the spin axes in space. In the former case, we integrated the orbits of the asteroids and of all their fictitious neighbors (from Table 2), while in the latter case we integrated orbits of the asteroids only, but with 15 different (spatially isotropic) orientations of the spin axis for each of them.

Table 4: Estimated maximum values of the semimajor axis secular drift (da/dt) due to the diurnal and seasonal variants of the Yarkovsky effect for the three asteroids studied in this paper (units are 10^{-6} AU/Myr; the seasonal drift is always negative, while the diurnal may be both positive or negative). Three values of the surface conductivity K considered; out of these the first – $K = 0.0015$ W/m/K – is the most likely for km-sized objects. Other parameters as in the text.

Asteroid	$K = 0.0015$ W/m/K		$K = 0.1$ W/m/K		$K = 1$ W/m/K	
	Diurnal	Seasonal	Diurnal	Seasonal	Diurnal	Seasonal
Vysheslavia	17.1	0.2	7.2	2.1	2.3	5.8
1991 UA_2	45.8	0.6	19.2	5.5	6.0	15.5
1993 FR_{58}	42.8	0.6	17.9	5.2	5.6	14.4

2.2.1. Fixed orientation of the spin axes

Let us first note that the 135° initial obliquity (assumed throughout this section) leads to an average decrease of the semimajor axis of the integrated orbits even in the diurnal Yarkovsky effect dominated cases (when the surface conductivity is low). We have chosen this value of the obliquity to force drifting of the nearby objects towards the 5 : 2 resonance, in order to perform a statistical study of the leakage of the Koronis family members and the family-aging process. We, however, note that the 135° value of the initial obliquity does not maximize neither the diurnal nor the seasonal Yarkovsky mobility.

Figure 2 shows the evolution of the mean semimajor axis as a function of time for Vysheslavia and its 14 FNs. Both planetary and Yarkovsky effects are included and a low value of the surface conductivity assumed ($K = 0.0015$ W/m/K). The general characteristics of the results have not changed significantly (though a somewhat shorter the median time to fall into the 5 : 2 resonance was observed: in the substantial case of CCs 10.6 Myr compared to 14.1 Myr without the Yarkovsky effect, see the Table 3), except for the behavior of the FN^0_{+3} particle. In contrast to the stable behavior shown in Figure 1 when only the planetary effects were included, the body now reaches the chaotic zone within 25 Myr due to the Yarkovsky mobility. We also notice that, in contrast with Figure 1, none of the integrated objects stayed in the chaotic zone longer than 35 Myr.

Figures 3 and 4 show the semimajor axis mobility of the asteroids 7340 (1991 UA_2) and 1993 FR_{58} together with their 28 FNs. The same thermal parameters and initial orientation of the spin axis are assumed as in the Vysheslavia's case. We remind that these bodies are further away from the 5 : 2 resonance, and that none of them have fallen there in the course of the 200 Myr integration without the Yarkovsky effect; some fictitious neighbors of 1991 UA_2, e.g. FN^0_{+2}, and the asteroid 1993 FR_{58} itself exhibited a temporary chaotic behavior, but the range of their semimajor axis fluctuations was smaller than in the Vysheslavia's case. These results changed completely when the Yarkovsky effect had been added into our simulation: all the bodies reached the 5 : 2 resonance within 55 to 180 Myr, for the most part by a smooth drift in the semimajor axis due to the Yarkovsky effect. Some of the FNs initially placed above the Vysheslavia's chaotic zone were trapped in it for a while before the final fall, while others just quickly passed through on their way to the 5 : 2 resonance. The typical semimajor axis interval crossed by these bodies (of the size $\simeq 7.5$ km) was 2.9×10^{-3} AU per 100 Myr, which corresponds well to the previous

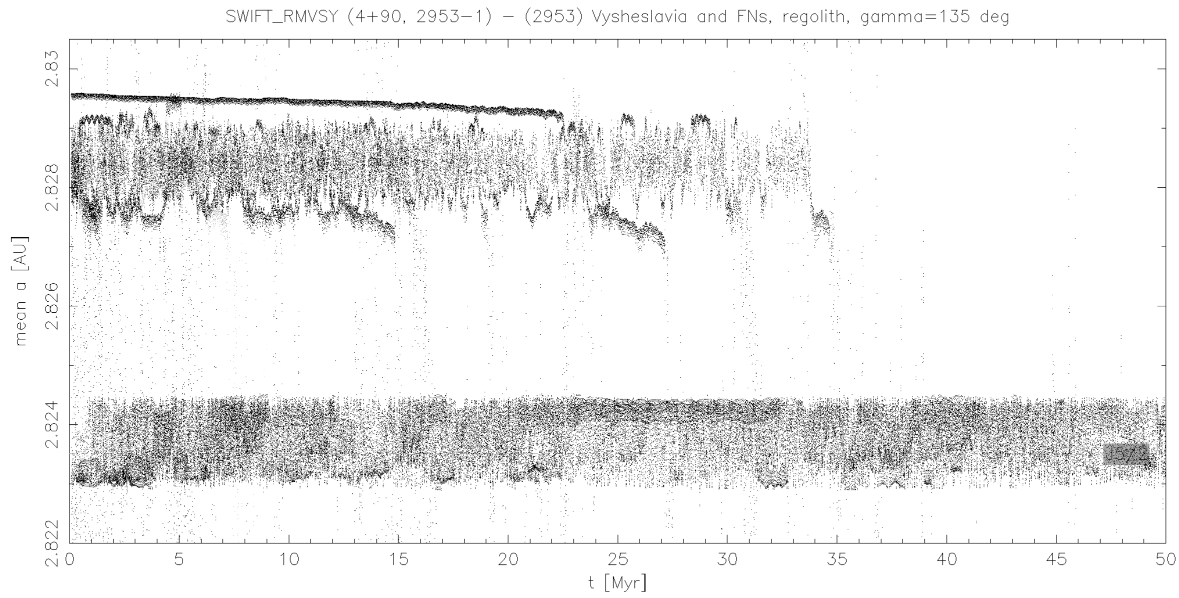

Fig. 2.— Mean semimajor axis (in AU) vs. time (in Myr) for Vysheslavia (grey curve) and its 14 fictitious neighbors (FN). Planetary perturbations (four outer planets only) together with the Yarkovsky effect included. Low–conductive surface ($K = 0.0015$ W/m/K) assumed and the initial obliquity set to 135°.

analytical estimates (Farinella *et al.* 1998; Farinella and Vokrouhlický 1999; Bottke *et al.* 2000) and also data given in Table 4. In a Gyr, comparable to the lifetime of the Koronis family and still shorter than the estimated collisional lifetime, these objects may thus shift for as much as 0.029 AU, which turns to be larger by at least one order of magnitude than the random fluctuations due to the encounters with Ceres and Pallas (see below).

Qualitatively the same results have been obtained when repeating the previous simulations with the high value of the surface conductivity ($K = 1$ W/m/K). A slower drift by a factor of about 2.2 was the only quantitative difference between these integrations, which again matches well the previous analytical estimates.

Note that all the results of this and of all other simulations (many more than it was possible to report herein) can be found at: `http://sirrah.troja.mff.cuni.cz/~mira/mp/`.

2.2.2. Isotropic distribution of the spin axes

The Yarkovsky mobility is dominated by the diurnal variant of the effect when the low surface conductivity is assumed. As it is well–known from the analytical theory (Vokrouhlický 1998, 1999), in such a case the semimajor axis secular decrease or increase depends on the orientation of the body's spin axis with respect to the orbital plane. If the obliquity value is between $\simeq 90°$ and 180° the orbit decays, while in the complementary half-space orientation of the spin axis the semimajor axis increases. In order to assess the transport of Koronis family members towards the 5 : 2 resonance, in the previous Sec. 2.2.1 we have assumed 135° for obliquity (note that the very fact of the Vysheslavia's present location in the chaotic zone actually indicates that the obliquity of its spin axis is probably $> 90°$). However, it might be also interesting

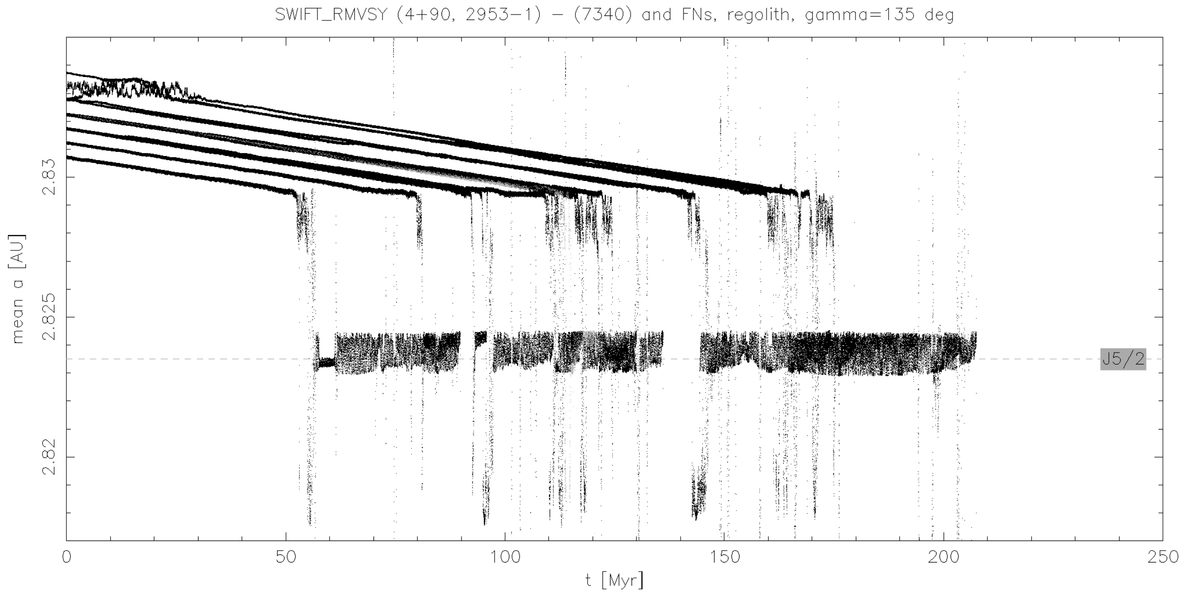

Fig. 3.— The same as on Figure 2 but for 7340 (1991 UA_2; grey curve) and its 14 fictitious neighbors.

to investigate the role of the Yarkovsky effect in the overall "mixing" of the small Koronis family members at the edge of this resonance. To that purpose we performed another set of simulations, where we assumed isotropic distribution of the spin axes of the integrated asteroids (no fictitious neighbors assumed in this case). Since we do not have any information about the real orientation of the spin axes of Vysheslavia, 7340 (1991 UA_2) and 1993 FR_{58}, each of the integrated "particles" might be a viable representation of the real body.

Figure 5 shows the mean semimajor axis versus time for the 15 Vysheslavia–like objects with different orientation of the spin axis. Low value of the surface conductivity ($K = 0.0015$ W/m/K) has been assumed and perturbations by the outer planets were included. No significant difference between these results and those in Figures 1 and 2 was observed. None of the "particles" escaped from the chaotic zone toward the stable region of the Koronis family at higher values of the semimajor axis, in spite of the fact that half of the objects have $(da/dt)_{\rm Yark} > 0$. This is due to the fact that the Yarkovsky drift for bodies of this size is rather small, and that therefore it has been overwhelmed and masked by the corresponding chaotic wandering. As in the previous simulations, majority of the bodies ultimately suffered an abrupt fall into the 5 : 2 resonance, with a median time of the fall of 13.9 Myr (CCs; see the Table 3). This value is closer to the corresponding median time in the simulation where the Yarkovsky effect was neglected. This can be easily understood, since half of the objects are drifting outward from the 5 : 2 resonance preventing thus a rapid fall.

Figures 6 and 7 show the same as above but for 7340 (1991 UA_2) and 1993 FR_{58}. Contrary to the case of Vysheslavia, half of the orbits, as expected, now shifts towards the larger values of the semimajor axes. Temporary interaction with weak resonances might be noticed, but it is the overall Yarkovsky mobility of the two asteroids in the course of time that turns out to be the most interesting result of this test. Starting from the same location in the Koronis family, the two fastest "particles" shifting in the opposite directions, build up in a Gyr of time their mutual distance to as much as 0.07 AU, which is a fair fraction of the total

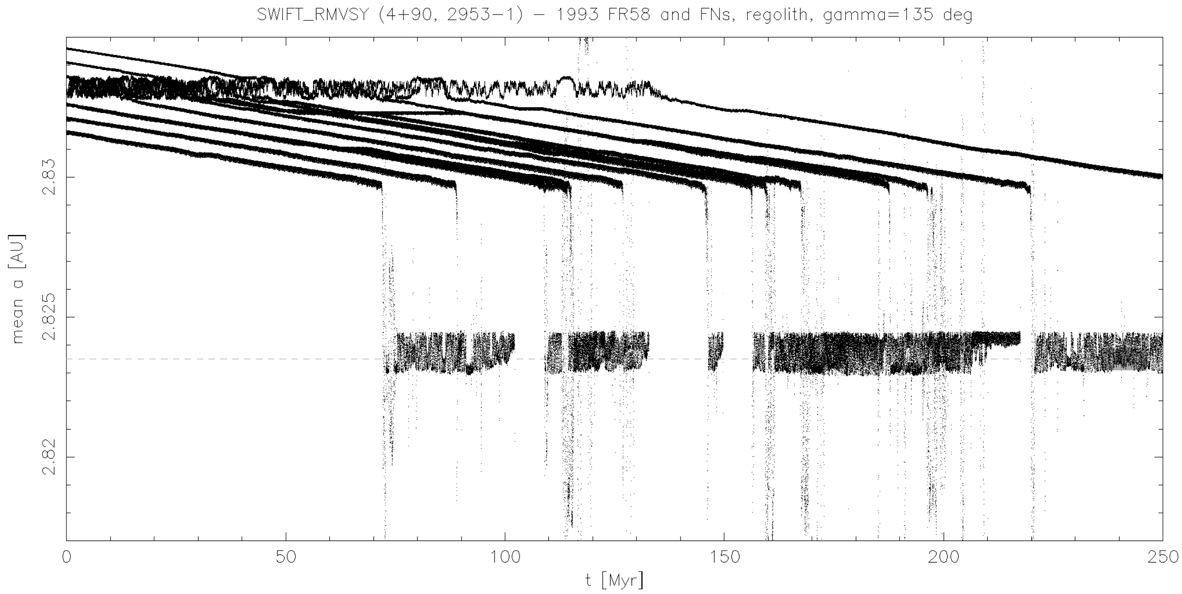

Fig. 4.— The same as on Figure 2 but for 1993 FR_{58} (grey curve) and its 14 fictitious neighbors. Despite of a possibly long–lasting capture in the chaotic zone near 2.833 AU, all particles eventually fall in the 5 : 2 resonance.

semimajor axis range occupied by this otherwise rather elongated family. Let us emphasize one more time that we are dealing with objects of about 7.5 km in size.

As in the previous Sec. 2.2.1 we have repeated all the above simulations for the case of the high surface conductivity ($K = 1$ W/m/K). The seasonal variant of the Yarkovsky effect then dominates the semimajor axis mobility and most of the orbits decay towards smaller values of a. The results are thus very similar to those we have got with the low value of the surface thermal conductivity and the initial obliquity in the 90° to 180° range, only the timescale is again "stretched" by a factor of about 2.2 due to a slower seasonal mobility. An example is shown on the Figure 8 where 15 objects of initially the same orbital parameters as 7340 (1991 UA_2), but isotropic spin axes, are shown. Only when the obliquity is set to zero, and hence the seasonal variant of the Yarkovsky effect vanishes, there occurs a very slow upward drift due to the diurnal variant (grey particle).

2.3. Additional tests

Hereafter we shall verify that our conclusions from the previous Sections are robust to several modifications of the (necessarily simplified) dynamical model we have been using above.

2.3.1. Terrestrial planets included

We first add in the simulation the perturbations due to the inner planets (Mercury through Mars). Figure 9 shows the mean semimajor axis of Vysheslavia and its 14 FNs as a function of time. One may

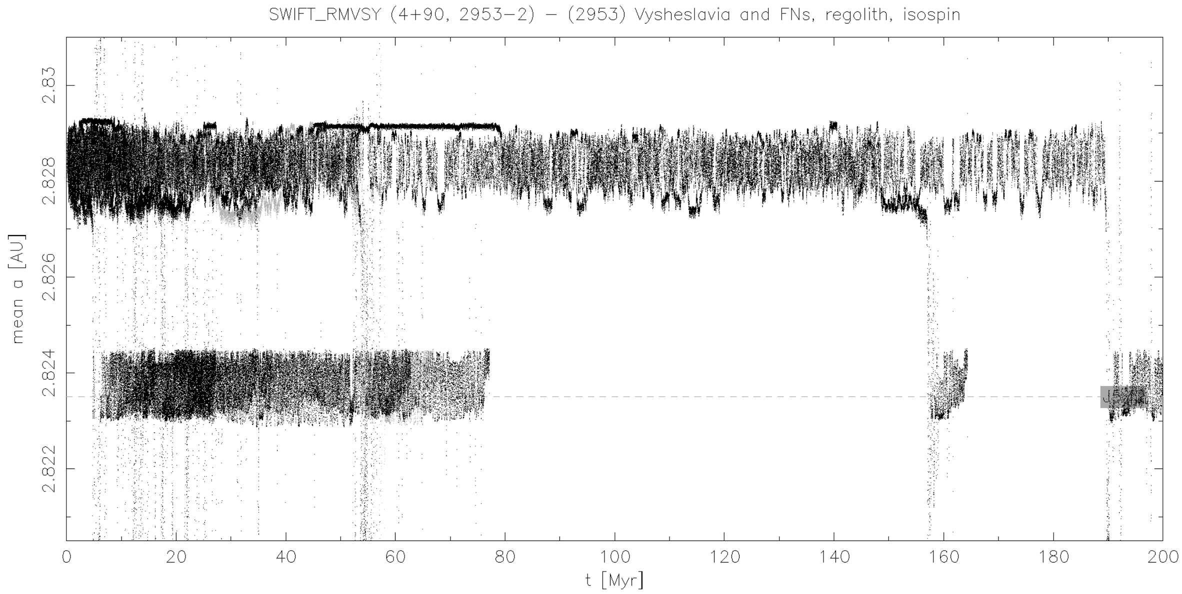

Fig. 5.— Mean semimajor axis (in AU) vs. time (in Myr) for 15 fictitious “Vysheslavia–like” asteroids. The initial conditions of each of the integrated bodies correspond to Vysheslavia. The same physical characteristics assumed (notably the low value $K = 0.0015$ W/m/K of the surface conductivity) except from orientation of the spin axis. The latter are distributed isotropically in space by taking equidistant steps in cosine of initial obliquity.

notice an obvious difference of these results and the results presented on Figure 1, notably the different residence time in the 5 : 2 resonance. While in the case where only four outer planets were considered (Figure 1) the bodies could stay in the 5 : 2 resonance typically for tens of Myr, in the integration where all the planets were considered (Figure 9) the typical residence time is about one Myr. We have verified that the principal mechanism that removes particles from the 5 : 2 resonance is a close encounter with one of the inner solar system planets (mainly Earth and Venus). These findings match well recent results of Gladman *et al.* (1997). On the other hand, and what is the most important in the context of our work, the general dynamical picture of Vysheslavia itself and of its clones seems to be insensitive to adding the inner solar system planets to the model. In particular, the Vysheslavia’s chaotic zone location is well preserved. This indicates that the chaotic zone, related to the resonance effects due to the outer planets, has been well represented by the four–body problem with appropriately applied barycentric correction. The median time to fall in the 5 : 2 resonance is 10.9 Myr, very close to the value obtained with the perturbing influence of the four outer planets only (Table 3). We may thus conclude that adding the inner solar system bodies practically does not alter our previous results.

2.3.2. The possible role of Ceres and Pallas

The main asteroid belt, where Vysheslavia is located, represents a vast reservoir of much less massive perturbers than the inner planets. However, the possibility of close encounters with these bodies may in part balance the deficiency in their mass as far as the total effect on the Vysheslavia orbit is concerned. Since already the most massive asteroids, Ceres and Pallas, might closely approach Vysheslavia, we decided

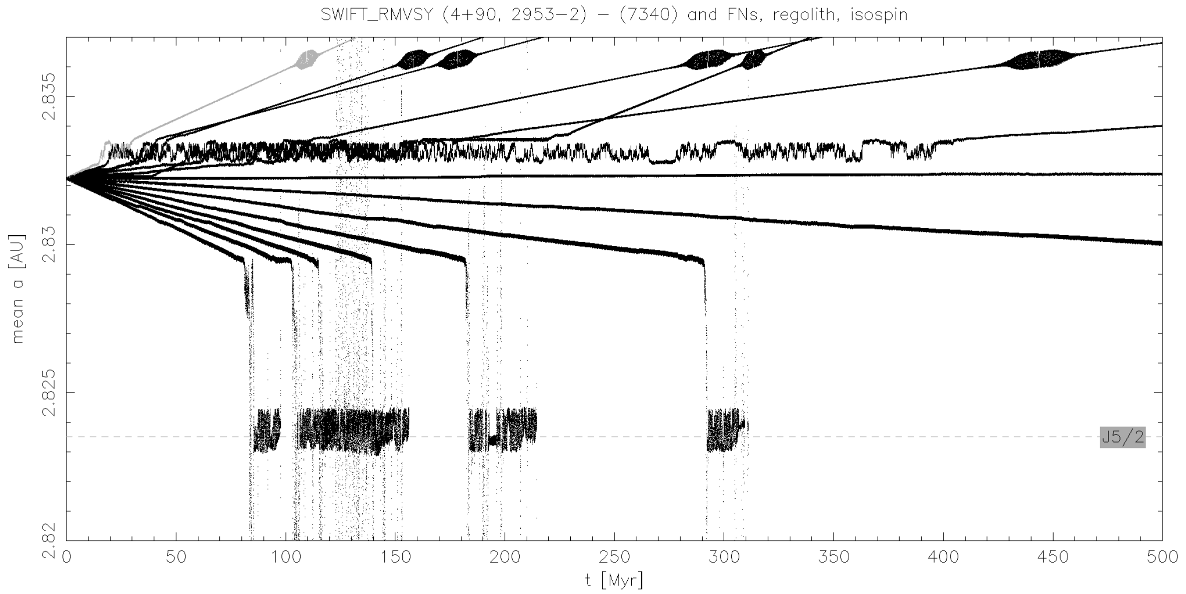

Fig. 6.— The same in Figure 5 but for the asteroid 7340 (1991 UA$_2$). Contrary to the Vysheslavia case, some particles may now escape from the chaotic region after a very long–lasting period of a capture.

to perform a test that consists of including these two asteroids together with all the planets (except Pluto) in our simulation. Masses and osculating elements of these asteroids were taken from the Bowell's catalogue (Bowell *et al.* 1994).

Before commenting on the results of numerical simulations, let us derive a simple estimate of the expected effect. Let us assume Vysheslavia on a circular orbit and a perturber of mass m' (expressed as a fraction of the solar mass) on a nearby circular orbit, so that the ratio of their semimajor axes $\alpha = a'/a$ is smaller but very close to unity. Then one might estimate a variation δa of the semimajor axis a of Vysheslavia related to the conjunction encounter with the perturber as $(\delta a/a) \simeq 2\,m'/(1-\alpha)/(1-\alpha^{3/2}) \simeq 2 \times 10^{-6}$ AU (the numerical value pertains to Ceres). Even smaller variations of the semimajor axis are expected due to the interaction with other members of the Koronis family, though the duration of the conjunction is quite different because of their smaller distance. Note, however, that this is just the lowest order–of–magnitude estimate of the effect. When approximating the orbits (of both the perturber and the perturbed asteroid) by precessing ellipses, we might get a better analytical estimate of the effects, but we shall in this case rather directly perform a complete numerical simulation. We shall thus see in the following that the gravitational influence of massive asteroids on the orbits of smaller objects (like Vysheslavia) is composed from: (i) random, very–small–scale fluctuations in semimajor axis that however can accumulate over a longer–time giving rise to a sort of chaotic diffusion (see Figure 11), and (ii) abrupt changes in the semimajor axis resulting from a close encounter (both Ceres and Pallas eccentricities are large enough to cross the Vysheslavia region). Milani and Farinella (1995) estimated that a close encounter with Ceres resulting in more than 0.001 AU shift of the Vysheslavia's semimajor axis occurring in the last 100 Myr is not very likely event. We shall try to investigate this conjecture by direct numerical integration.

Numerical integration with the massive asteroids that may approach very close to the orbits of integrated objects (e.g. the closest detected approaches to Vysheslavia or to its clones were at about 1.1×10^{-4} AU), is a delicate problem. The close approaches last very short (typically a day), so that

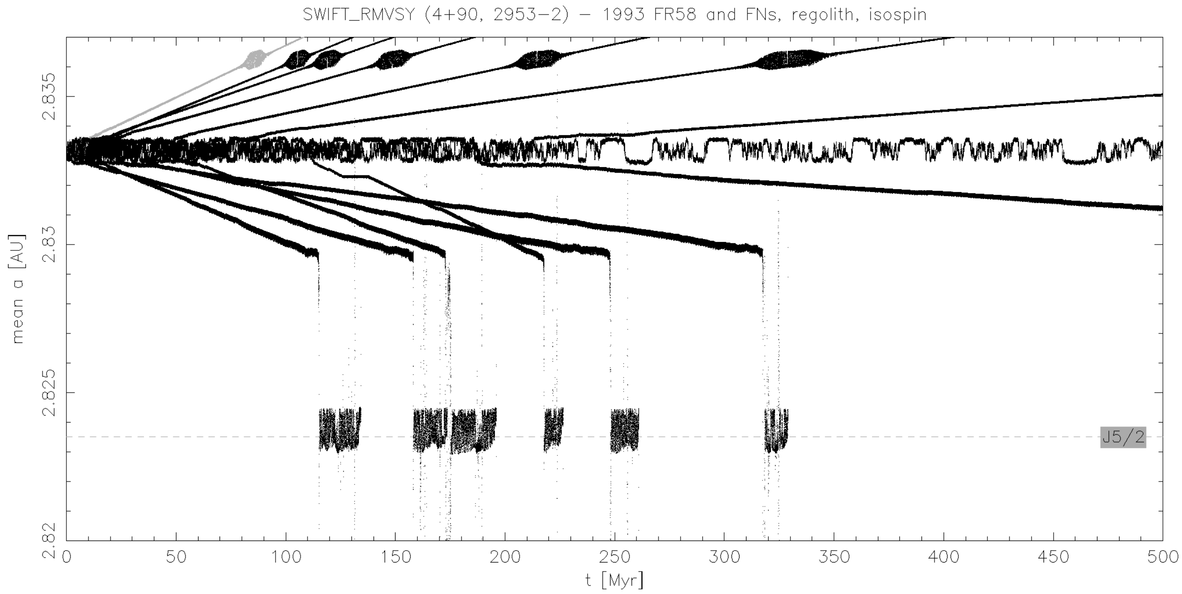

Fig. 7.— The same in Figure 5 but for the asteroid 1993 FR_{58}.

a "blind" application of an integrator like `swift` might result in misleading or incorrect conclusions (by definition the close approach to one on the perturbing bodies is a "short-periodic" effect). Either a too long integration timestep from the previous simulations is kept, and then the close approaches are not well represented, or if the Hill sphere of the gravitational influence of the massive asteroids is forced to be too large, the symplecticity of the `swift` gets largely violated (by the time we did most of the simulations we were not aware of availability of the symplectic integrators that permit a correct modelling of the close encounters; e.g. Chambers 1999). Being aware of this problem, we have integrated the orbit of Vysheslavia and its 14 FNs (and similarly for the asteroids 7340 (1991 UA_2) and 1993 FR_{58}) in the gravitational field of 8 planets (except Pluto), Ceres and Pallas by using the `Bulirsh-Stoer` integrator (Yarkovsky perturbation was not considered in this case). Obviously, a tight control of the numerical precision results in large expenses as far as the computation time is concerned. The integrations that will be presented in this section took much more CPU time than all the other integrations reported above.

Figure 10 shows the mean semimajor axes of all integrated asteroids (and their FNs) during the 20 Myr time interval. The results are basically similar to those reported in Sec. 2.3.1, where the perturbations of all 8 planets were considered. However, a closer look at the orbits that are not locked in a seemingly chaotic zone reveals semimajor–axis long–term fluctuations at the level of about $\leq 10^{-3}$ AU. As it has been discussed above, these are induced by the gravitational influence of the two massive asteroids considered in the simulation (Ceres and Pallas). We can also notice rapid jumps in the semimajor axes of some particles (e.g. FN^0_{-3} of 7340 at about 15.3 Myr) that are triggered by a very close encounters with Ceres or Pallas. Notice, however, that none these effects is large enough to place the Vysheslavia's neighbor FN^0_{+3}, originally above the chaotic zone, to the chaotic zone. The medium time to fall in the 5 : 2 resonance computed for Vysheslavia and its fictitious neighbors is 11.6 Myr, essentially the same as in the integration without the gravitational influence of Ceres and Pallas.

Acknowledging somewhat shorter timespan of the previous simulation we have integrated three FNs, notably FN^0_{+3} of Vysheslavia, FN^0_{-3} of 7340 (1991 UA_2) and FN^0_{+3} of 1993 FR_{58}, over a much longer

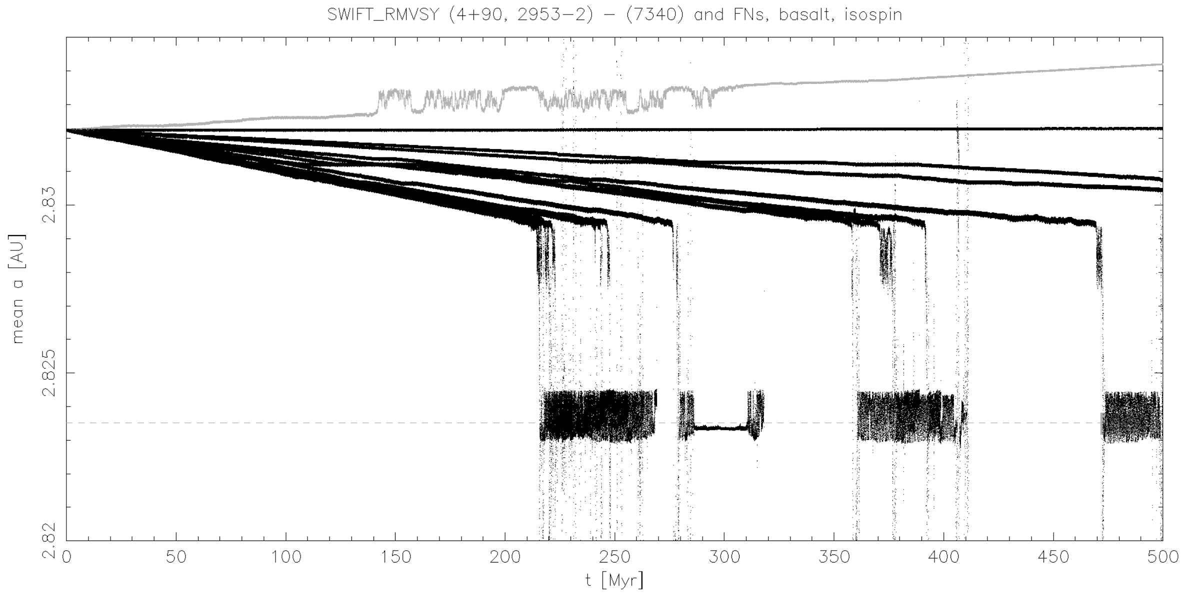

Fig. 8.— The same in Figure 6 but for bodies with higher value of the surface thermal conductivity ($K = 1$ W/m/K). Except for the grey particle (corresponding to zero obliquity), the dominating seasonal contribution from the seasonal variant of the Yarkovsky effect drags the orbits to smaller values of the semimajor axis. The strength of the drift is smaller than in the diurnal–dominated situation on the Figure 6. Notice a long–lasting capture of the grey particle in the tiny chaotic zone, where the asteroid 1993 FR_{58} is currently located.

timespan. Their orbits were again integrated by the `Bulirsh-Stoer` integrator with gravitational effects of 8 planets and the 2 massive asteroids. Resulting evolution of the mean semimajor axes, that now covers 100 Myr, is shown on Figure 11. Notice again the FN^0_{+3} Vysheslavia neighbor's semimajor axis fluctuates at the level of 10^{-3} AU, but that this is still not enough to get trapped by the chaotic zone. Strictly speaking we cannot exclude the possibility that Vysheslavia has been transferred to its present location by the gravitational influence of Ceres or Pallas, but the fact that we did not observe this to occur in our integration puts a limit on the corresponding probability. There are three interesting features to be noticed in Figure 11: (i) an abrupt change in the semimajor axis of the Vysheslavia's FN^0_{+3} at about 5.1 Myr, (ii) a similar change in the semimajor axis of FN^0_{-3} that belongs to 1993 FR_{58}, at about 97.5 Myr, and (iii) still identical effect in the semimajor axis of FN^0_{+3} that belongs to 1993 FR_{58}, at about 68.2 Myr. In all these cases we have detected a very close encounter of the integrated orbit with Ceres. The minimum separation of the two bodies was only 1.1×10^{-4} AU in the last case. Notice also, that due to chaoticity of the orbits, the close encounter of 7340's FN^0_{-3} at approximately 15.3 Myr (see Figure 10) is not reproduced in this integration. The noticeable ($\simeq 0.0001 - 0.001$ AU) changes of the semimajor axis require very close encounters with small cross–section, so that two different integrations preserve statistical characteristics of the encounters only. Note that we have used two different computers (with different fortran compilers) to perform simulations in the Figures 10 and 11.

As a by-product of our investigation, we mention an interesting finding about the orbit of Pallas. Figure 12 shows the mean semimajor axis of Ceres and Pallas as a function of time in the course of 20 Myr as obtained from the integration with all the planets (except Pluto) included. In contrast to the orbit of

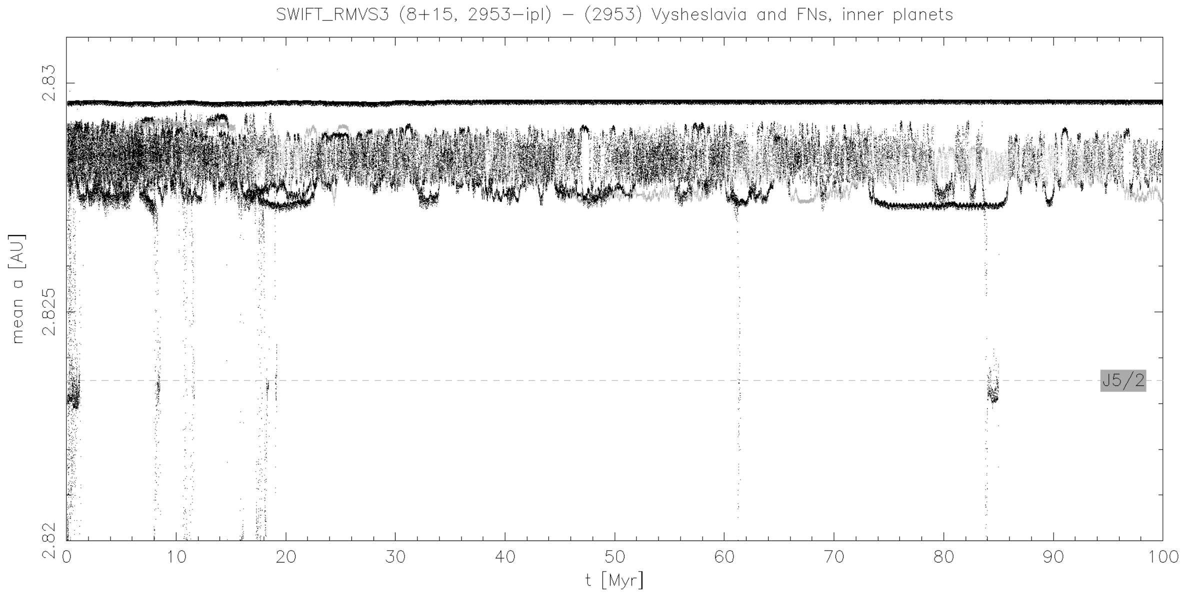

Fig. 9.— Mean semimajor axis (in AU) vs. time (in Myr) for Vysheslavia and its 14 fictitious neighbors (FN). As in the Figure 1 the Yarkovsky effect is not included in this simulation, but perturbations due to all the planets (except Pluto) are considered. Notice a much shorter lifetime of the objects residing temporarily in the 5 : 2 resonance.

Ceres which appears to be very stable, the semimajor axis of Pallas undergoes chaotic fluctuations on the order of 0.0008 AU. Šidlichovský (1999) pointed out that Pallas is located in the high–order 18 : 7 resonance with Jupiter and this seems responsible for the observed chaos.

2.4. Evolutionary implications and the future work

There might be several reasons why the Vysheslavia's orbit is more evolved than the orbits of 7340 (1991 UA_2) or 1993 FR_{58}: (i) it might be originally placed closer to the 5 : 2 resonance (since 7340 (1991 UA_2) and 1993 FR_{58} are smaller objects they might arrive at the 5 : 2 proximity by drifting from a farther original location) and/or (ii) its spin axis orientation and the rotation period results in faster Yarkovsky mobility of the semimajor axis to mention just the most obvious; future observations may help to check this possibility.

In a forthcoming paper we shall present an additional, thought indirect, argument favoring the Yarkovsky–driven origin of the Vysheslavia orbit. Notably, we have identified about a dozen of other asteroids, most probably Koronis members, that are located in or very close to the Vysheslavia's chaotic zone. These objects are smaller, typically 4–8.5 km in size, thus representing a more numerous category of family members, which should also have faster Yarkovsky mobility of semimajor axes. It is natural to expect at every time some of these bodies to be located in the Vysheslavia's chaotic zone. On the other hand, these objects are large enough to place stronger constraint on their and Vysheslavia's collisional origin (as the possible offsprings of the recent, secondary collision in the family) making this hypothesis less probable.

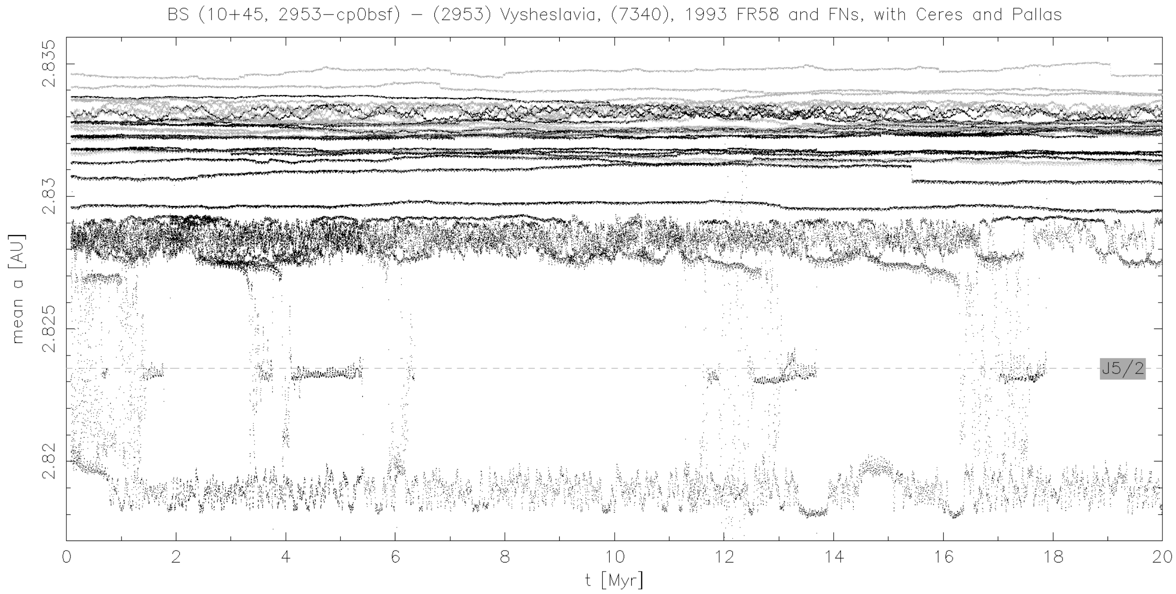

Fig. 10.— Mean semimajor axis (in AU) vs. time (in Myr) for Vysheslavia, 7340 (1991 UA_2), 1993 FR_{58} and their 42 fictitious neighbors defined in the Table 2 (the asteroid 1993 FR_{58} and its clones are grey). Perturbations due to all the planets (except Pluto) and two massive asteroids (Ceres and Pallas) considered. Notice the tiny, long–term fluctuations ($\leq 10^{-3}$ AU) and occasional jumps of the semimajor axes of the previously perfectly stable orbits. Both effects are due to the gravitational influence of Ceres and Pallas. Simulation performed with the `Bulirsh-Stoer` integrator.

Our conjecture is that other asteroid families located very close to the powerful main–belt resonances might suffer a similar kind of Yarkovsky erosion as Koronis family. Of particular interest could be Maria and Hestia families that are located very close the 3 : 1 mean motion jovian resonance, and possibly even Themis and Hygiea families located close to the 2 : 1 mean motion jovian resonance (e.g. Zappalà *et al.* 1995, 1997). In fact, the inner part of the Maria family, for example, is even cut by the 3 : 1 resonance, so that a significant number of originally created bodies must have been placed right there or very close to it. A detailed study of the process of possible secondary mass–loss for these families is, however, beyond the scope of this study.

3. Conclusions

The main results of this paper can be summarized as follows:

- Performing long–term (up to 0.5 Gyr) integrations we have verified that the Yarkovsky effect might be efficient in altering positions of small asteroids (km in size) up to 0.01 – 0.03 AU within their collisional lifetime.
- We have argued, that such a "small scale" mobility may eventually drive asteroids onto dynamically unstable or metastable orbits related to some nearby resonances. A significant difference of the dynamical lifetime of the body at the present orbit and of the family age (as it is the case of

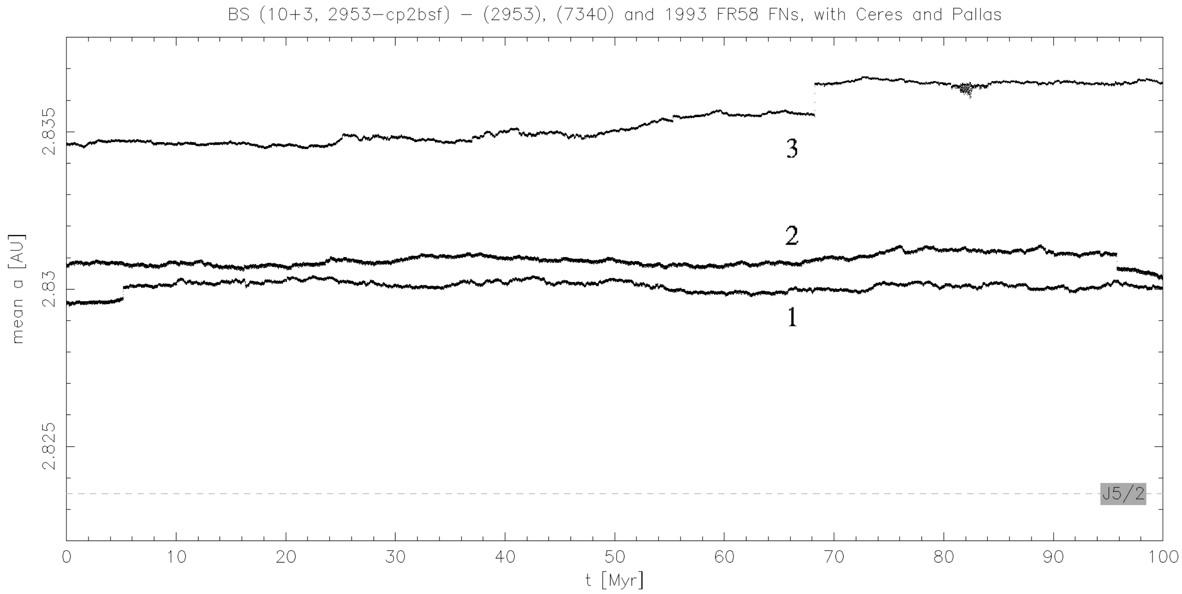

Fig. 11.— The same as in Figure 10 but for a longer timespan. Only three bodies are considered: FN^0_{+3} of Vysheslavia – curve 1, FN^0_{-3} of 7340 (1991 UA_2) – curve 2 – and FN^0_{+3} of 1993 FR_{58} – curve 3. Notice three deepest close encounters of the integrated particles with Ceres that both resulted in a significant change of the semimajor axis: (i) particle 1 at about 5.1 Myr, (ii) particle 2 at about 97.5 Myr, and (iii) particle 3 at about 68.2 Myr. The closest approach to Ceres in the third case was 1.1×10^{-4} AU only.

Vysheslavia) might be a strong indication for the evolutionary processes taking place in the family.

- A secondary collision in the family placing the bodies on the dynamically metastable orbits still cannot be ruled out as one of the possible explanations. However, in the Vysheslavia's case the probability of such a possibility is low. Similarly, our simulations indicate low probability for gravitational origin of the Vysheslavia's orbit due to the influence of massive asteroids (Ceres and Pallas were studied above). The Yarkovsky–driven transport is the last and most probable possibility. Though we cannot determine exactly the history of Vysheslavia's orbit it seems most likely that it has been placed onto its present orbit by a slow drift from its original location in the stable region further from the 5 : 2 resonance.

- Photometric and spectral observations of Vysheslavia (and other small members of the Koronis family located close to the 5 : 2 resonance) are needed to constrain the uncertainties and possibly discriminate between the different hypotheses. In particular, photometric data should yield a very useful information about its rotation. The precise value of the rotation period and of the orientation of the polar axis would improve the Yarkovsky effect modelling, while more reliable information about the rotation state (a possible tumbling) may offer a hint about the age of Vysheslavia. We have mentioned that Šarounová (2000; personal communication) has recently observed Vysheslavia and estimated the rotation period to about 6.29 hr. Complementing these observations with other observations at different phase angles might constrain obliquity of its spin axis (even if determination of the precise spin axis orientation is difficult). Assuming diurnal effect to be the dominant Yarkovsky force, such a constraint may significantly contribute to understanding Vysheslavia's history. In particular, if the inward drift of its semimajor axis is confirmed, the hypothesis of this paper –

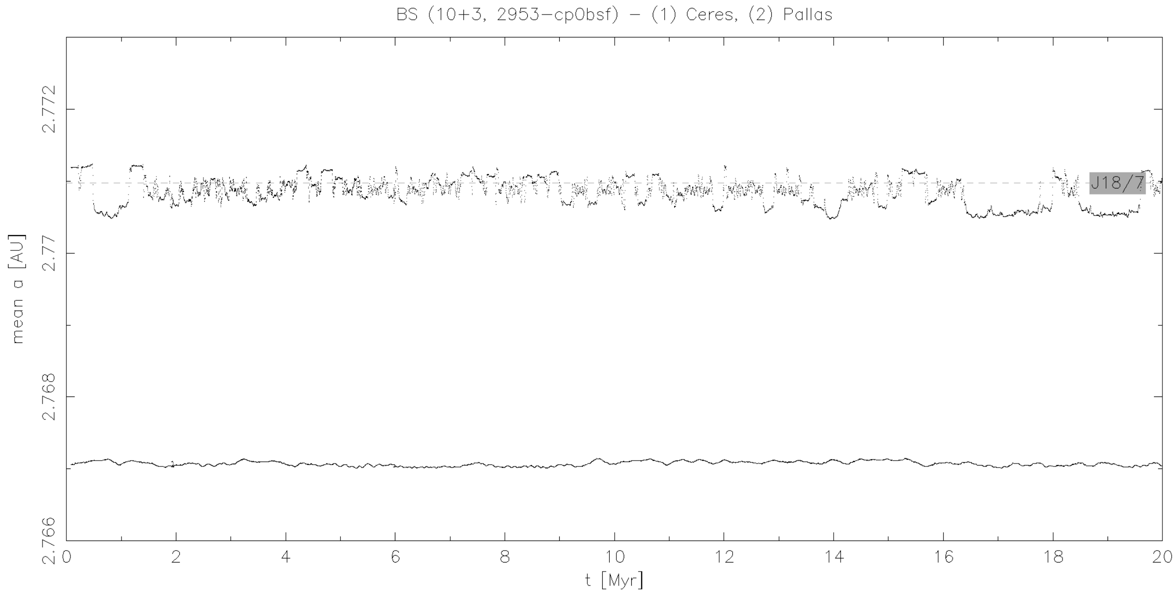

Fig. 12.— Long–term evolution of the mean semimajor axes of Ceres and Pallas (time in Myr at the abscissa). Notice the difference between the stable behavior of the Ceres semimajor axis (lower curve) and the chaotic fluctuations of the Pallas semimajor axis (upper curve). The latter seems to be locked in a similar chaotic zone (width of about 0.0008 AU) as Vysheslavia. This chaotic region seems to be associated to the high–order 18 : 7 resonance with Jupiter.

Yarkovsky–driven origin of the Vysheslavia's orbit – would turn to near certitude.

- A detailed study of the Yarkovsky–driven diffusion in the asteroid families and their leakage to the principal resonances in the main asteroid belt appears as challenging prospect for the future work.

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