Yarkovsky effects
as a source of mobility for the asteroid fragments

Miroslav Brož [1,2] and David Vokrouhlický [1,3]

[1] Institute of Astronomy, Charles University, V Holešovičkách 2, 182 00 Prague, Czech Republic
[2] (mailto: miroslav.broz@usa.net)
[3] (mailto: vokrouhl@mbox.cesnet.cz)

Abstract:

Yarkovsky effects are variants of the radiation pressure recoil which acts on anisotropically emitting spinning bodies, heated by sunlight to different temperatures in different parts of their surface. After having discussed the basic principles, we recall here the principal new results obtained in the last few years. In particular, we comment on the importance of the Yarkovsky effects for delivery of the asteroid fragments to the Earth vicinity. We show that the new scenario – extended by the Yarkovsky effects – may explain a number of unsolved problems in meteoritics and dynamics of NEAs.

Structure of the poster:

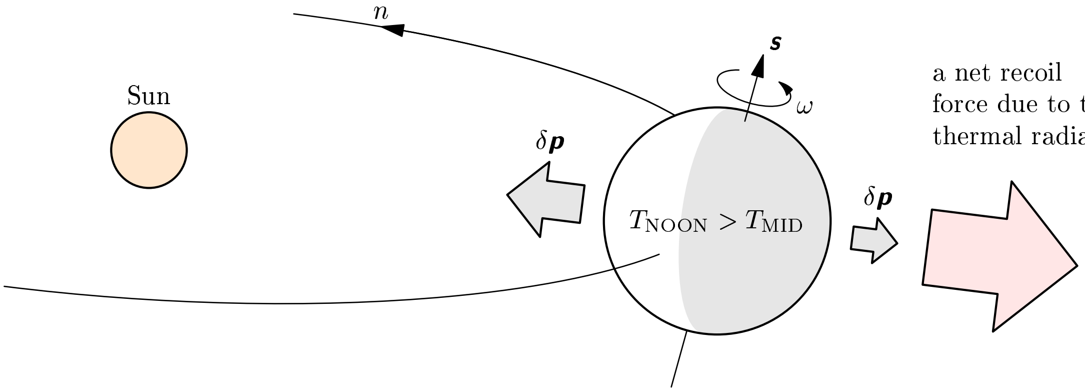
- modelling of the underlying physics
- orbital effects
- astronomical consequences
- delivery of the Hebe (6) fragments to the Earth
- references

For the new results and updates please look at

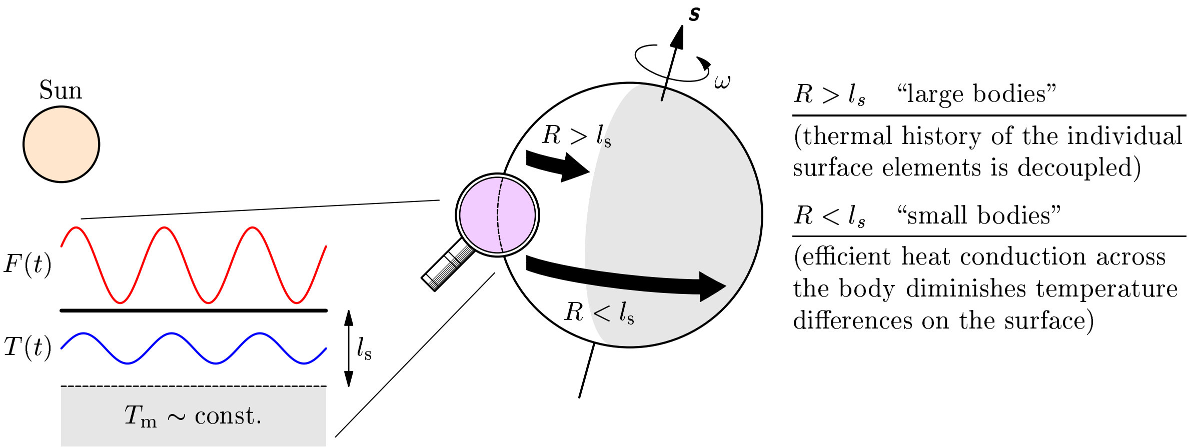
http://www.kolej.mff.cuni.cz/~broz/mp/

1. Modelling of the underlying physics: principles

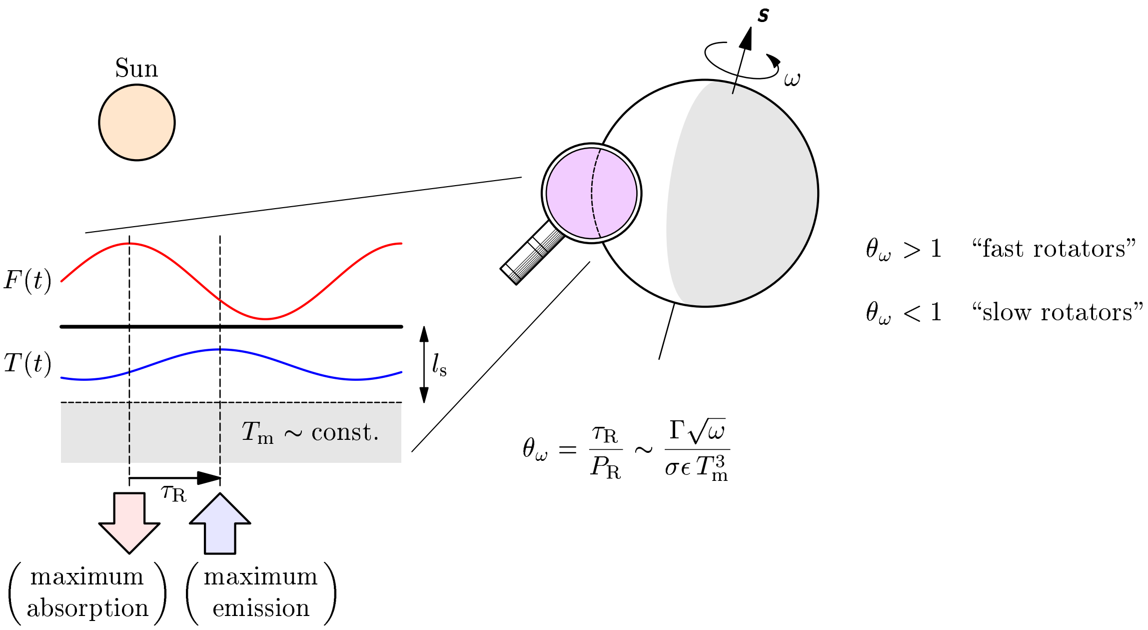
(a) a first look (principle):



(b) a second look (concept of largeness vs. smallness of the body):



(c) a third look (concept of rapid vs. slow rotation):



1. Modelling of the underlying physics: mathematical details

Mathematical modelling of the thermal effects is "a boundary constraint problem". The energy conservation on the surface of the body reads:

$$\begin{pmatrix}\text{energy thermally}\\ \text{emitted per unit}\\ \text{of surface area}\end{pmatrix} + \begin{pmatrix}\text{energy conducted}\\ \text{towards deep layers}\\ \text{of the body}\end{pmatrix} = \begin{pmatrix}\text{energy absorbed}\\ \text{per unit of time and}\\ \text{unit of surface area}\end{pmatrix}$$

$$\varepsilon\,\sigma\,T^4 + K\left(\frac{\partial T}{\partial r}\right) = \alpha\,F$$

Given a Fourier development of the radiation energy flux $F = \sum f_n \zeta^n \quad (\zeta = \exp(\mathrm{i}\nu t))$, we seek a similar development of the temperature $T = \sum t_n \zeta^n$. Notice, however, that the fourth-power emission law is a serious obstacle for the analytical approach. Two methods have been used to tackle this problem:

- **Linearized theory:** $T = T_{\rm av} + \Delta T, \quad T^4 \approx T_{\rm av}^4 + 4\,T_{\rm av}^3\,\Delta T\,, \quad T_{\rm av} = \mathrm{const.}$ Rubincam (1995, 1998), Vokrouhlický (1998a,b), Vokrouhlický and Farinella (1998)
- **Complete theory:** numerical approach, Vokrouhlický and Farinella (1998b,c)

Fourier spectrum of $F(t)$ for a given surface element contains basically two frequencies (and their multiplies; mixed terms are negligible in the astronomical applications, Vokrouhlický, unpublished):

$\omega \dots \tau_R(\omega)$ $2\,\omega \dots \tau_R(2\,\omega)$ $3\,\omega \dots \tau_R(3\,\omega)$ $\vdots$	$n \dots \tau_R(n)$ $2\,n \dots \tau_R(2\,n) \sim e^1$ $3\,n \dots \tau_R(3\,n) \sim e^2$ $\vdots$
diurnal effect	**seasonal effect**
(original form of Öpik, Radzevskii, Peterson, ...)	(newly discussed version by Rubincam, Farinella, ...)

total effect

Once the temperature T on the surface is known, we can compute the thermal acceleration by

$$\boldsymbol{f} = \frac{2}{3}\frac{\epsilon\sigma}{mc}\int_S T^4 \boldsymbol{n}\,\mathrm{d}S$$

where ϵ ... thermal emissivity of the surface, σ ... Stefan-Boltzmann constant, $\boldsymbol{n}$ ... normal unit vector to the surface element $\mathrm{d}S$, and the integration $\int_S$ is performed over the whole surface of the body.

2. Orbital effects: analytical results of the linearized theory

The principal perturbation consists of a secular change in the semimajor axis. Quantitative results have been obtained by Rubincam (1995, 1998), Vokrouhlický (1998a,b), Vokrouhlický and Farinella (1998b).

diurnal effect	seasonal effect
$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{8\alpha}{9}\frac{\Phi}{n}\,\psi^d(R',\theta_\omega)\cos\gamma + \mathrm{o}(e)$ with $\Phi = \frac{\pi R^2 F}{mc}$ $\quad\gamma$ ... obliquity	$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{4\alpha}{9}\frac{\Phi_a}{n}\sum_{k\geq 1}\psi_k^s(R',\theta_n)\,[s_P^2\alpha_k^2 + s_Q^2\beta_k^2]$ $= -\frac{4\alpha}{9}\frac{\Phi_a}{n}\,\psi_1^s(R',\theta_n)\sin^2\gamma + \mathrm{o}(e)$ with $\alpha_k, \beta_k \sim \mathrm{o}(e^k)$ ... eccentricity functions $\theta_n = \frac{\Gamma\sqrt{n}}{\epsilon\sigma T_{\rm m}^3}$ ... seasonal thermal parameter $s_P = \boldsymbol{s}\cdot\boldsymbol{P}\,, \quad s_Q = \boldsymbol{s}\cdot\boldsymbol{Q}$

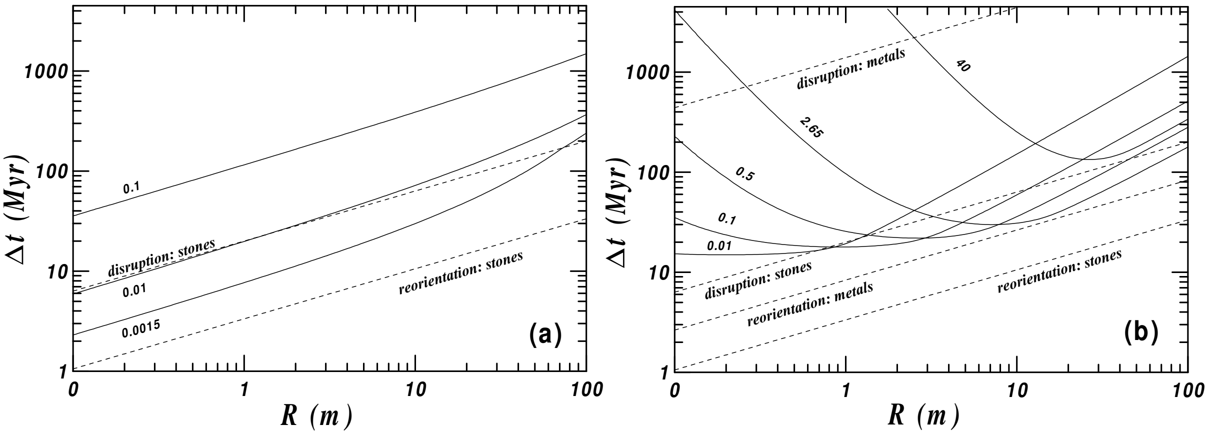
Notice an important **difference** between two variants of the Yarkovsky effect — dependence of $\mathrm{d}a/\mathrm{d}t$ the on the obliquity γ. While the diurnal effect can both increase or decrease the semimajor axis, the seasonal variant is always a drag like. If the timescale for a collisional reorientation of the body's spin axis is shorter than the time over which the body drifts in the main belt, the diurnal effect represents a random walk in a ($\mathrm{d}a \propto \sqrt{\mathrm{d}t}$).

2. Orbital effects: examples of the semimajor axis mobility

The above given formulae allow to estimate the semimajor axis drift Δa within a time Δt. Notice, however, that random changes of the obliquity γ due to collisions have to be considered when the timescale of the reorientation of the fragments (1 – 10 Myr) is shorter than Δt.

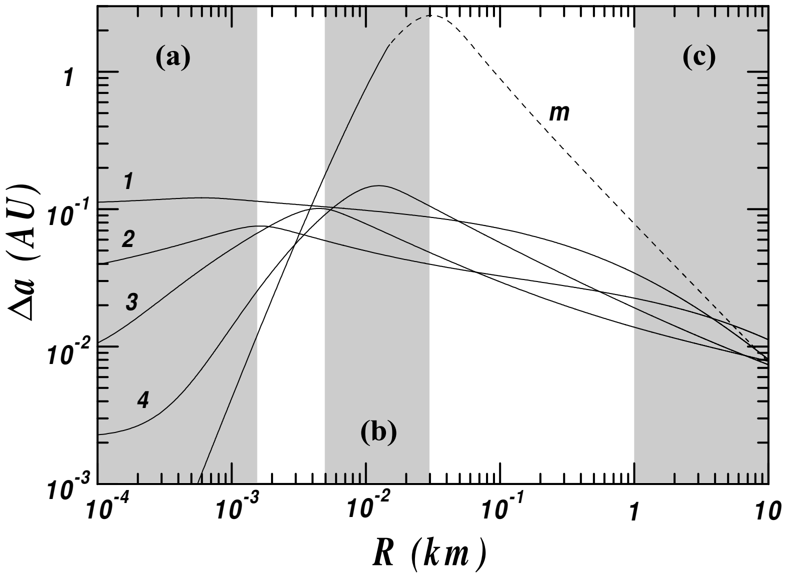
Two experiments have been performed to illustrate the thermal mobility of the asteroid fragments.

- Δa fixed – we solve for time Δt necessary to move the body by Δa



Characteristic time Δt to move a body by $\Delta a = 0.05\,\mathrm{AU}$ vs. radius R in meters. Diurnal effect (left figure - a) and seasonal effect (right figure - b) considered separately. The numbers along with the curves indicate the thermal conductivity K (in W/m/K) of the surface layer. Low conductivity curves ($K < 0.01$) correspond to particulate surfaces or surfaces covered with a thin regolith layer. High conductivity cases ($K > 0.1$) correspond to bare-rock fragments; $K = 40$ corresponds to iron-rich fragments. The effects of random collisional reorientation of the spin axis have been taken into account by a simple Monte Carlo model.

- Δt fixed – we solve for the corresponding Δa



Thermal drift Δa within the collisional lifetime of the asteroid fragments τ_{coll} ($= \Delta t$) vs. radius R in meters; τ_{coll} estimated by the equilibrium Dohnanyi (1969) theory, $\tau_{\mathrm{coll}} \approx 16.8\sqrt{R}\,\mathrm{Myr}$. Both diurnal and seasonal versions of the Yarkovsky effect considered. Different values of the surface thermal conductivity assumed: (i) $K = 0.002$ W/m/K – curve 1, (ii) K = 0.02 W/m/K – curve 2, (iii) $K = 0.2$ W/m/K - curve 3, (iv) $K = 2$ W/m/K – curve 4, and (v) $K = 40$ W/m/K – curve m (iron-rich fragments). The shaded strips (a) – (c) show the astronomically relevant classes of bodies (see Sec. 3 below).

Notice that in the (a) and (b) regions the resulting shift Δa depends strongly on the surface conductivity K, while in the case of the large fragments – (c) – the result is nearly insensitive to this parameter.

3. Astronomical consequences: a general overview

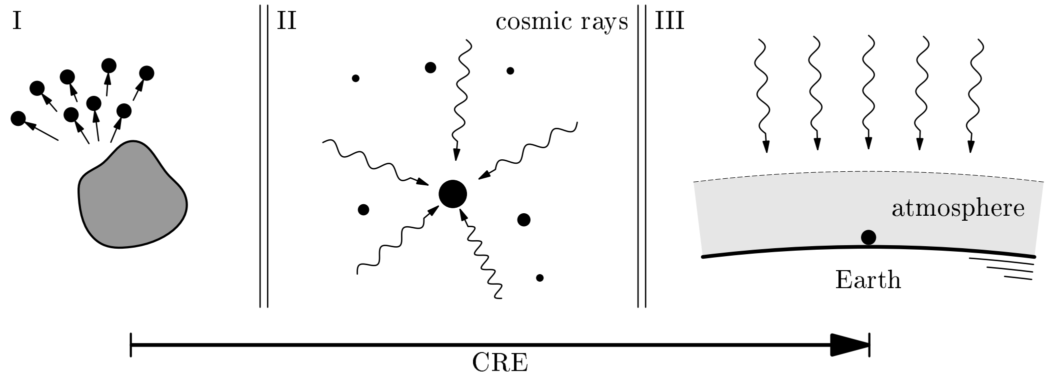
The Yarkovsky effects may play a crucial role in delivery of the asteroid fragments towards the inner solar system (particularly the Earth). They substantially extend the classical model of the asteroid fragment delivery to the Earth by a slow-track phase outside the main belt resonances. The new scenario thus looks as follows:

(1) Fragment born outside a resonance (in the main belt).
(2) Slow phase dominated by Yarkovsky effects & collisional dynamics (a few Myr – 100 Myr).
(3) Fast phase of the residence in the resonance (the fragment becomes planet-crosser, Sun-grazer on a typical timescale of 1 – 10 Myr).

The semimajor axis mobility due to the Yarkovsky effects depends strongly on the fragment's size (see Sec. 2 above). We thus distinguish three different groups of fragments according to their size:

(a) **meter–sized bodies** (0.1 – 2 m) — meteorite parents
(b) **decameter–sized bodies** (5 – 50 m) — "Rabinowitz population of NEAs" or "Tunguska like objects"
(c) **kilometer–sized bodies** (1 – 10 km) — large NEAs

(ad a) Main observational constraint are CRE (cosmic ray exposure) timescales for different taxonomic classes of meteorites.

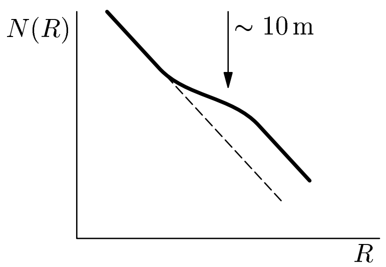


Important questions are:

- Why CRE's are much longer than the typical residence timescales in the main resonances ($\nu_6, 3:1 \sim 1 - 10\,\mathrm{Myr}$)?
- Why different taxonomic classes have qualitatively and quantitatively different CRE's?

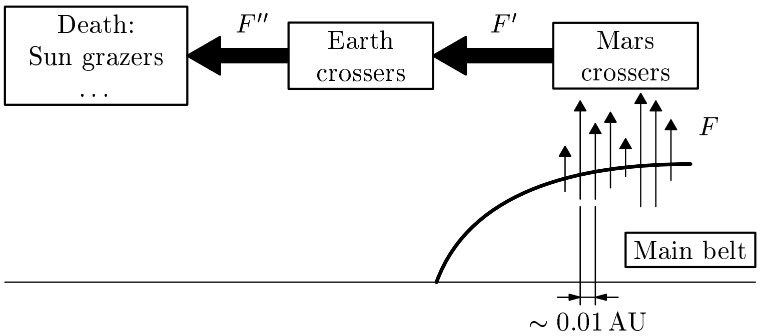
The extended scenario (1) – (3), accounting for the Yarkovsky effects, offers a natural solution of these problems, since during the Yarkovsky dominated phase (1) the meteorite parent bodies have enough time to accumulate long CRE's (a quantitative model by Farinella and Vokrouhlický, to be submitted).

(ad b) Rabinowitz (1993, 1994) reports a possible overabundance of the decameter NEAs. Chapman reports possible defficiency of $\sim$ 100 m craters on Gaspra, which may be caused by missing population of decameter–sized bodies in the main belt.



Since the seasonal mobility of the bare-rock fragments is maximum at about 7 – 20 m, it can explain defficiency of these bodies in the main belt and their relative overabundancy in the Earth-crossing population of bodies.

(ad c) Migliorini *et al.* (1998) observed a problem of maintaining a population of $(5-10)$ km NEAs.



The population of Mars crossers is maintained by a "slow leaking" from the main belt via high-order resonances with Jupiter, outer resonances with Mars or mixed resonances with Jupiter and Saturn. However, these resonances do not contain enough bodies to maintain the population of km-NEAs in equilibrium.

The high-order resonances may be efficiently fed by kilometer-sized main-belt bodies drifting up to 0.01 AU within their collisional lifetime, sustaining thus the flux F at a high level (see Farinella and Vokrouhlický, 1998b).

4. Astronomical consequences: Hebe (6) fragments to the Earth

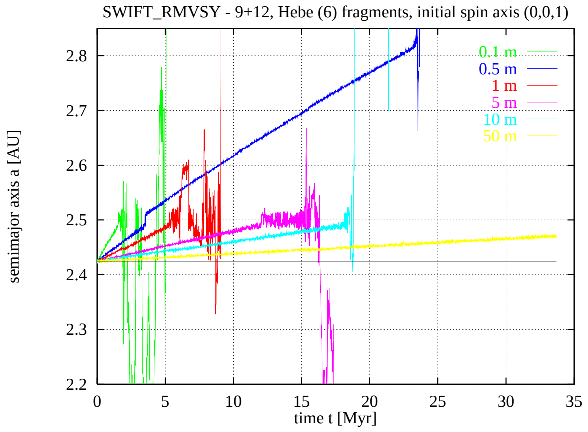
As a particular example of the previously discussed problems we are going to develop a detailed model of a delivery of fragments from the asteroid Hebe (6) (see also Farinella *et al.*, 1993). On contrary to the previous models we intend to implement the Yarkovsky effects in our integration. A preliminary results, taking into account the diurnal version of the Yarkovsky effects, are shown below. The fragments are assumed to be covered by a thin, insulating regolith layer. The effects of the collisional reorientation of the fragments spin axes are not included yet. Physical parameters are as follows:

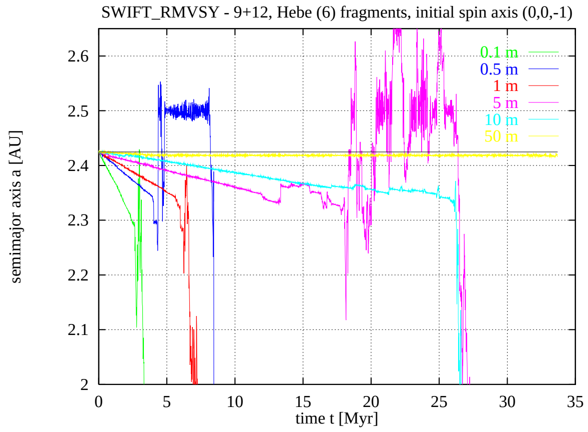
- density $\varrho = 3500\,\mathrm{kg}\cdot\mathrm{m}^{-3}$, surface density $\varrho = 1500\,\mathrm{kg}\cdot\mathrm{m}^{-3}$, surface thermal conductivity $K = 0.005\,\mathrm{W}\cdot\mathrm{m}^{-1}\cdot\mathrm{K}^{-1}$, specific heat $C = 680\,\mathrm{J}\cdot\mathrm{kg}^{-1}\cdot\mathrm{K}^{-1}$, surface albedo $A = 0$

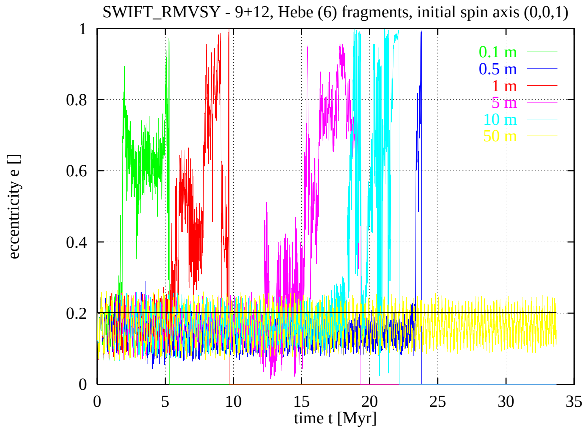
We consider two sets of six fragments with radii in the range **0.1 – 50 m**. The first group of fragments have **initial** obliquity $\gamma = 0°$, while fragments of the second group have initially $\gamma = 180°$. The black line indicates the initial orbital elements (2000.0) for asteroid Hebe (6) (taken from *MPC* 24219):

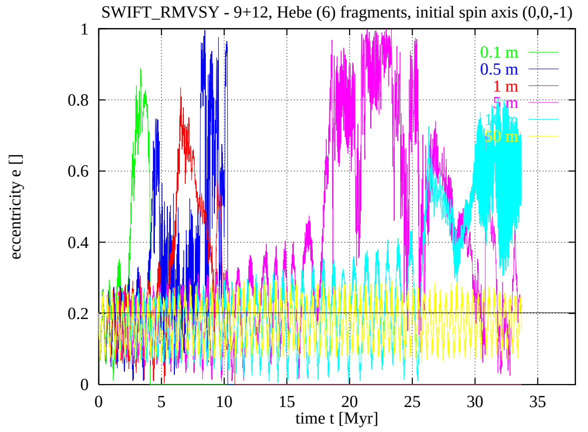
- $a = 2.4247741 AU$, $e = 0.2018431$, $i = 14.77118°$, $peri = 239.07195°$, $node = 138.88979°$, $M = 211.60690°$, epoch JDT 2450800.5

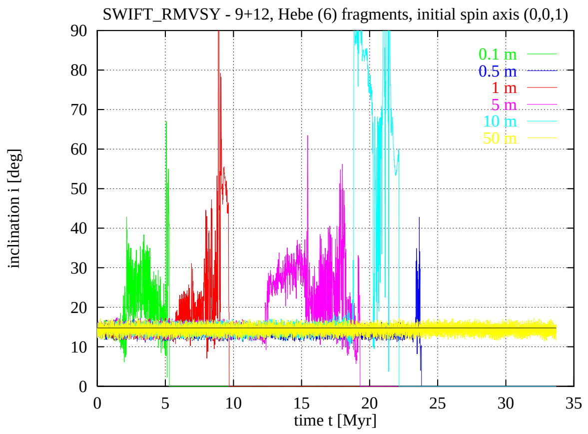
The evolution was computed by `swift_rmvsy` program on a P100/16M computer, `f2c/gcc` compiler. 10 of 12 fragments have been discarded from the run after **33 Myr**, since they became either Sun grazers or were farther than 1000 AU from the Sun. Evolution of the osculating elements for each of the fragments shown in the figures below. Notice the thermal drift in the semimajor axes and its dependence on the fragments size. Strong perturbations of the eccentricity are due to interaction with mean-motion and secular resonances with giant planets (the first eight planets taken into account in our model).

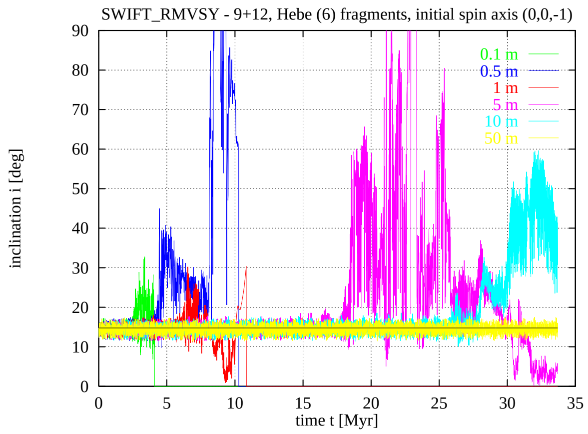


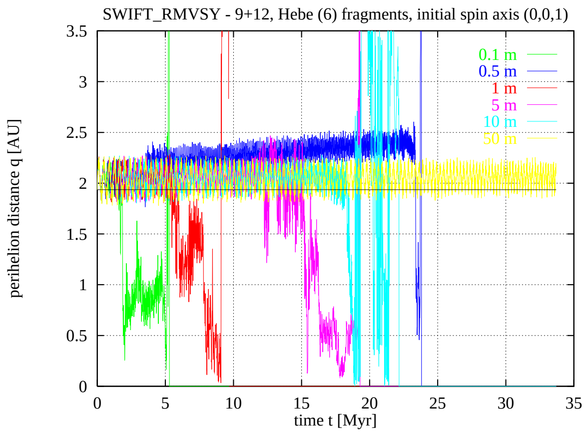


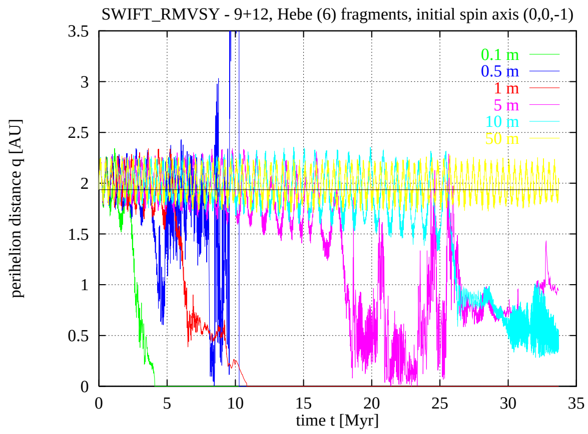




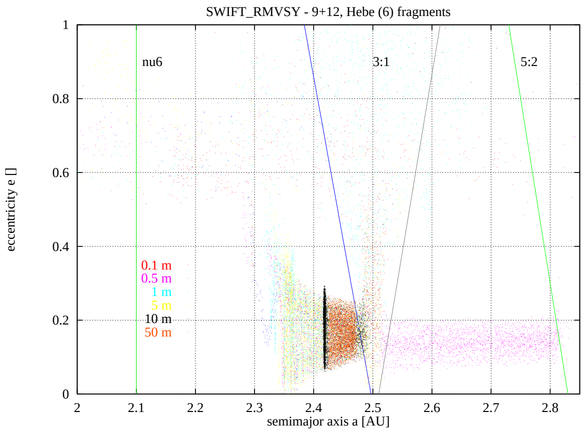


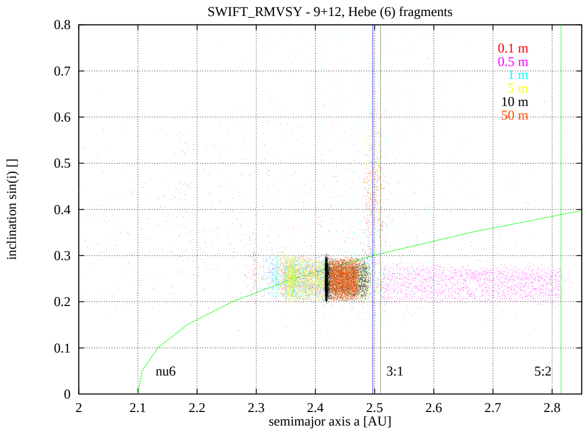






The last two figures show the orbital evolution of the fragments in the (a,e) and (a, sin i) planes of the osculating elements. The approximate location of the main resonances are marked (border of ν_6 secular resonance in (a, e) is for $\sin i = 0$).





Please look for the updates at `http://www.kolej.mff.cuni.cz/~broz/mp/hebe6.html`.

5. References related to this work

papers on the thermal effects

- Afonso, G. B., Gomes, R. S., Florczak, M. A.: **Asteroid fragments in Earth-crossing orbits.** Planet. Space Sci., Vol. 43, No. 6, pp. 787–795, 1995
- Burns, J. A., Lamy, P. L., Soter, S.: **Radiation forces on small particles in the solar system.** Icarus 40, 1–48, 1979
- Farinella, P., Vokrouhlický, D., Hartmann, W. K.: **Meteorite delivery via Yarkovsky orbital drift.** Icarus 132, 378–387, 1998
- Farinella, P., Vokrouhlický, D.: **Semimajor axis mobility of asteroidal fragments.** Science, submitted, 1998
- Hartmann, W. K., Farinella P., Vokrouhlický, D., *et al.*: **Reviewing the Yarkovsky effect: New light on the delivery of stones and irons from the asteroidal belt.** Meteorit. Planet. Sci., in press, 1998
- Öpik, E.J.: **Collision probabilities with the planets and the distribution of interplanetary matter.** Proc. R. Irish Acad. 54A, 165–199, 1951
- Peterson, Ch.: **A source mechanism for meteorites controlled by the Yarkovsky effect.** Icarus 29, 91–111, 1976
- Rubincam, D. P.: **Asteroid orbit evolution due to thermal drag.** JGR 100(E1), 1585–1594, 1997

- Rubincam, D. P.: **Yarkovsky thermal drag on small asteroids and Mars-Earth delivery.** JGR 103(E1), 1725–1732, 1998
- Vokrouhlický, D.: **Diurnal Yarkovsky effect for meter-sized asteroidal fragments mobility. I. Linear theory.** Astron. Astrophys. 335, 1093–1100, 1998a
- Vokrouhlický, D.: **Diurnal Yarkovsky effect as a source of mobility of meter-sized asteroidal fragments. II. Non-sphericity effects.** Astron. Astrophys., in press, 1998b
- Vokrouhlický, D., Farinella, P.: **Orbital evolution of asteroidal fragments into the ν_6 resonance via Yarkovsky effects.** Astron. Astrophys. 335, 351–362, 1998
- Vokrouhlický, D., Farinella, P.: **A non-linear model of the seasonal Yarkovsky effect on spheres.** Icarus, submitted, 1998a
- Vokrouhlický, D., Farinella, P.: **The Yarkovsky seasonal effect on asteroidal fragments: A non-linearized theory for the plane-parallel case.** Astron. J., in press, 1998b

papers on the delivery of the asteroid fragments to the Earth

---

- Farinella, P., Froeschlé, Ch., Gonczi, R.: **Meteorites from the asteroid Hebe.** Celest. Mech. 56, 287–305, 1993
- Farinella, P., Froeschlé, C., Gonzi, R.: **Meteorite delivery and transport.** Asteroids, Comets, Meteors 1993, 205–222, 1994
- Gladman, B. J. *et al.*: **Dynamical lifetimes of objects injected into asteroid belt resonances.** Science, Vol. 277, 197–201, 11 July 1997
- Migliorini, F. *et al.*: **Origin of earth crossing asteroids: the new scenario.** Science, in press, 1998
- Migliorini, F. *et al.*: **Vesta fragments from ν_6 and 3:1 resonances: Implications for V-type near-Earth asteroids and howardite, eucrite and diogenite meteorites.** Meteorit. Planet. Sci. 32, 903–916, 1997
- Morbidelli, A., Gladman, B.: **Orbital and temporal distributions of meteorites originating in the asteroid belt.** Meteorit. Planet. Sci., in press, 1998
- Morbidelli, A. *et al.*: **Delivery of meteorites through the ν_6 secular resonance.** Astron. Astrophys. 282, 955–979, 1994
- Rabinowitz D. L.: **The size distribution of the Earth-approaching asteroids.** Astrophys. J. 407, 412–427, 1993.
- Rabinowitz D. L.: **The size and shape of the near-Earth asteroid belt.** Icarus 111, 364-377, 1994.

others

- Dohnanyi, J. W.: **Collisional models of asteroids and their debris.** JGR 74, 2531–2554, 1969
- Welten, K. *et al.*: **Cosmic-ray exposure ages of diogenites and the recent collisional history of the howardite, eucrite and diogenite parent body/bodies.** Meteorit. Planet. Sci. 32, 891–902, 1997

Typeset by plainTEX, METAPOST, gnuplot