smoothed particle → **SPH models**

basic equations lagrangian formulation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\boldsymbol{v}\,,$$

eq. of continuity

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla P - \nabla\Phi + \frac{1}{\rho}\nabla\cdot\boldsymbol{S}\,,$$

Navier–Stokes

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -P\nabla\cdot\boldsymbol{v} + \boldsymbol{S}\cdot\tfrac{1}{2}\left[\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T\right]\,,$$

1^{st} law of thermodynamics

$$\nabla^2\Phi = 4\pi G\rho\,,$$

Poisson

eq. of state (Tillotson 1962)

$$P = \begin{cases} A\left(\frac{\rho}{\rho_0}-1\right) + B\left(\frac{\rho}{\rho_0}-1\right)^2 + a\rho U + \frac{b\rho U}{\frac{U}{U_0}\frac{\rho_0^2}{\rho^2}+1} & \text{pro } U < U_{\mathrm{iv}}\,, \\ a\rho U + \left[\frac{b\rho U}{\frac{U}{U_0}\frac{\rho_0^2}{\rho^2}+1} + A\left(\frac{\rho}{\rho_0}-1\right)\mathrm{e}^{-\beta\left(\frac{\rho_0}{\rho}-1\right)}\right]\mathrm{e}^{-\alpha\left(\frac{\rho_0}{\rho}-1\right)} & \text{pro } U > U_{\mathrm{cv}}\,, \end{cases}$$

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \not{2}\mu_1\,\tfrac{1}{\not{2}}\left[\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T\right] + \left(\mu_2 - \tfrac{2}{3}\mu_1\right)\nabla\cdot\boldsymbol{v}\,\boldsymbol{I}\,.$$

constitutive relation (for solids)

SPH models (cont.)

- yielding criterion (von Mises 1913)

$$\mathbf{S} = f\mathbf{S}\,, \quad f = \min\left[\frac{Y^2}{3J_2}, 1\right]\,, \quad J_2 = S^{\alpha\beta}S^{\alpha\beta}\,,$$

- flaws distribution (Weibull 1938) → cracks, damage *D*

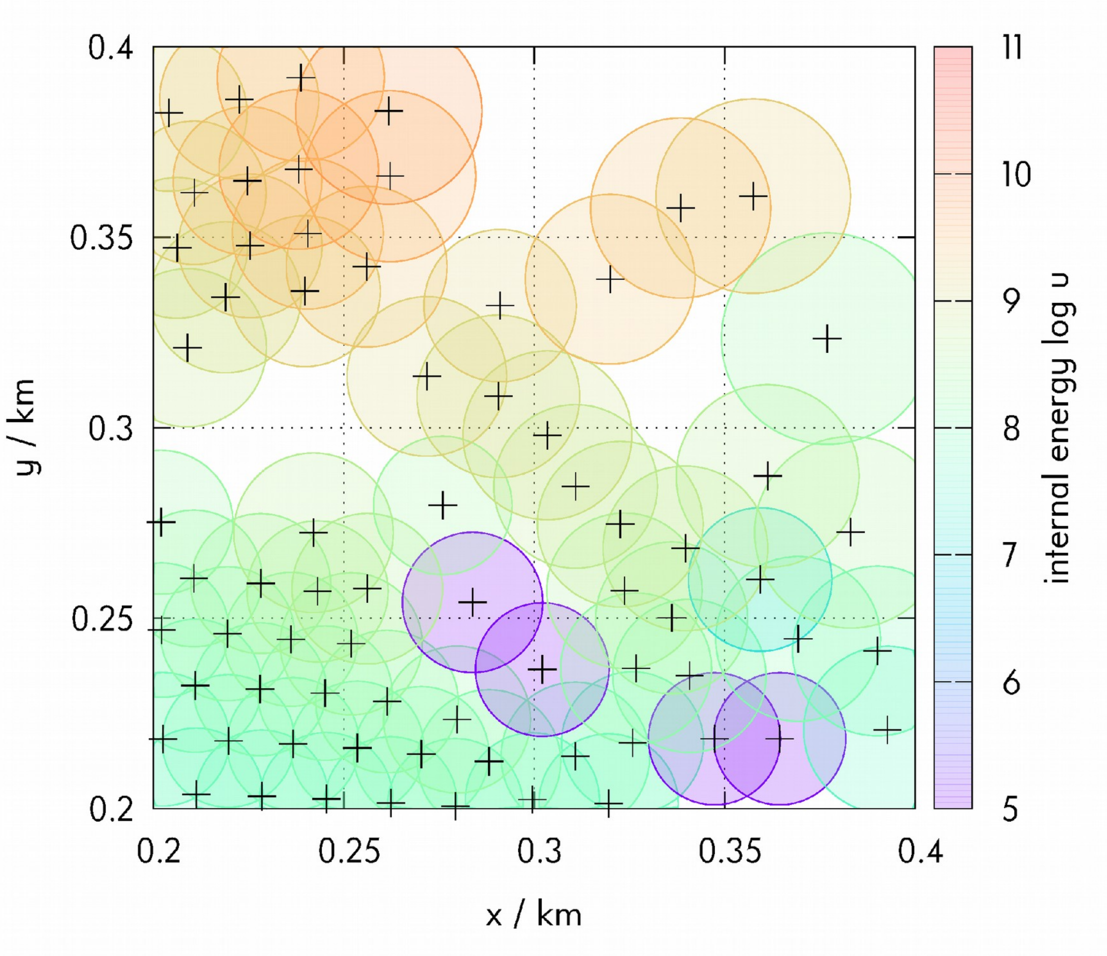
$$\sigma_{\alpha\beta} = \begin{cases} -P\delta_{\alpha\beta} + (1-D)S_{\alpha\beta} & \text{pro } P \geq 0\,, \\ -(1-D)P\delta_{\alpha\beta} + (1-D)S_{\alpha\beta} & \text{pro } P < 0\,. \end{cases}$$

$$\frac{\mathrm{d}D^{\frac{1}{3}}}{\mathrm{d}t} = \left[\left(\frac{c_g}{R_\mathrm{s}}\right)^3 + \left(\frac{m+3}{3}\alpha^{\frac{1}{3}}\epsilon^{\frac{m}{3}}\right)^3\right]^{\frac{1}{3}}\,,$$

Grady & Kipp (1980)

SPH approximation

- continuum → a finite set of extended particles ("vehicles"), cf. Cossins (2010), Price (2012)



SPH formulation

- spatial derivatives → summations over nearest *neighbours*
- discretization in time (Euler or predictor/corrector)

$$\rho_i^{n+1} = \rho_i^n - \Delta t \rho_i^n \sum_j \boldsymbol{v}_j^n \cdot \nabla W_{ij}(h) \frac{m_j}{\rho_j^n} ,$$

$$\boldsymbol{v}_i^{n+1} = \boldsymbol{v}_i^n - \frac{\Delta t}{\rho_i^n} \sum_j P_j^n \nabla W_{ij}(h) \frac{m_j}{\rho_j^n} + \frac{\Delta t}{\rho_i^n} \sum_j \boldsymbol{S}_j^n \cdot \nabla W_{ij}(h) \frac{m_j}{\rho_j^n} ,$$

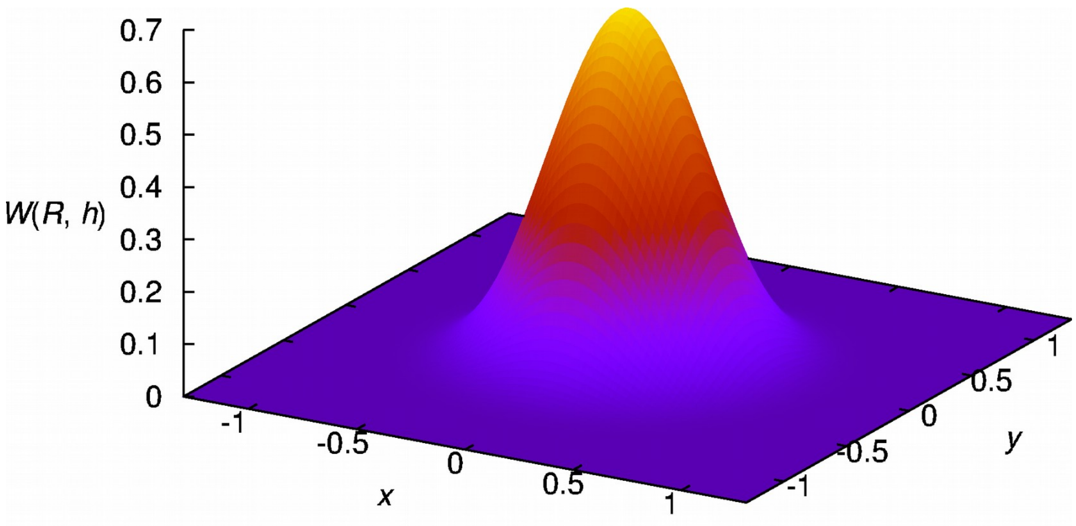
$$U_i^{n+1} = U_i^n - \Delta t P_i^n \sum_j \boldsymbol{v}_j^n \cdot \nabla W_{ij}(h) \frac{m_j}{\rho_j^n} + $$

$$+ \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} S_{\alpha\beta}^n \, \tfrac{1}{2} \sum_j \left[v_{\beta j}^n \frac{\partial}{\partial x_\alpha} W_{ij}(h) + v_{\alpha j}^n \frac{\partial}{\partial x_\beta} W_{ij}(h) \frac{m_j}{\rho_j^n} \right] .$$

Smoothing Kernel

- suitable function: normal, compact, $\lim_{h \to 0} W(h) = \delta$, positive, decreasing, symmetric, smooth

$$W(R,h) = \frac{3}{2\pi h^3} \begin{cases} \frac{2}{3} - 4R^2 + 4R^3 & \text{pro } 0 \leq R < \frac{1}{2}\,, \\ \frac{4}{3} - 4R + 4R^2 - \frac{4}{3}R^3 & \text{pro } \frac{1}{2} \leq R < 1\,, \\ 0 & \text{pro } R \geq 1\,. \end{cases}$$

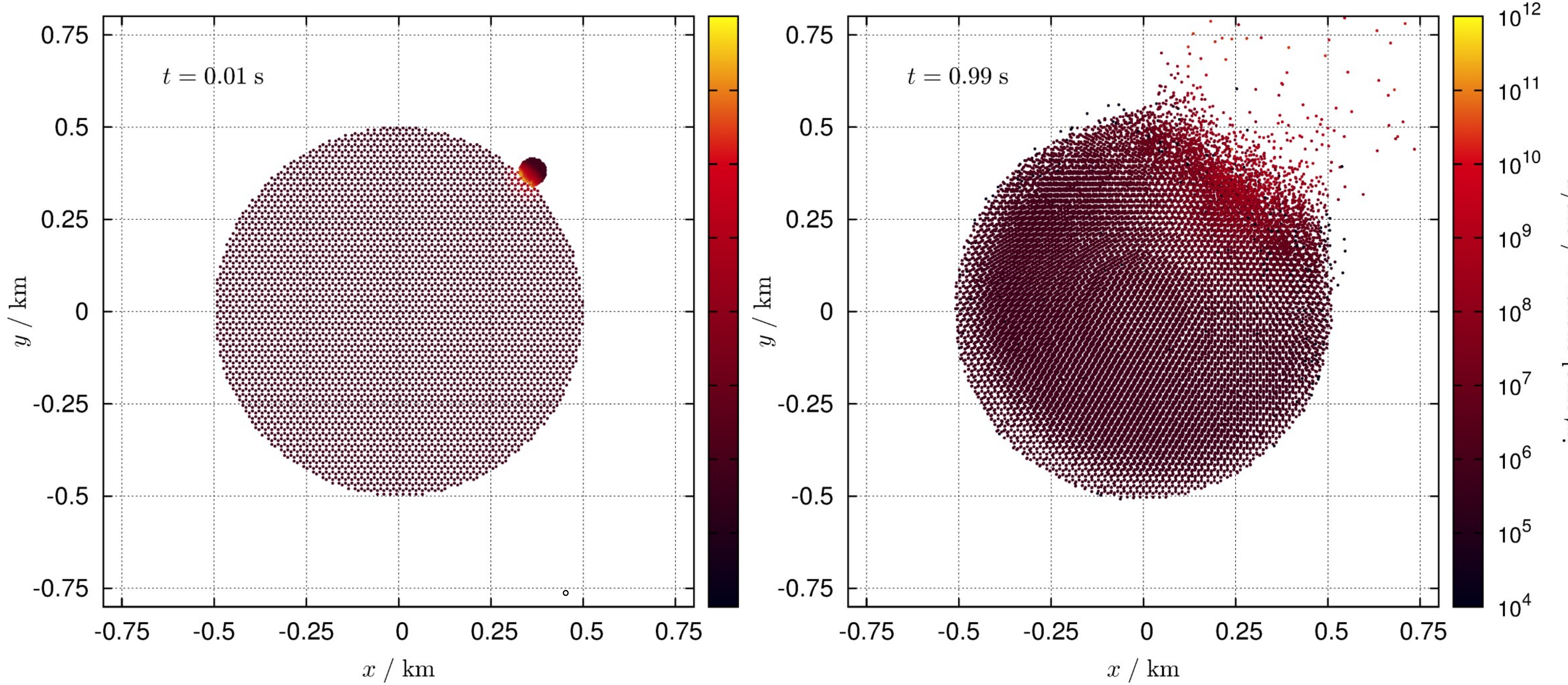


Obr. 2 — Kubický spline $W(R,h)$ dle rovnice (15).

Fragmentation phase

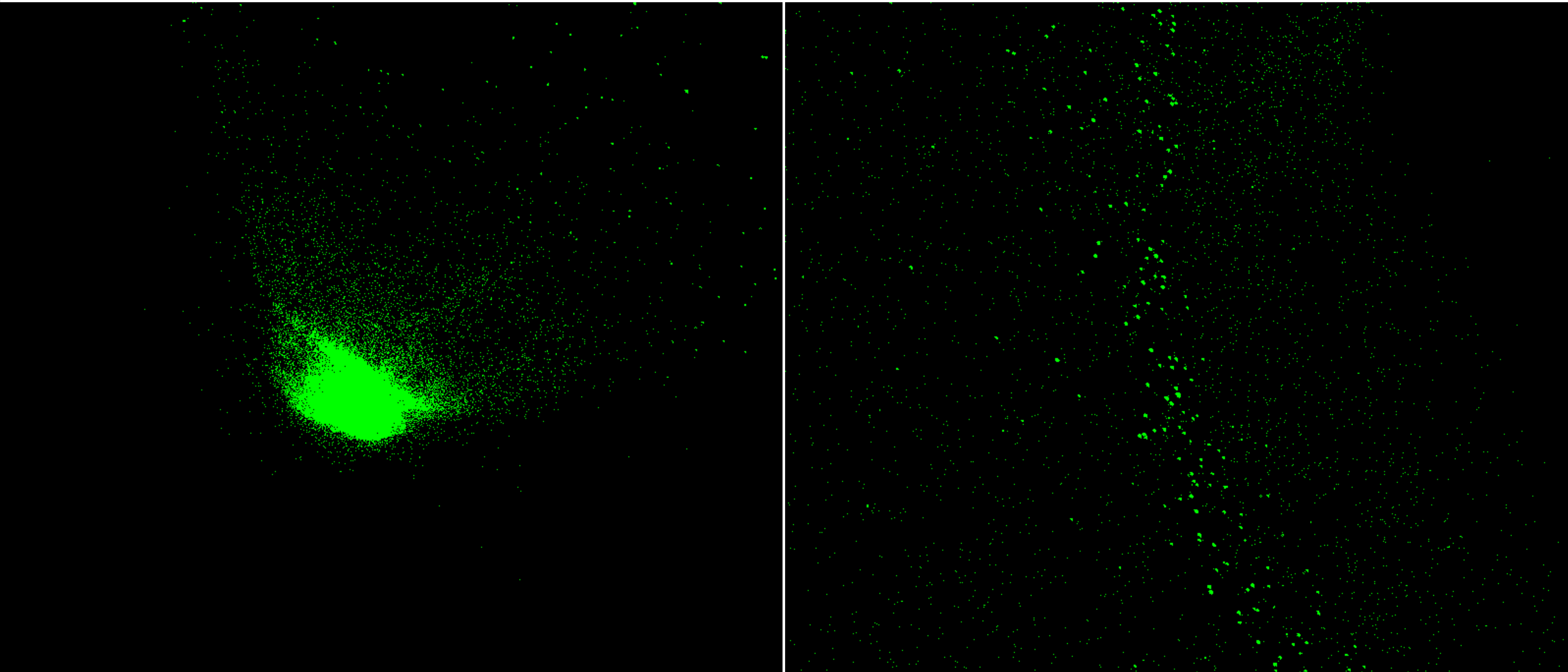
- hydrodynamic approach, SPH5 code (Benz & Asphaug 1994)

$D = 1$ km, $d = 0.074$ km, $v_{imp} = 5$ km/s, $\varphi_{imp} = 45°$, $Q/Q^*_D = 9.837$



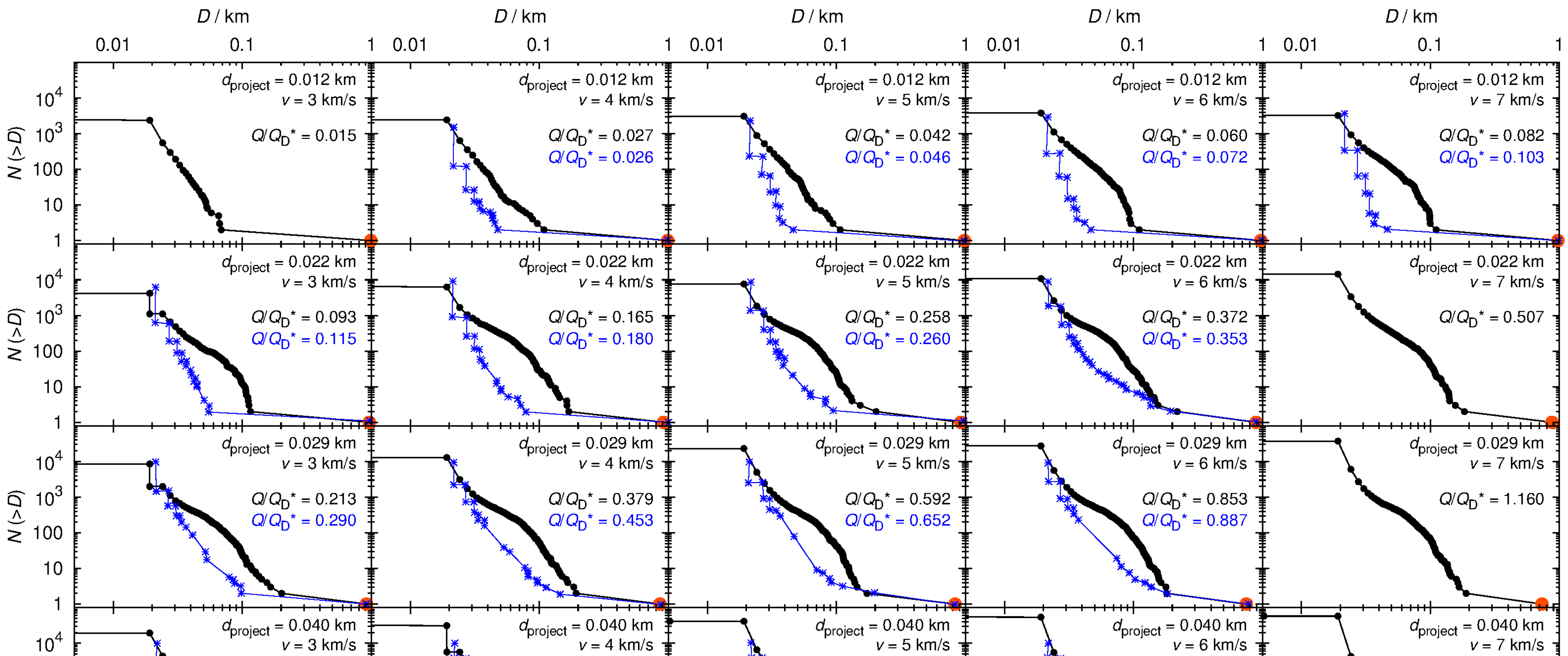
Reaccumulation phase

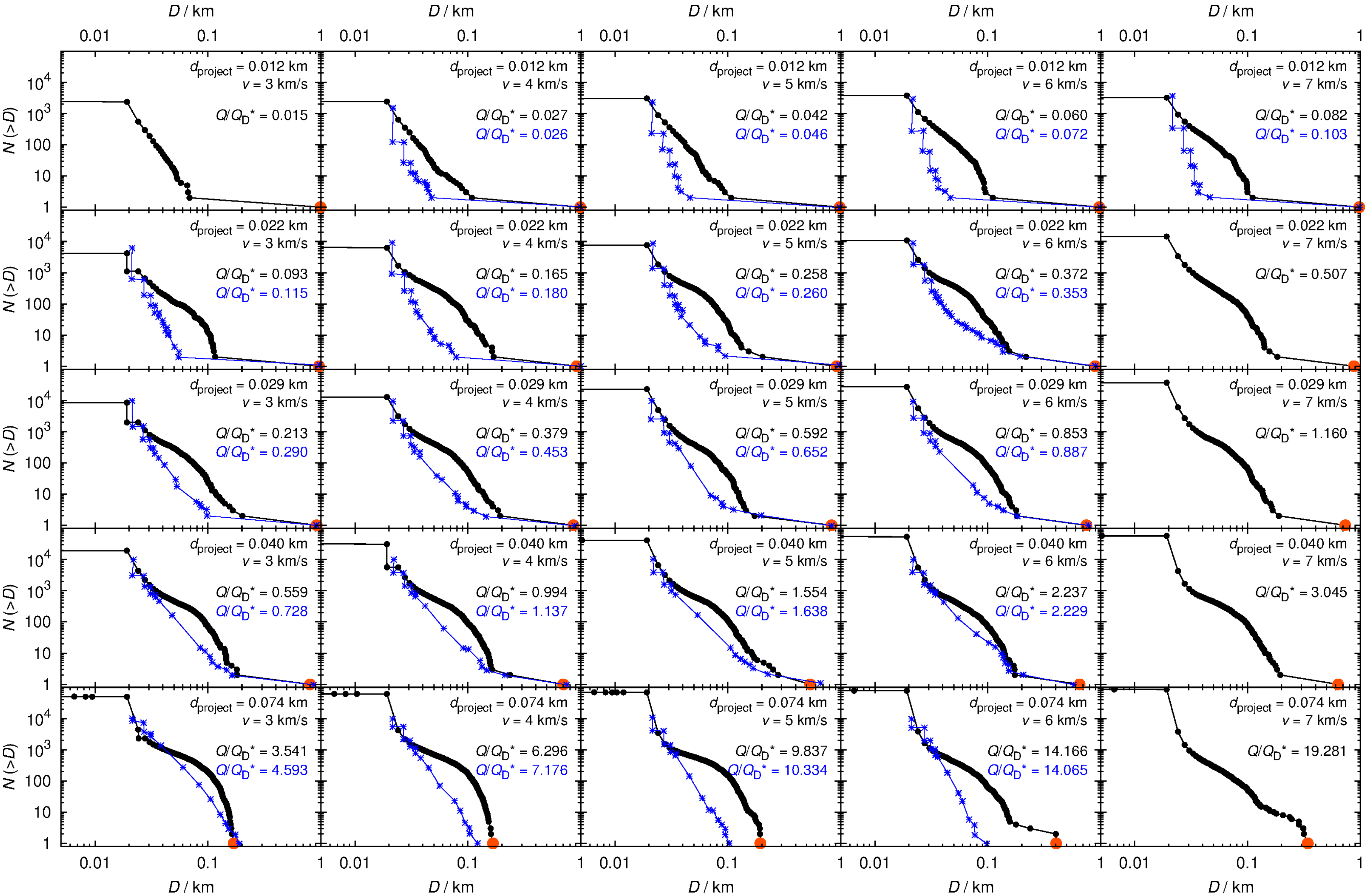
- *N*-body approach, *k*-d tree, only spheres, perfect merging, pkdgrav code (Richardson et al. 2009)



Results for D = 1 km targets

- $d_{project}$ = 0.012 to 0.074 m, v_{imp} = 3 to 7 km/s, φ_{imp} = 45°, size-frequency distributions and a comparison with D = 100 km targets
- substantial differences for large Q/Q^*_D (i.e. super-catastropnic)

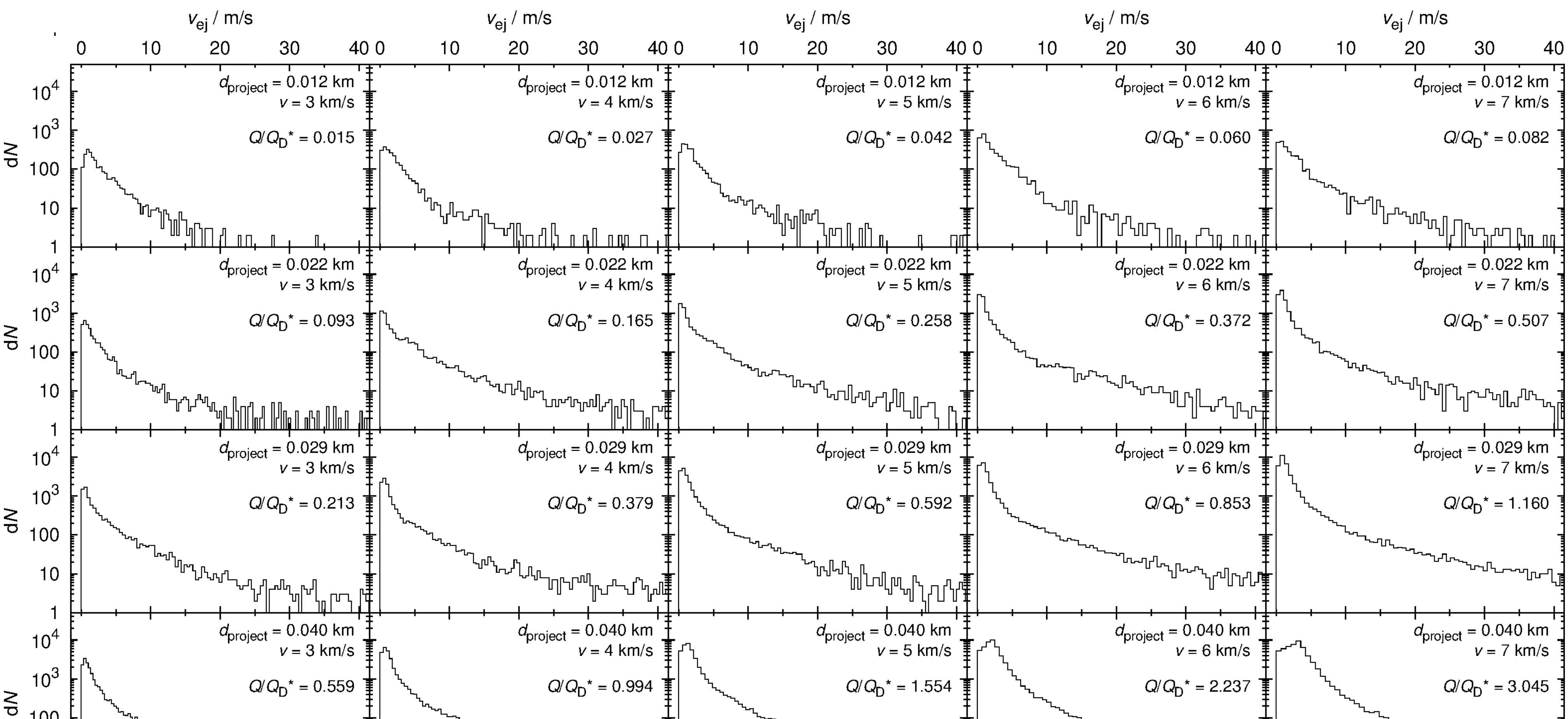


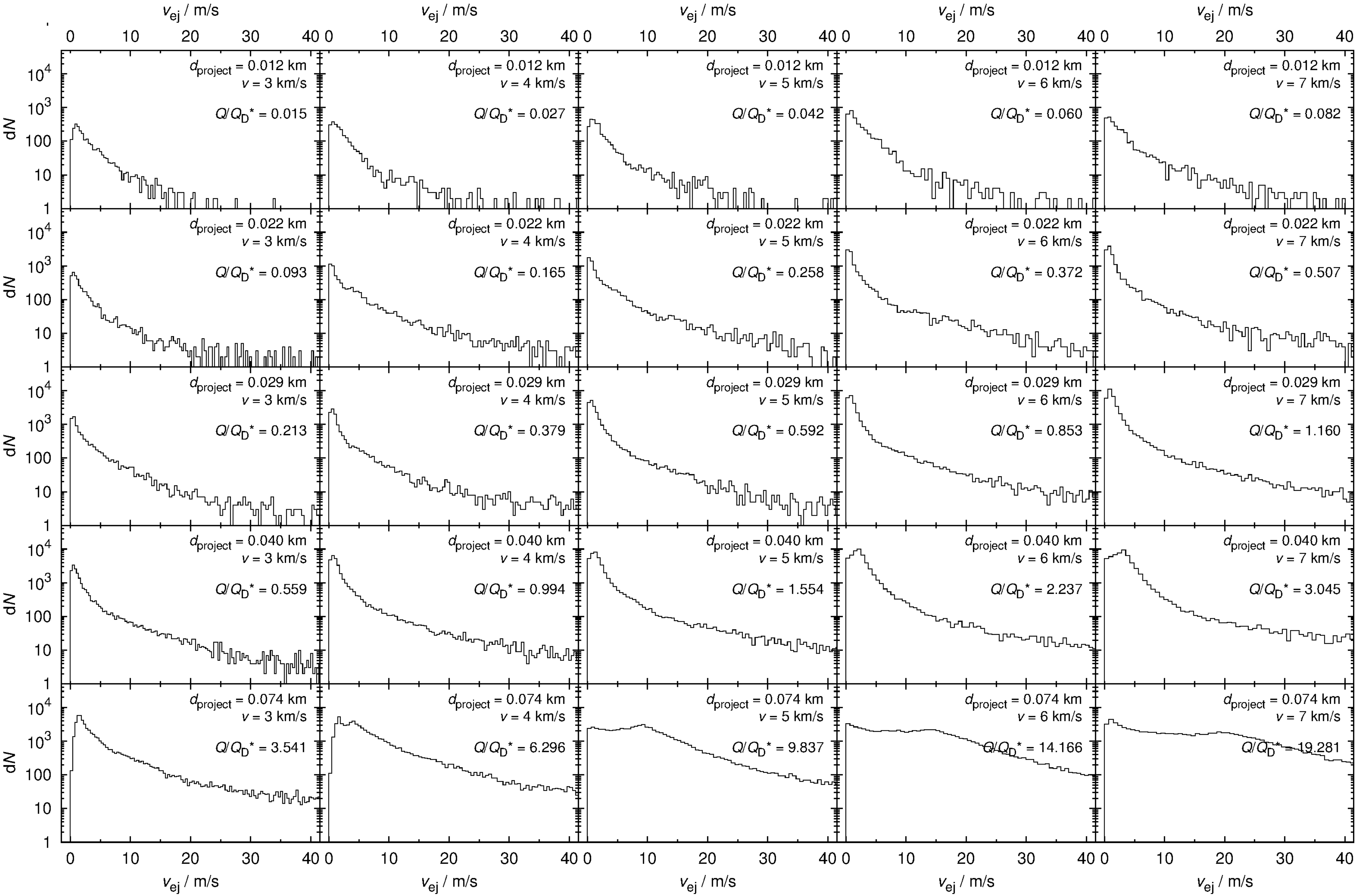


D / km
0.01
0.1
1
N (>D)
10^4
10^3
100
10
1
$d_{project}$ = 0.012 km
v = 3 km/s
Q/Q_D^* = 0.015
$d_{project}$ = 0.012 km
v = 4 km/s
Q/Q_D^* = 0.027
Q/Q_D^* = 0.026
$d_{project}$ = 0.012 km
v = 5 km/s
Q/Q_D^* = 0.042
Q/Q_D^* = 0.046
$d_{project}$ = 0.012 km
v = 6 km/s
Q/Q_D^* = 0.060
Q/Q_D^* = 0.072
$d_{project}$ = 0.012 km
v = 7 km/s
Q/Q_D^* = 0.082
Q/Q_D^* = 0.103
$d_{project}$ = 0.022 km
v = 3 km/s
Q/Q_D^* = 0.093
Q/Q_D^* = 0.115
$d_{project}$ = 0.022 km
v = 4 km/s
Q/Q_D^* = 0.165
Q/Q_D^* = 0.180
$d_{project}$ = 0.022 km
v = 5 km/s
Q/Q_D^* = 0.258
Q/Q_D^* = 0.260
$d_{project}$ = 0.022 km
v = 6 km/s
Q/Q_D^* = 0.372
Q/Q_D^* = 0.353
$d_{project}$ = 0.022 km
v = 7 km/s
Q/Q_D^* = 0.507
$d_{project}$ = 0.029 km
v = 3 km/s
Q/Q_D^* = 0.213
Q/Q_D^* = 0.290
$d_{project}$ = 0.029 km
v = 4 km/s
Q/Q_D^* = 0.379
Q/Q_D^* = 0.453
$d_{project}$ = 0.029 km
v = 5 km/s
Q/Q_D^* = 0.592
Q/Q_D^* = 0.652
$d_{project}$ = 0.029 km
v = 6 km/s
Q/Q_D^* = 0.853
Q/Q_D^* = 0.887
$d_{project}$ = 0.029 km
v = 7 km/s
Q/Q_D^* = 1.160
$d_{project}$ = 0.040 km
v = 3 km/s
Q/Q_D^* = 0.559
Q/Q_D^* = 0.728
$d_{project}$ = 0.040 km
v = 4 km/s
Q/Q_D^* = 0.994
Q/Q_D^* = 1.137
$d_{project}$ = 0.040 km
v = 5 km/s
Q/Q_D^* = 1.554
Q/Q_D^* = 1.638
$d_{project}$ = 0.040 km
v = 6 km/s
Q/Q_D^* = 2.237
Q/Q_D^* = 2.229
$d_{project}$ = 0.040 km
v = 7 km/s
Q/Q_D^* = 3.045
$d_{project}$ = 0.074 km
v = 3 km/s
Q/Q_D^* = 3.541
Q/Q_D^* = 4.593
$d_{project}$ = 0.074 km
v = 4 km/s
Q/Q_D^* = 6.296
Q/Q_D^* = 7.176
$d_{project}$ = 0.074 km
v = 5 km/s
Q/Q_D^* = 9.837
Q/Q_D^* = 10.334
$d_{project}$ = 0.074 km
v = 6 km/s
Q/Q_D^* = 14.166
Q/Q_D^* = 14.065
$d_{project}$ = 0.074 km
v = 7 km/s
Q/Q_D^* = 19.281

Results on velocity fields

- differential histograms, usual peak @ about v_{esc}
- additional `shift' for increasing Q/Q^*_D




v_{ej} / m/s
dN
$d_{project}$ = 0.012 km
v = 3 km/s
Q/Q_D^* = 0.015
$d_{project}$ = 0.012 km
v = 4 km/s
Q/Q_D^* = 0.027
$d_{project}$ = 0.012 km
v = 5 km/s
Q/Q_D^* = 0.042
$d_{project}$ = 0.012 km
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$d_{project}$ = 0.074 km
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Q/Q_D^* = 19.281

Uncertainties related to SPH

- material parameters (moduli, flaws)
- state equation, phase transitions (e.g. ANEOS, SESAME)
- chemical reactions (!) in gaseous phase
- *total* damage → dust clouds?
- bouncing and friction in reaccumulation phase
- no information on fragment shapes and rotation yet
- laboratory experiments, e.g. for icy projectiles

Livermore ↓