1 Geochemie a radiometrie

wolfram hafnium siderofilní litofilní

rezervoáry: 1 .. mlhovina (CAI), 2 .. plášť, 3 .. jádro

veličiny: N počet atomů M hmotnost C koncentrace $\gamma \equiv \frac{M_3}{M}$ hmotnostní poměr jádra $D \equiv \frac{C_3}{C_2}$ rozdělovací koeficient [1]rozhraní jádro/plášt (CMB)

1.1 Geochemické rovnice

transport stabilních izotopů zákon zachování wolframu Jacobsen (2009)

$$\dot{N}_1 + \dot{N}_2 + \dot{N}_3 = 0$$

$$\dot{N}_1 = -C_1 \dot{M} \tag{1}$$

$$\dot{N}_2 = -\dot{N}_1 - \dot{N}_3 = C_1 \dot{M} - DC_2 \dot{M}_3 = (C_1 - \gamma D C_2)\dot{M} \tag{2}$$

$$\dot{N}_3 = \gamma D C_2 \dot{M} \tag{3}$$

koncentrace

$$C_2 = \frac{N_2}{M_2}$$

$$\dot{C}_2 = \dot{N}_2 \frac{1}{M_2} - N_2 \frac{1}{M_2^2} \dot{M}_2 = \frac{C_1 - (\gamma D + 1 - \gamma) C_2}{(1-\gamma) M} \dot{M}$$

frakcionace Hf/W

$$f_2 \equiv \frac{\cancel{N_{\mathrm{Hf},2}}/N_{\mathrm{W},2}}{\cancel{N_{\mathrm{Hf},1}}/N_{\mathrm{W},1}} - 1 = \frac{N_{\mathrm{W},1}}{N_{\mathrm{W},2}} - 1 = \frac{N_{\mathrm{W},1} - N_{\mathrm{W},2}}{N_{\mathrm{W},2}} = \frac{N_{\mathrm{W},3}}{N_{\mathrm{W},2}} \left(= \frac{N_3}{N_2} \right)$$

neboť Hf je silně litofilní ($N_{\mathrm{Hf},3} \doteq 0$) stabilní izotopy ^{180}Hf, ^{183}W

$$\dot{f}_2 = \frac{\dot{N}_3}{N_2} - N_3 \frac{1}{N_2^2} \dot{N}_2 \,,$$

dle (2), (3):

$$\dot{f}_2 = \gamma D \cancel{C_2} \dot{M} \frac{1}{\cancel{C_2} M_2} - \frac{N_3}{N_2} \frac{1}{C_2 M_2} (C_1 - \gamma D C_2) \dot{M} = \frac{\gamma D - f_2 \left(\frac{C_1}{C_2} - \gamma D \right)}{(1-\gamma) M} \dot{M}$$

[1] je-li $D(t)$ a $\dot{M} > 0$, neplatí to neustále, ale jen pro $\dot{M}$

radiogenní izotop ^{182}W odchylka od mlhoviny (tzv. chonriditického uniformního rezervoáru; CHUR)

$$\varepsilon_2 \equiv 10^4 \left(\frac{N_{182\mathrm{W},2}/N_{183\mathrm{W},2}}{N_{182\mathrm{W},1}/N_{183\mathrm{W},1}} - 1 \right)$$

radioaktivní rozpady (s poločasy $T_{1/2} = 9\,\mathrm{Myr}$ a $144\,\mathrm{d}$):

$$^{182}\mathrm{Hf} \rightarrow {}^{182}\mathrm{Ta} + \mathrm{e}^+ + \nu$$
$$^{182}\mathrm{Ta} \rightarrow {}^{182}\mathrm{W} + \mathrm{e}^+ + \nu$$

rozpadový zákon

$$\dot{N}_{182\mathrm{Hf}} = -\lambda N_{182\mathrm{Hf}}$$
$$\dot{N}_{182\mathrm{W}} = +\lambda N_{182\mathrm{Hf}}$$
$$N_{182\mathrm{Hf}} = N_{182\mathrm{Hf}}(0)\,\mathrm{e}^{-\lambda t}$$
$$N_{182\mathrm{W}} = N_{182\mathrm{W}}(0) + N_{182\mathrm{Hf}}(0)(1-\mathrm{e}^{-\lambda t}) \doteq N_{182\mathrm{W}}(0)$$

jen několik ppm pak lze odvodit (Jacobsen 2005):

$$\dot{\varepsilon}_2 = Q^\star f_2 - \frac{C_1}{C_2(1-\gamma)M}\dot{M}\varepsilon_2$$

$$Q^\star = 10^4 \lambda\,\mathrm{e}^{-\lambda t}\,\frac{C_{182\mathrm{Hf},1}(0)}{C_{180\mathrm{Hf},1}(0)}\,\frac{C_{180\mathrm{Hf},1}(0)}{C_{182\mathrm{W},1}(t)}$$

$C_{182\mathrm{W},1}(t) \doteq C_{182\mathrm{W},1}(0)$; liší se jen o několik ε

$$\dot{N}_{182\mathrm{W},2} = +\lambda N_{182\mathrm{Hf},2} + (C_1 - \gamma D C_2)\dot{M}\,,$$
$$\dot{N}_{182\mathrm{W},1} = +\lambda N_{182\mathrm{Hf},1} - C_1\dot{M}\,,$$
$$\dot{N}_{183\mathrm{W},1} = -C_1\dot{M}\,,$$
$$\dot{N}_{183\mathrm{W},2} = (C_1 - \gamma D C_2)\dot{M}\,,$$

$$\dot{\varepsilon}_2 = 10^4 \overbrace{\frac{N_{182\mathrm{W},2}}{N_{182\mathrm{W},1}}\frac{N_{183\mathrm{W},1}}{N_{183\mathrm{W},2}}}^{\doteq 1} \left(\frac{\dot{N}_{182\mathrm{W},2}}{N_{182\mathrm{W},2}} - \frac{\dot{N}_{182\mathrm{W},1}}{N_{182\mathrm{W},1}} + \frac{\dot{N}_{183\mathrm{W},1}}{N_{183\mathrm{W},1}} - \frac{\dot{N}_{183\mathrm{W},2}}{N_{183\mathrm{W},2}} \right) =$$
$$= \ldots \left(\lambda\frac{N_{182\mathrm{Hf},2}}{N_{182\mathrm{W},2}} + \frac{C_{182\mathrm{W},1}\dot{M}}{N_{182\mathrm{W},2}} - \cancel{\frac{\gamma D C_{182\mathrm{W},2}\dot{M}}{N_{182\mathrm{W},2}}} - \lambda\frac{N_{182\mathrm{W},1}}{N_{182\mathrm{W},1}} + \cancel{\frac{C_{182\mathrm{W},1}\dot{M}}{N_{182\mathrm{W},1}}} - \right.$$
$$\left. - \cancel{\frac{C_{183\mathrm{W},1}\dot{M}}{N_{183\mathrm{W},1}}} - \frac{C_{183\mathrm{W},1}\dot{M}}{N_{183\mathrm{W},2}} + \cancel{\frac{\gamma D C_{183\mathrm{W},2}\dot{M}}{N_{183\mathrm{W},2}}} \right) =$$
$$= \ldots\lambda \left(\frac{N_{182\mathrm{Hf},2}}{N_{182\mathrm{W},2}} - \frac{N_{182\mathrm{W},1}}{N_{182\mathrm{W},1}} \right) + \ldots\dot{M} \left(\frac{C_{182\mathrm{W},1}}{N_{182\mathrm{W},2}} - \frac{C_{183\mathrm{W},1}}{N_{183\mathrm{W},2}} \right) =$$
$$= 10^4\lambda\frac{C_{182\mathrm{Hf},1}(0)\,\mathrm{e}^{-\lambda t}}{C_{182\mathrm{W},1}(t)}\,\frac{C_{180\mathrm{Hf},1}}{C_{180\mathrm{Hf},1}} \left(\frac{N_{182\mathrm{Hf},2}}{N_{182\mathrm{W},2}}\frac{N_{182\mathrm{W},1}}{N_{182\mathrm{Hf},1}} - 1 \right) +$$
$$+ 10^4\frac{C_{183\mathrm{W},1}}{C_{183\mathrm{W},2}}\,\frac{1}{(1-\gamma)M}\dot{M} \left(\frac{C_{182\mathrm{W},1}}{C_{182\mathrm{W},2}}\frac{C_{183\mathrm{W},2}}{C_{183\mathrm{W},1}} - 1 \right) =$$
$$= Q^\star f_2 + \frac{C_1}{C_2(1-\gamma)M}\dot{M}\varepsilon_2 \qquad \text{c.b.d.}$$

počáteční podmínky, rovnováha

$$C_2 = \frac{C_1}{\gamma D + 1 - \gamma}$$
$$f_2 = \frac{\gamma D}{1 - \gamma}$$
$$\varepsilon_2 = 0$$

1.2 Diferenciace Země

1.3 Měsíční impakt

definice

$$a \equiv \frac{1}{1-\gamma}\frac{C_1}{C_2} = \frac{\gamma D + 1 - \gamma}{1-\gamma} = \frac{\gamma D}{1-\gamma} + 1 = f_2 + 1$$

za prvé vezměme ε_2, bez radioaktivity

$$\dot{\varepsilon}_2 \doteq -a\frac{\dot{M}}{M}\varepsilon_2$$

$$\frac{\dot{\varepsilon}_2}{\varepsilon_2} = -a\frac{\dot{M}}{M}$$

neurčitý integrál

$$\ln \varepsilon_2 = -a \ln M + C$$

$$\varepsilon_2 = C' M^{-a}$$

konstanta před impaktem $C' = \varepsilon_2 M^a$ čili po impaktu

$$\varepsilon_2' = \varepsilon_2 \left(\frac{M}{M'}\right)^a$$

obdobně

$$\dot{C}_2 = \left(\frac{C_1}{1-\gamma} - aC_2\right)\frac{\dot{M}}{M} \tag{4}$$

partikulární řešení

$$\dot{C}_2 = -aC_2\frac{\dot{M}}{M}$$

totéž jako ε

$$C_2 = C' M^{-a}$$

variace konstant

$$C_2 = C'(M) M^{-a}$$

$$\frac{\mathrm{d}C_2}{\mathrm{d}M} = \frac{\mathrm{d}C'}{\mathrm{d}M} M^{-a} + C'(-a) M^{-a-1} = \left(\frac{C_1}{1-\gamma} - aC' M^{-1} \right) M^{-1}$$

kam jsme dosadili (4)

$$\frac{\mathrm{d}C'}{\mathrm{d}M} = \frac{C_1}{1-\gamma} M^{a-1}$$

$$C'(M) = \frac{C_1}{1-\gamma} \frac{M^a}{a} + C''$$

kde C'' je druhá integrační konstanta řešení

$$C_2 = \left(\frac{C_1}{1-\gamma} \frac{M^a}{a} + C'' \right) M^{-a}$$

má-li pro M, vyjít právě C_2, pak:

$$C'' = C_2 M^a - \frac{C_1}{1-\gamma} \frac{M^a}{a}$$

$$C_2' = \left(\frac{C_1}{1-\gamma} \frac{M'^a}{a} + C_2 M^a - \frac{C_1}{1-\gamma} \frac{M^a}{a} \right) M'^{-a}$$

$$C_2' = \frac{C_1}{1-\gamma} \frac{1}{a} \left[1 - \left(\frac{M}{M'} \right)^a \right] + C_2 \left(\frac{M}{M'} \right)^a$$

obdobně obdobně

$$\dot{f}_2 = \frac{\gamma D - f_2 \left(\frac{C_1}{C_2} - \gamma D \right)}{(1-\gamma) M} \dot{M}$$

partikulárně

$$\frac{\dot{f}_2}{f_2} = -b \frac{\dot{M}}{M}$$

kde $b = (\frac{C_1}{C_2} - \gamma D)/(1-\gamma)$

$$f_2 = C' M^{-b}$$

variabilně

$$f_2 = C'(M)M^{-b}$$

$$\frac{\mathrm{d}f_2}{\mathrm{d}M} = \frac{\mathrm{d}C'}{\mathrm{d}M}M^{-b} - \cancel{C'bM^{-b-1}} = -\cancel{f_2bM^{-1}} + \frac{\gamma D}{1-\gamma}M^{-1}$$

$$\frac{\mathrm{d}C'}{\mathrm{d}M} = \frac{\gamma D}{1-\gamma}M^{b-1}$$

$$C' = \frac{\gamma D}{1-\gamma}\frac{M^b}{b}$$

$$f_2 = \frac{\gamma D}{1-\gamma}\frac{M^b}{b}M^{-b} = \frac{\gamma D}{\cancel{1-\gamma}}\frac{\cancel{1-\gamma}}{\frac{C_1}{C_2} - \gamma D} = \frac{\gamma D}{\frac{C_1}{C_2} - \gamma D} = f_2$$

což nezávisí na M, čili $f_2' = f_2$

1.4 Variabilní rozdělování

rozdělovací koeficient $D \neq$ konst.

fugacita $f_{\mathrm{O}2}$

1.5 Akrece dle N-částicových modelů

1.6 Systém U/Pb

1.7 Systém D/H

[1] Jacobsen, 2005.
[2] Rudge , 2010.
[3] Yu, Jacobsen, 2011.