ICARUS 47, 1–35 (1981)

The Tides of Io

CHARLES F. YODER

Jet Propulsion Laboratory, Pasadena, California 91103

AND

STANTON J. PEALE[1]

Joint Institute of Laboratory Astrophysics, University of Colorado and National Bureau of Standards, Boulder, Colorado 80309

Received August 29, 1980; revised June 16, 1981

The Galilean satellites Io, Europa, and Ganymede interact through several stable orbital resonances where $\lambda_1 - 2\lambda_2 + \tilde{\omega}_1 = 0$, $\lambda_1 - 2\lambda_2 + \tilde{\omega}_2 = 180°$, $\lambda_2 - 2\lambda_3 + \tilde{\omega}_2 = 0$ and $\lambda_1 - 3\lambda_2 + 2\lambda_3 = 180°$, with λ_i being the mean longitude of the ith satellite and $\tilde{\omega}_i$ the longitude of the pericenter. The last relation involving all three bodies is known as the Laplace relation. A theory of origin and subsequent evolution of these resonances outlined earlier (C. F. Yoder, 1979b, *Nature* **279**, 747–770) is described in detail. From an initially quasi-random distribution of the orbits the resonances are assembled through differential tidal expansion of the orbits. Io is driven out most rapidly and the first two resonance variables above are captured into libration about 0 and 180° respectively with unit probability. The orbits of Io and Europa expand together maintaining the 2:1 orbital commensurability and Europa's mean angular velocity approaches a value which is twice that of Ganymede. The third resonance variable and simultaneously the Laplace angle are captured into libration with probability ~0.9. The tidal dissipation in Io is vital for the rapid damping of the libration amplitudes and for the establishment of a quasi-stationary orbital configuration. Here the eccentricity of Io's orbit is determined by a balance between the effects of tidal dissipation in Io and that in Jupiter, and its measured value leads to the relation $k_1 f_1/Q_1 \approx 900 k_J/Q_J$ with the k's being Love numbers, the Q's dissipation factors, and f a factor to account for a molten core in Io. This relation and an upper bound on Q_1 deduced from Io's observed thermal activity establishes the bounds $6 \times 10^4 < Q_J < 2 \times 10^6$, where the lower bound follows from the limited expansion of the satellite orbits. The damping time for the Laplace libration and therefore a minimum lifetime of the resonance is $1600 Q_J$ years. Passage of the system through nearby three-body resonances excites free eccentricities. The remnant free eccentricity of Europa leads to the relation $Q_2/f_2 \gtrsim 2 \times 10^{-4}$ Q_J for rigidity $\mu_2 = 5 \times 10^{11}$ dynes/cm². Probable capture into any of several stable 3:1 two-body resonances implies that the ratio of the orbital mean motions of any adjacent pair of satellites was never this large.

A generalized Hamiltonian theory of the resonances in which third-order terms in eccentricity are retained is developed to evaluate the hypothesis that the resonances were of primordial origin. The Laplace relation is unstable for values of Io's eccentricity $e_1 > 0.012$ showing that the theory which retains only the linear terms in e_1 is not valid for values of e_1 larger than about twice the current value. Processes by which the resonances can be established at the time of satellite formation are undefined, but even if primordial formation is conjectured, the bounds established above for Q_J cannot be relaxed. Electromagnetic torques on Io are also not sufficient to relax the bounds on Q_J. Some ideas on processes for the dissipation of tidal energy in Jupiter yield values of Q_J within the dynamical bounds, but no theory has produced a Q_J small enough to be compatible with the measurements of heat flow from Io given the above relation between Q_1 and Q_J. Tentative observational bounds on the secular acceleration of Io's mean motion are also shown not to be consistent with such low values of Q_J. Io's heat flow may therefore be episodic. Q_J may actually be determined from improved analysis of 300 years of eclipse data.

[1] Permanent address: Department of Physics, University of California, Santa Barbara, Calif. 93106.

0019-1035/81/070001-35$02.00/0

1. INTRODUCTION

The Galilean satellites Io (1), Europa (2), and Ganymede (3) interact through several orbital resonance locks. The most well known lock involves the mean motions n_i of the three satellites which satisfy the relation $\dot{\phi} = n_1 - 3n_2 + 2n_3 = 0$ in a time-average sense to less than one part in 10^8. (We have listed all notation in an Appendix for easy reference.) The corresponding relation among the mean longitudes λ_i is $\phi = \lambda_1 - 3\lambda_2 + 2\lambda_3 \simeq 180°$, with a libration amplitude $\phi_m = 0.066 \pm 0.013°$ and a period of 2074 days (Lieske, 1980). The other sets of resonance variables involve the 2:1 near commensurabilities of the mean motions of the inner pair and outer pair respectively and the forced precession of the longitudes of pericenter, $\tilde{\omega}_1$ and $\tilde{\omega}_2$. The mean daily motion of $\tilde{\omega}_1$ and $\tilde{\omega}_2$ is $-0.7395°$ day^{-1}. Sinclair (1975) and Chao (1976) noted that the arguments $\lambda_1 - 2\lambda_2 + \tilde{\omega}_1$, $\lambda_1 - 2\lambda_2 + \tilde{\omega}_2$ and $\lambda_2 - 2\lambda_3 + \tilde{\omega}_2$ execute small-amplitude librations about either 0 or 180°. In fact, it is just the interaction of the last pair of variables which predominantly controls the three-body couple.

The stability of these resonances has been understood since the time of Laplace (1805), but the establishment of the resonances and the efficient damping of the libration amplitudes has only recently been unraveled. Sinclair (1975) investigated the reduction of the amplitude of the Laplace libration from the effect of differential tidal expansion of the satellite orbits increasing the magnitude of the restoring acceleration in the pendulum equation describing the libration. The libration action is adiabatically conserved in such an evolution and the amplitude could not be reduced to less than 30° over the age of the solar system. This cast doubt on the capability of tidal torques from Jupiter establishing the orbital resonances. However, Yoder (1979b) showed how the relative expansion of the orbits by tidal torques from Jupiter together with dissipation in Io (Peale *et al.*, 1979) led to a self-consistent explanation of the origin of the various resonance locks and their subsequent evaluation to the present configuration (also see Lin and Papaloizou, 1979).

This article describes in more detail in Section 4 the dynamical model outlined in Yoder's (1979b) paper after preliminary development of the effects of tidal dissipation in Section 2 and a more comprehensive Hamiltonian theory of the resonance interactions in Section 3. The latter theory allows the systematic incorporation of terms up to third order in eccentricity, which must be included if eccentricities larger than twice the current values are considered. The principal results of the earlier paper confirmed in Section 4 depend on tidal torques from Jupiter being sufficient to expand the satellites by more than a few percent over the age of the solar system. The amplitude of the Laplace libration of $0\overset{\circ}{.}066$ may in fact be forced or may be the result of a recent asteroidal impact on one of the satellites. If this amplitude is not a remnant of the damping process, the resonance lifetime of $1600Q_J$ years is only a lower bound.

In Section 5 we investigate the effects of passage through other orbital resonances. Probable capture into any of several 3:1 two-body commensurabilities implies that the ratio of the mean motions for any adjacent pairs of satellites was never as low as 3:1. We also show that the system passed through a series of three-body resonances just prior to its capture into the Laplace resonance which, although tidally unstable, excite significant free eccentricities. Interpreting the existing free eccentricity of Europa as the remnant of tidal damping from the time of the establishment of the Laplace relation leads to a relation between Q_2 and Q_J consistent with that between Q_1 and Q_J and the Laplace libration of $0\overset{\circ}{.}066$ (Lieske, 1980) being a remnant of the tidal damping.

Analysis of the hypothesis that the Laplace relation is really primoridal is shown in Section 6 not to relax the bounds on Q_J established by the hypothesis that the resonances were assembled by differential tidal expansion of the orbits. This result agrees with that of Peale and Greenberg (1980), although this latter work used a theory linear in e, which is not valid for values of e larger than twice the current values. Stability of the resonant configuration as a function of the magnitude of the forced eccentricities is determined here based on the generalized third-order theory developed in Section 3. The effects of tidal dissipation are incorporated into the higher-order theory to yield expressions for the tidal evolution of the system which are valid for large eccentricity and which reduce to those of Section 4 when eccentricities approach current values.

Because several theoretical analyses of dissipation in Jupiter yield values of Q_J which exceed the upper bound established by the dynamics, we outline the work in this area in Section 7. Some recent ideas are consistent with the dynamical bounds $6 \times 10^4 \lesssim Q_J \lesssim 2 \times 10^6$, although the relatively low values of Q_J so derived are not very secure. None of these theories can yield a value of Q_J sufficiently small to be consistent with the high heat flux estimates for Io by Matson *et al.* (1981), Sinton (1981), and Morrison and Telesco (1980), where Q_J is related to Q_I through a balance of effects on Io's eccentricity.

The possibility of an observational determination of Q_J and thereby whether or not the current high heat flux is episodic is evaluated in Section 8. Estimates of upper bounds on the secular acceleration of Io's mean motion (Goldstein, 1975) are not secure but so far appear inconsistent with a Q_J sufficiently low to accommodate the heat flow measurements for Io. Improvements in the determination of Io's mean motion through analysis of a more complete set of 17th century eclipse observations are feasible (Lieske, private communication, 1980).

2. CONSEQUENCES OF TIDAL INTERACTION

Tidal dissipation in a satellite in an eccentric orbit which is rotating synchronously with its mean orbital angular velocity arises from two effects. The magnitude of the tide varies as the distance of the satellite from its primary changes, which is referred to as the radial tide, and the tidal bulge oscillates back and forth across the mean subprimary point during each orbit period since the rotation is uniform but the orbital motion is not. The second effect actually results in somewhat more dissipation. At pericenter, where dissipation and the resultant torque are greatest, the orbital angular motion is $n(1 + 2e)$. As seen from the planet, the satellite seems to rotate retrograde with angular velocity $\sim -2en$. Dissi-

TABLE I

ORBITAL PARAMETERS FOR THE GALILEAN SATELLITES

	Io	Europa	Ganymede	Callisto
$M/M_J \times 10^5$	4.684 ± 0.022	2.523 ± 0.025	7.803 ± 0.030	5.661 ± 0.019
n(°/day)	203.4890	101.1747	50.3176	21.5711
$\dot{\omega}_s$(°/day)	0.161	0.048	0.007	0.002
$\dot{\Omega}_s$(°/day)	−0.134	−0.033	0.007	−0.002
a(km)	422,000	671,400	1,071,000	1,884,000
e_{forced} (2:1)	0.0041	0.0101	0.0006	
e_{free}	$1 \pm 2 \times 10^{-5}$	$9.2 \pm 1.9 \times 10^{-5}$	0.0015	0.0073
$\sin I_{\mathrm{free}}$	$7.0 \pm 1.9 \times 10^{-4}$	0.0082	0.0034	0.0049
R(km)	1816 ± 5	1569 ± 10	2631 ± 10	2400 ± 10
ρ(g/cm³)	3.53	3.03	1.93	1.79

pation in the satellite causes a time delay ($\simeq 1/Qn$) of high tide leading to a displaced tidal bulge, which leads the instantaneous subplanet point. The torque by the planet acts to spin up the satellite but is prevented from doing so because of the existence of a permanent bulge. The effect of dissipation is to displace the rigid bulge until the resulting torque by the planet exactly cancels the average tidal torque (cf. Goldreich and Peale, 1966).

Table I contains some of the pertinent physical data used in the calculations herein: mass M (Null, 1976), radius R (Davies and Katayama, 1980), mean density ρ, semimajor axis a, mean daily motion n, both free and forced eccentricities e, and the free inclination I of the orbit with respect to the Jovian equator (Lieske, 1980). The free or proper eccentricity represents that eccentricity which would remain if we could slowly remove all perturbing forces. The libration amplitude of the 2:1 resonance (in radians) is equal to the ratio of the free to the forced eccentricity.

Since the rotational energy of a synchronously rotating satellite cannot be diminished, the tidal energy dissipated because of a nonzero eccentricity must come from its orbit. This rate of change of the orbital energy $E(= -\frac{1}{2}Mn^2a^2$, where M is the satellite mass) is given by the following formula

$$dE/dt = -(21/2)(kf/Q)M_J n^3 a^2 (R/a)^5 e^2 \quad (1)$$

to lowest order in e^2 (cf. Cassen *et al.*, 1979a). The radial component of the tide accounts for only 3/7 of the total dissipation. For small homogeneous satellites the Love number $k \simeq 3\rho g R/(19\mu)$, where ρ is the density, g the surface gravity, and μ the rigidity. The value of the rigidity μ for the determination of k is probably in the range 2×10^{11}–6×10^{11} dynes/cm^2. The low end of the range corresponds to the value for rocks typical of the Earth's crust and the deep lunar interior; the high end corresponds to values of the Earth's rigidity equivalent to 60 kbar pressure and to values appropriate to the outer layers of the Moon (Bullen, 1975; Nakamura *et al.*, 1976; Cheng and Toksoz, 1978). For this range of μ, $0.025 < k_1 < 0.088$. We shall choose a nominal value of $\mu = 5 \times 10^{11}$ dynes/cm^2 with $k = 0.035$ for the calculations herein. If the satellite has a liquid core but with properties otherwise identical to those of the homogeneous, solid satellite, the dissipation per unit volume in the solid mantle is higher because of the greater deformation under a given tidal potential. This increase in volumetric dissipation can dominate the reduction in the volume of the mantle as the (nondissipative) core increases in size leading to an increase in the total dissipation by a factor which Peale *et al.* (1979) have called f. Generally $f = 1$ for a completely solid satellite, increases to a maximum as the core radius increases and drops to 0 in the limit of a completely molten body. For Io, $f \sim 10$ at a core radius of about $0.95\,R$. The factor f also depends on the parameter $\mu/\rho g R$ as well as the core radius and decreases with this parameter for a fixed core radius (Cassen *et al.*, 1980). We find

$$dE_1/dt = 2.17 \times 10^{21}\,(f_1/Q_1)\ \text{ergs/sec} \quad (2)$$

for Io. If the volumetric radiogenic heating of Io is comparable to that of the moon (Cassen *et al.*, 1979b), tidal heating exceeds the radiogenic heating rate of 6×10^{18} ergs/sec if $Q_1/f_1 \lesssim 370$. The rate of change of the mean motion n due to tidal dissipation in the satellite can be obtained from (2) using Kepler's third law $G(M_J + M) = n^2a^3$, where M_J is Jupiter's mass and the expression for orbital energy $(-GM_JM/2a)$. The corresponding rate of change of the eccentricity is found by relating de/dt to da/dt through the conserved orbital angular momentum,

$$\mathscr{L} = M[G(M_J + M)a(1 - e^2)]^{1/2}.$$

From these considerations, the equations describing the evolution of the satellite orbit due to both the inelastic planetary and satellite tides are (Kaula, 1964)

$$\frac{1}{n}\frac{dn}{dt_T} = -c(1 - (7D - 12.75)e^2), \quad (3)$$

$$\frac{de^2}{dt_T} = -\tfrac{2}{3}c(7D - 4.75)e^2, \quad (4)$$

$$D = \frac{k}{k_J}\left(\frac{R}{R_J}\right)^5\left(\frac{M_J}{M}\right)^2\frac{Q_J}{Q}f, \quad (5)$$

$$c = \tfrac{9}{2}\frac{k_J}{Q_J}\frac{M}{M_J}\left(\frac{R}{a}\right)^5 n. \quad (6)$$

The Jovian whole-body $k_J = 0.38$ (Gavrilov and Zharkov, 1977), although different values have been adopted by other investigators. Note that dissipation in Jupiter tends to increase e, whereas dissipation in the satellite tends to reduce e. We find $c_1 = 4.6 \times 10^{-13} Q_J^{-1}$ sec^{-1}, while $c_2 \simeq 0.026\, c_1$ if there is no marked change in Q_J with the change in either tidal frequency or amplitude. Thus the Jovian tidal torque acting on Europa is negligible compared to that acting on Io. We should perhaps point out here that the tide raised on Jupiter by Io can cause a secular acceleration of Europa proportional to e_1 because of their nearly commensurate mean motions. Io similarly responds to the tide raised by Europa. However, these accelerations are $\sim 10^{-3}$–10^{-4} smaller than the principal tidal acceleration.

The parameter D is the ratio of the tidal scale factor of a satellite to that of Jupiter. We find $D_1 \simeq 2000$ if we adopt "plausible" values for $Q_J = 4 \times 10^5$ and $Q_1 = 100$ and $f_1 = 1$. We shall argue later that $D_1 \simeq 4200$–4600. The Jovian contribution to the tidal equations (3, 4) proportional to e^2 have been calculated assuming that Q_J is independent of frequency. Although this premise is unlikely to hold true, we find that the Jovian e^2 contribution is negligible compared to the effect of the satellite tide and shall hereafter be ignored. The frequency dependence of the satellite Q factors is not critical, since the most important terms in the expression for the dissipation all have periods equal to the orbit period.

3. DYNAMICAL EQUATIONS

The perturbed orbital motion of satellites about their primaries is usually described in terms of a variation of the Keplerian ellipse involving the set of variables $\{a, e, I, \lambda, \tilde{\omega}$, and $\Omega\}$. Here Ω refers to the ascending node of the orbit on some reference plane (the primary equator for close satellites). In the strictly two body problem the mean longitude λ is a linear function of time and is related to the true anomaly v by $\lambda \simeq v + \tilde{\omega} + 2e \sin v$ if e is small. If the gravitationally interacting satellites are well spaced and have relatively small masses compared to their primary, then the major variations in the perturbed orbit can be expected to occur on time scales which are long compared to their orbital periods if not small in magnitude.

The simplest form of the dynamical equations involves the canonical set $\{L, \lambda; \Gamma, \tilde{\omega}; Z, \Omega\}$

$$L = M(\mu a)^{1/2}; \qquad \mu = G(M_J + M)$$

$$\Gamma = L[(1 - e^2)^{1/2} - 1]$$

$$Z = L(1 - e^2)^{1/2}(\cos I - 1), \quad (7)$$

where L contains a factor M not usually included in the definition in order to simplify the following development (cf. Hagihara, 1972).

The equations of motion are expressed in terms of the partial derivatives of a disturbing function R.

$$dL/dt = \partial R/\partial\lambda; \qquad d\lambda/dt = n - \partial R/\partial L$$

$$d\Gamma/dt = \partial R/\partial\tilde{\omega}; \qquad d\tilde{\omega}/dt = -\partial R/\partial\Gamma$$

$$dZ/dt = \partial R/\partial\Omega; \qquad d\Omega/dt = -\partial R/\partial Z \quad (8)$$

The dynamic response of an *inner* satellite (subscript "1") to the gravitational action of an *outer* satellite ("2") is obtained from the following disturbing function

$$R_2^1 = GM_1M_2(\Delta^{-1} - \vec{r}_1 \cdot \vec{r}_2 r_2^{-3}), \quad (9)$$

where $\Delta = |\vec{r}_1 - \vec{r}_2|$ is the distance of separation. Both masses have been included in (9) such that R_2^1 has the dimensions of energy consistent with our definition of L in (7). The terms in R proportional to Δ^{-1} and $\vec{r}_1 \cdot \vec{r}_2 r_2^{-3}$ are known as the direct and indirect parts, respectively. The direct part of R corresponds to the negative of the potential energy. The indirect part results from the transformation from an inertial to a planet-centered reference frame. This transformation is required since the Keplerian variables are defined with respect to this frame. The disadvantage of this formulation is that the "outer" disturbing function R_1^2, obtained by interchanging the subscripts in (9), does *not* equal R_2^1.

The expansion of R is formally

$$R = (GM_1M_2/a_2) \sum e_1^n e_2^m C^j_{nmhk}(\alpha) \times \cos[j(\lambda_1 - \lambda_2) + h(\lambda_1 - \tilde{\omega}_1) + k(\lambda_2 - \tilde{\omega}_2)] \quad (10)$$

for noncrossing coplanar orbits. We shall show later that the restriction to coplanar orbits is a good approximation. Also, $n \geq |h|$ and $m \geq |k|$. The coefficient $C^j_{nmhk}(\alpha)$ is a sum of *Laplace coefficients* $b_{1/2}{}^j(\alpha)$, and their derivatives, and the $b_{1/2}{}^j(\alpha)$ are in turn an infinite series in $\alpha = a_1/a_2$ (cf. Brouwer and Clemence, 1961; Newcomb, 1895).

The inner and outer coefficients $C^1_{010-1}(\alpha)$, corresponding to the cosine argument $\lambda_1 - 2\lambda_2 + \tilde{\omega}_2$, are not equal in general because of the unequal contribution from the indirect part. For example,

$$C^1_{010-1}(\alpha) = \tfrac{1}{2}\left(3 + \alpha \frac{d}{d\alpha}\right) b_{1/2}{}^1(\alpha) - 1/2\alpha^{-2} \quad \text{(Outer)},$$

$$C^1_{010-1}(\alpha) = \tfrac{1}{2}\left(3 + \alpha \frac{d}{d\alpha}\right) b_{1/2}{}^1(\alpha) - 2\alpha \quad \text{(Inner)}. \quad (11)$$

The numerical values of these coefficients are equal only if $\alpha = (0.5)^{2/3} \simeq 0.63$. The observed ratio is slightly smaller since the commensurability is not exact. We shall ignore this small difference and evaluate $C^j_{nmhk}(\alpha)$ at $\alpha = 0.63$. Given these approximations we find $R_1^2 = R_2^1$ if we restrict R to those terms which depend on the 2:1 commensurable argument $V_1 = \lambda_1 - 2\lambda_2$. We show in Section 6 that variations involving $\partial R/\partial a$ are relatively small, and the fact that $\partial R_1^2/\partial a_i \neq \partial R_2^1/\partial a_i$ is unimportant since these derivatives are never used. This allows us to display only the e_i and the angle variables explicitly in the following expressions for $R_1^2 = R_2^1$ through third order in the eccentricities, and the variations of both the inner and outer satellites are derivable from the same disturbing function.

$$\begin{aligned} R = (GM_1M_2/a_2)\{&(-1.19 - 0.20e_1^2 + 0.87e_2^2)e_1 \cos \phi_1 \\ &+ (0.43 + 2.20e_1^2 + 1.17e_2^2)e_2 \cos \phi_2 - 0.58e_1e_2 \cos(\phi_1 - \phi_2) \\ &+ 1.70e_1^2 \cos 2\phi_1 - 4.97e_1e_2 \cos(\phi_1 + \phi_2) + 3.59e_2^2 \cos 2\phi_2 \\ &- 1.65e_1e_2^2 \cos(2\phi_2 - \phi_1) - 0.75e_1^2e_2 \cos(2\phi_1 - \phi_2) \\ &- 2.97e_1^3 \cos 3\phi_1 + 13.13e_1^2e_2 \cos(2\phi_1 + \phi_2) \\ &- 19.25e_1e_2^2 \cos(\phi_1 + 2\phi_2) + 9.35e_2^3 \cos 3\phi_2\}. \end{aligned} \quad (12)$$

Here $\phi_1 = V_1 + \tilde{\omega}_1$ and $\phi_2 = V_1 + \tilde{\omega}_2$. We shall also ignore the direct gravitational interaction between Io and Ganymede. The 4:1 interaction is third order in the eccentricities and has considerably smaller coefficients $C^j(\alpha)$ due to smaller $\alpha = (0.63)^2$. The principal reason we have retained third-order terms in e even though the observed eccentricities ≤ 0.01 is that we shall explore the consequence of the relaxation model in Section 6, which presumes that the orbital eccentricities may have been considerably larger in the past.

The Europan disturbing function equals $R_1^2 + R_3^2$. Since in our model Io and Ganymede do not directly interact, we can deduce that the motions of all satellite variables can be obtained from the combined disturbing function $R = R_1^2 + R_3^2$. Next, we can add the following zero-order Hamiltonian

$$H^0 = \sum_{i=1}^{3} \mu_i M_i/2a_i - \dot{\tilde{\omega}}_{si}\Gamma_i \qquad (13)$$

from which the secular motions of λ and $\tilde{\omega}$ can be obtained. A dot over a symbol implies d/dt. The dynamical equations are now completely canonical; i.e.,

$$dx/dt = \partial H/\partial\phi; \quad d\phi/dt = \partial H/\partial x, \quad (14)$$

where $\{x, \phi\}$ represent any pair of canonical variables defined in (7) and $H = H^0 + R$. The primary contribution to the "secular" pericenter motion is caused by the Jovian oblateness.

$$\dot{\tilde{\omega}}_{si} = \tfrac{3}{2} J_2(R_J/a_i)^2 n_i. \qquad (15)$$

We find $\dot{\tilde{\omega}}_{s1} = 0\overset{\circ}{.}16$/day, $\dot{\tilde{\omega}}_{s2} = 0\overset{\circ}{.}036$/day, and $\tilde{\omega}_{s3} = 0\overset{\circ}{.}007$/day at their present positions if we include the purely secular intersatellite contributions plus terms which are second order in the mass ratios M_i/M_J which are important because of the 2:1 commensurabilities (Chao, 1976). The secular motion of the node $\dot{\Omega}_{si}$ due to Jovian oblateness is $\simeq -\dot{\tilde{\omega}}_{si}$.

The Hamiltonian $H = H^0 + R$ is a constant of the motion in the absence of dissipation. By restricting the problem to coplanar orbits we have eliminated Z_i and Ω_i as variables and are left with six action and six angle variables. However, the Hamiltonian depends only on the following four angle variables

$$\begin{aligned}
\phi_1 &= \lambda_1 - 2\lambda_2 + \tilde{\omega}_1,\\
\phi_2 &= \lambda_1 - 2\lambda_2 + \tilde{\omega}_2,\\
\phi_3 &= \lambda_2 - 2\lambda_3 + \tilde{\omega}_2,\\
\phi_4 &= \lambda_2 - 2\lambda_3 + \tilde{\omega}_3,
\end{aligned} \qquad (16)$$

which suggests that we transform to this set of angle variables plus two more linearly independent variables ϕ_5, ϕ_6. For example, we can choose $\phi_5 = \lambda_1$ and $\phi_6 = \lambda_2$. Since $H \neq H(\phi_5, \phi_6)$ the conjugate action variables corresponding to these angles are constants in the absence of tides and the problem is thereby simplified to effectively four degrees of freedom. The action variables x_i conjugate to the ϕ_i are most easily found by requiring $d\phi_i/dt = -\partial H/\partial x_i = \sum_i (\partial H/\partial L_i)(\partial L_i/\partial x_i) + (\partial H/\partial\Gamma_i)(\partial\Gamma_i/\partial x_i)$. Explicit expressions for $d\phi_i/dt$ follow from (16) and (8) which fix $\partial L_i/\partial x_i$ and $\partial\Gamma_i/\partial x_i$ and thereby the following relations between the sets x_i and L_i, Γ_i.

$$\begin{aligned}
L_1 &= L_{10} + x_1 + x_2 + x_5,\\
L_2 &= L_{10} - 2(x_1 + x_2) + x_3 + x_4 + x_6,\\
L_3 &= L_{30} - 2(x_3 + x_4),\\
\Gamma_1 &= \Gamma_{10} + x_1,\\
\Gamma_2 &= \Gamma_{20} + x_2 + x_3,\\
\Gamma_3 &= \Gamma_{30} + x_4,
\end{aligned} \qquad (17)$$

where L_{i0} and Γ_{i0} are constants and the x_i represent small variations caused by the resonant interactions. The action variables x_5 and x_6 are explicitly included in (17) since they are not constant once tides are added. Their presence is essential for a correct description of the effect of tides on the rate of evolution of the resonance configuration developed in Section 6.

The Hamiltonian formalism developed here proves to be a useful tool in determining the stability of the system when the eccentricities are relatively large and the resonance variables are strongly interacting. It shall form the basis of the study on the relaxation hypothesis. If the eccentricities are very small, then R can be restricted to just the linear terms in e. We find that in this case an adequate description can be developed based on the dynamical equations for n and the Poincaré eccentric variables $p = e \exp(-i\tilde{\omega})$ and $q = p^*$, where p^* is the complex conjugate of p. This is the approach adopted by Yoder (1979b) where the dynamical equations are

$$\frac{dn}{dt} = -\frac{3}{Ma^2}\frac{\partial R}{\partial \lambda}, \tag{18}$$

$$\frac{dp}{dt} = -\frac{2i}{Mna^2}\frac{\partial R}{\partial q}, \tag{19}$$

In the next section we shall use these equations to describe the simple tidal scenario for the formation and evolution of the two-body and Laplace resonances.

4. CLASSICAL TIDAL SCENARIO

Suppose that Io were formed well inside the orbit of Europa about 4.6 by ago. An initially free eccentricity would be quickly damped by tides raised by Jupiter on Io, and Io's orbit would thereafter expand, driven by the dissipative tide it raises on Jupiter. Europa's orbit would also expand, but because of Io's greater mass and smaller orbit, Io would spiral out faster. Io will approach six possible 2:1 resonance interactions with Europa involving the angle variables $2V_1 + 2\Omega_1$, $2V_1 + \Omega_1 + \Omega_2$, $2V_1 + 2\Omega_2$, $V_1 + \tilde{\omega}_2$, $2V_1 + \tilde{\omega}_1 + \tilde{\omega}_2$, and $V_1 + \tilde{\omega}_1$ with relative strengths or restoring accelerations proportional to $\sin^2 I_1$, $\sin I_1 \sin I_2$, $\sin^2 I_2$, e_2, e_1e_2, e_1, respectively. These resonances are encountered in the given order if the secular motion of the nodes and pericenters are dominated by Jovian oblateness. However, the resonance-induced retrograde motions of the Ω_i are relatively small and nearly constant as the resonances are approached whereas $\dot{\tilde{\omega}}_i$ are proportional to $-1/e_i$. Because the eccentricities are initially damped to very small values, the rates $\nu_1 + \dot{\tilde{\omega}}_1$ and $\nu_1 + \dot{\tilde{\omega}}_2$, vanish first (since ν_1 is positive) and the corresponding angle variables $V_1 + \tilde{\omega}_1$ and $V_1 + \tilde{\omega}_2$, are automatically captured into libration (Yoder, 1973). The frequencies of the remaining resonance variables remain sufficiently far from zero that we need retain only the terms with arguments $V_1 + \tilde{\omega}_1$ and $V_1 + \tilde{\omega}_2$ in R, which is equivalent to considering only coplanar orbits in Section 3. The near-resonant interaction is thus adequately described by the following set of differential equations [(3)–(6), (12), (13), (18), (19)].

$$dn_1/dt = 3(M_2/M_J)n_2{}^2\alpha^{-2} [e_1C_1 \sin(V_1 + \tilde{\omega}_1) + e_2C_2 \sin(V_1 + \tilde{\omega}_2)] + dn_1/dt_T, \tag{20}$$

$$dn_2/dt = -6(M_1/M_J)n_2{}^2 [e_1C_1 \sin(V_1 + \tilde{\omega}_1) + e_2C_2 \sin(V_1 + \tilde{\omega}_2)] + dn_2/dt_T, \tag{21}$$

$$dp_1/dt = -in_2\,\alpha^{-1/2}(M_2/M_J)C_1 \exp iV_1 - (\tfrac{7}{3}D_1c_1 + i\dot{\tilde{\omega}}_{s1})p_1, \tag{22}$$

$$dp_2/dt = -in_2(M_1/M_J)C_2 \exp iV_1 - (\tfrac{7}{3}D_2c_2 + i\dot{\tilde{\omega}}_{s2})p_2, \tag{23}$$

where $C_1 = -1.19$ and $C_2 = +0.43$. The effect of the tidal contribution dn_2/dt_T is small and was omitted in the earlier analysis.

A solution for the forced motion of p can be obtained by assuming that $\dot{V}_1 = \nu_1$ varies slowly such that $\dot{\nu}/\gamma^2$ is small. This approximation is valid as long as ν/n is large compared to $(eM_1/M_J)^{1/2}$ or the free eccentricity is small compared to the forced eccentricity. From (22), the formal solution for the forced motion of p_1 is simply

$$p_1 = (d/dt + \tfrac{7}{3}D_1c_1 + i\dot{\tilde{\omega}}_{s1})^{-1} \times (-in_2\,\alpha^{-1/2}(M_2/M_J)C_1 \exp iV_1). \tag{24}$$

If ν_1 were constant, the correct solution is obtained by replacing d/dt by $i\nu_1$. This suggests that we replace the operator $d/dt + \frac{7}{3}D_1c_1 + i\dot{\tilde{\omega}}_{s1}$ with $(d/dt - i\nu_1) + (i\nu_1 + \frac{7}{3}D_1c_1 + i\dot{\tilde{\omega}}_{s1})$ in (24) and expanding in powers of $(d/dt - i\nu_1)$. For p_1 and p_2, we find

$$p_1 = e_{12} \exp i(V_1 + \delta_1 - \dot{\nu}_1/\gamma_1{}^2) + e_{10} \exp(-i\dot{\tilde{\omega}}_{s1} - \gamma_1\delta_1)t, \tag{25}$$

$$p_2 = -e_{21} \exp i(V_1 + \delta_2 - \dot{\nu}_1/\gamma_2{}^2) + e_{20} \exp(-i\dot{\tilde{\omega}}_{s2} - \gamma_2\delta_2)t.$$

The forced eccentricities e_{12} and e_{21} and the phase lags δ_1 and δ_2 are given by

$$e_{12} = -\frac{M_2}{M_J}\frac{n_2}{\gamma_1}\alpha^{-1/2}C_1,$$
$$e_{21} = \frac{M_1}{M_J}\frac{n_2}{\gamma_2}C_2,$$
$$\delta_1 = \tfrac{3}{3}D_1c_1\gamma_1^{-1},$$
$$\delta_2 = \tfrac{3}{3}D_2c_2\gamma_2^{-1}, \qquad (26)$$

with $\gamma_1 = \nu_1 + \dot{\tilde{\omega}}_{s1}$ and $\gamma_2 = \nu_1 + \dot{\tilde{\omega}}_{s2}$.

Since $p = e \exp -i\tilde{\omega}$, we find from (25) that the longitudes of the pericenters are approximately described by

$$\tilde{\omega}_1 \simeq -(V_1 + \delta_1),$$
$$\tilde{\omega}_2 \simeq -(V_1 + \delta_2) + \pi, \qquad (27)$$

if the small $\dot{\nu}/\gamma^2$ and free eccentricity terms are neglected. Since V_1 increases linearly with time, the mutual perturbation leads to a forced retrograde motion of the pericenters. In the absence of satellite dissipation we find conjunction (i.e., $\lambda_1 = \lambda_2$) exactly occurs when Io is at its pericenter [i.e., $r_1 = a_1(1 - e_1)$] and Europa is at its apocenter. This configuration tends to maximize the mean distance of separation of this pair of satellites. Dissipation in Io induces a phase lag $\delta_1 > 0$ in Io's pericenter. Thus conjunctions occur when Io is slightly past its pericenter. This can be understood from a physical point of view since the forced motion $d\tilde{\omega}/dt$ is proportional to $-1/e$ and satellite dissipation tends to reduce e. If conjunctions occurred exactly at the orbit extremes, the components of the intersatellite force which are tangent to the orbit average to zero and no net angular momentum is transferred. In the case where dissipation is occurring in the satellites, the satellites are closer together after conjunction and the differential angular velocities are reduced, so a larger tangential force is applied for a longer period after conjunction than before (e.g., Peale, 1976a). Thus, dissipation in the satellites tends to cause a net transfer of angular momentum from Io to Europa.

Substituting the above solutions for p_1 and p_2 into (20) and (21) for $\dot{n}_1$ and $\dot{n}_2$, we obtain

$$dn_1/dt = -3(M_2/M_J)\,\alpha^{-2}n_2^2 \{C_1e_{12}(\delta_1 - \dot{\nu}_1/\gamma_1^2) - C_2e_{21}(\delta_2 - \dot{\nu}_1/\gamma_2^2)\} + dn_1/dt_T, \qquad (28)$$

$$dn_2/dt = +6(M_1/M_J)n_2^2 \{C_1e_{12}(\delta_1 - \dot{\nu}_1/\gamma_1^2) - C_2e_{21}(\delta_2 - \dot{\nu}_1/\gamma_2^2)\} + dn_2/dt_T, \qquad (29)$$

The dynamical equation for ν_1 is (3)–(6), (26)–(29)

$$(1 + K_1)\frac{d\nu_1}{dt} = -n_1c_1[1 - 35D_1e_{12}^2] + n_1c_2[1 + 12D_2e_{21}^2] \simeq -n_1c_1[0.97 - (35D_1 + 0.086D_2(\gamma_1/\gamma_2)^2)e_{12}^2], \qquad (30)$$

where we have used $c_2 \simeq 0.026\,c_1$ and $e_{21} = 0.53\,\gamma_1e_{12}/\gamma_2$. The parameter K_1 is

$$K_1 = 3.4 \times 10^{-8}(n_2/\gamma_1)^3 + 6.5 \times 10^{-9}(n_2/\gamma_2)^3 \simeq (86e_{12})^3 + (93e_{21})^3. \qquad (31)$$

K_1 is small as long as γ_1 and $\gamma_2 = \gamma_1 + \dot{\tilde{\omega}}_{s2} - \dot{\tilde{\omega}}_{s1}$ are greater than $n_2/310$ and $n_2/540$, respectively.

As Io tidally evolves outwards, Io's forced eccentricity increases until it reaches the critical value $\simeq 1/(35\,D_1)^{1/2} \simeq 0.0026$, where $\dot{\nu}_1$ vanishes ($D_1 \approx 4200$ is determined later). Europa's limiting eccentricity is $\simeq 0.0014$. Thereafter, the relative outward acceleration of Europa maintains its mean motion at half that of Io. This stable state is maintained until Europa encounters the 2:1 commensurability with Ganymede. The 4:1 commensurability of Io with Ganymede is third order in the eccentricities and represents a considerably weaker interaction. Like the dissipation in Io which caused a net transfer of angular momentum to Europa, the inelastic tide raised on Europa will repel Ganymede. Still, the acceleration of Europa by Io is so great that Europa's forced eccentricity would have to be at least three times larger than its present value (0.0101) for this mechanism to push Ganymede out so that

$\dot{n}_2 = 2\dot{n}_3$. Before Europa's eccentricity can be pumped up to this value the 2:1 frequency $\nu_2 = n_2 - 2n_3$ of the outer pair approaches that of the inner pair, $\nu_1 = n_1 - 2n_2$. The vanishing of the difference frequency $(\nu_1 - \nu_2)$ describes the presently observed three-body resonance.

The near-resonant gravitational couple between Europa and Ganymede can be obtained by increasing the subscripts (20)–(29) by one. If we neglect dissipation in Europa, then the resulting distortion in the shape of Europa's orbit by both Io and Ganymede is

$$p_2 = -e_{21}\exp(iV_1 - i\dot{\nu}_1/\gamma_2^2) + e_{23}\exp(iV_2 - i\dot{\nu}_2/\gamma_2^2), \quad (32)$$

where $e_{23} = -(M_3/M_J)\,\alpha\, C_1(n_2/\gamma_3)$, $V_2 = \lambda_2 - 2\lambda_3$, $\nu_2 = \dot{V}_2 = n_1 - 2n_2$, and $\gamma_3 = \nu_2 + \dot{\omega}_{s2}$.

Equation (20) for dn_1/dt remains unchanged but (21) is replaced by

$$dn_2/dt = -6(M_1/M_J)n_2^2[e_1C_1\sin(V_1 + \tilde{\omega}_1) + e_2C_2\sin(V_1 + \tilde{\omega}_2)] + 3(M_3/M_J)n_2^2\,\alpha\,[e_2C_1\sin(V_2 + \tilde{\omega}_2) + e_3C_2\sin(V_2 + \tilde{\omega}_3)] + dn_2/dt_T. \quad (33)$$

In addition

$$dn_3/dt = -6(M_2/M_J)n_3^2[e_2C_1\sin(V_2 + \tilde{\omega}_2) + e_3C_2\sin(V_2 + \tilde{\omega}_3)]. \quad (34)$$

When (32) is substituted back into (20), (33), and (34), we obtain the interaction involving the resonance argument $\phi = V_1 - V_2$, where only the dissipative terms for Io are retained.

$$\dot{\nu}_1 + (A_1 - 2A_2)n_2^2\sin\phi + n_1c_1(1 - 35D_1e_{12}^2) + K_2\dot{\nu}_1 + K_3\dot{\nu}_2 = 0, \quad (35)$$

$$\ddot{\phi} + An_2^2\sin\phi + n_1c_1(1 - 45D_1e_{12}^2) + K_4\dot{\nu}_1 + K_5\dot{\nu}_2 = 0. \quad (36)$$

The coefficient A equals $A_1 - 3A_2 + 2A_3$ and the A_i are

$$A_1 = 3C_1C_2\alpha^{-1}\frac{M_2M_3}{M_J^2}\frac{n_2}{\gamma_3},$$
$$A_2 = -3C_1C_2\alpha\frac{M_1M_3}{M_J^2}\left(\frac{2n_2}{\gamma_3} + \frac{n_2}{\gamma_2}\right),$$
$$A_3 = 6C_1C_2\alpha^3\frac{M_1M_2}{M_J^2}\frac{n_2}{\gamma_2}. \quad (37)$$

The coefficients K_i are:

$$K_2 = (86e_{12})^3 + (1 + 1.1\cos\phi)(93e_{21})^3,$$
$$K_3 = -(1 + 0.9\cos\phi)(47e_{23})^3,$$
$$K_4 = (95e_{12})^3 + (1 + 1.4\cos\phi)(103e_{21})^3,$$
$$K_5 = -(1 + 0.7\cos\phi)(57e_{23})^3. \quad (38)$$

We assume that the 2:1 resonance between Io and Europa has reached the equilibrium configuration by the time the resonance with Ganymede is approached. Free eccentricities are also likely to be damped such that librations about the resonance are small and ν_1 is nearly constant. However, as the resonance with Ganymede is approached, the growing perturbations will introduce a variation in ν_1, proportional to $\dot{\phi}$, which can be found from (35) and (36). The variation in ν_1 is important since it defines the scheme by which the system is captured into the three-body Laplace resonance. For current values of the parameters, the coefficients $K_2 = 0.049$, $K_3 = -0.001$, $K_4 = 0.060$, and $K_5 = -0.023$, and they were smaller in the past. Hence, for our present purposes, we can neglect these terms.

If we eliminate the sin ϕ term from (35) and (36) we find

$$\frac{d\nu_1}{dt} = 0.68\frac{d\dot{\phi}}{dt} - 0.32c_1n_1(1 - 13D_1e_{12}^2), \quad (39)$$

where the numerical coefficient multiplying $d\dot{\phi}/dt$ equals $(A_1 - 2A_2)/A$. Integration of (39) yields

$$\nu_1 = \nu_{10} + 0.68\,\dot{\phi}, \quad (40)$$

where ν_{10} is only slowly varying due to the dissipative term in (39). Define $\delta\nu = 0.68\,\delta\dot{\phi}$, where $\delta\dot{\phi}$ is the periodic part of $\dot{\phi}$. Clearly, the periodic variation of the mean motions δn_i equals $(A_i/A)\delta\phi_i$ if the K_i are small. We find $\delta n_1 = 0.125\delta\dot{\phi}$, $\delta n_2 = -0.276\delta\dot{\phi}$, $\delta n_3 = 0.023\delta\dot{\phi}$, and $\delta\nu_2 = -0.32\delta\dot{\phi}$.

The pendulum equation (36) depends on ν_1 and ν_2 through the parameters A and e_{12}. If we replace ν_i and $\nu_{i0} + \delta\nu_i$ in the relations [(26), (37)] defining A and e_{12} and expand to first order, we find that their variations proportional to $\delta\dot{\phi}$ are

$$\begin{aligned}\delta A &\simeq 0.042A\gamma_2^{-1}\delta\dot{\phi},\\ \delta e_{12}^2 &= -1.36e_{12}^2\gamma_1^{-1}\delta\dot{\phi}.\end{aligned} \tag{41}$$

The variation in A is relatively small compared to that in e_{12}^2 and can be shown to increase capture probability and damping by less than 1%. If we include only the e_{12}^2 variation in the pendulum equation, we find

$$\begin{aligned}\ddot{\phi} + An_2^2 \sin\phi = &-n_1c_1(1 - 45D_1e_{12}^2)\\ &- 61c_1n_1D_1e_{12}^2\gamma_1^{-1}\delta\dot{\phi},\end{aligned} \tag{42}$$

where the $\delta\dot{\phi}$ dependence is now removed from ν_1 in the definitions of A and e_{12}. Equation (42) is that for a pendulum with an applied torque which is valid for ϕ being either an angle of circulation (pendulum rotating around its support) or an angle of libration. Incidentally, the linearized solution for the three-body restoring force ($= An_2^2 \sin\phi$) as given by (37) is too large by ~40%. The primary source of this reduction involves both the K_i contributions to the acceleration and the e^2 terms of R (2:1) in (12).

Initially, $\dot{\phi} = \nu_1 - \nu_2 < 0$ since $\nu_2 = n_2 - 2n_3$ must be large. The resonance ($\langle\dot{\phi}\rangle = 0$) can only be approached if the first term on the right-hand side of (42) is positive (i.e., $\langle\ddot{\phi}\rangle > 0$), which requires $(1 - 45D_1e_{12}^2) < 0$ or

$$e_{12} > 1/(45D_1)^{1/2}. \tag{43}$$

But this is assured since the equilibrium value of $e_{12} \simeq 1/(35D_1)^{1/2}$ in the 2:1 Io–Europa resonance is reached before the Ganymede resonance is approached. The situation is thus reduced to that of a pendulum rotating over the top of its support with an applied torque slowly reducing the magnitude of the angular velocity. The dynamical equation describing the motion and tidal evolution of the three-body resonance is nearly identical to that of spin–orbit resonance (Goldreich and Peale, 1966; Yoder, 1979a).

Like the spin orbit case, capture into libration is not certain but depends on conditions as the equivalent pendulum passes over the top of its support the last time before reversing directions. Since these conditions cannot be known precisely, the capture is probabilistic in general.

As the system passes through the resonance the mean value of $\dot{\phi}$ is nearly zero and $\delta\dot{\phi} \approx \dot{\phi}$. If we thus replace $\delta\dot{\phi}$ by $\dot{\phi}$, (42) is now identical to that of Goldreich and Peale (1966) except for the form of the coefficients and the initial sign of $\dot{\phi}$, and we can use their expression for the capture probability P_c.

$$P_c = \frac{2}{1 + (\pi/4)(\ddot{\phi}_T/\sigma|An_2^2|^{1/2})}. \tag{44}$$

Here $\ddot{\phi}_T = -n_1c_1\ [1 - 45D_1e_{12}^2]$ is the constant part of the tidal acceleration and $\sigma = 61n_1c_1\gamma_1^{-1}D_1e_{12}^2$ is the coefficient of the $\dot{\phi}$ term in (42).

If $e_{12} = 1/(35D_1)^{1/2}$ at the time of capture we find

$$P_c \simeq \frac{2}{1 + (D_1/3700)^{3/4}} \tag{45}$$

and capture into resonance is certain if $D_1 < 3700$. P_c is only weakly dependent on D_1 if e_{12} had attained its 2:1 equilibrium value prior to transition. After capture into the resonance the fact that the $\dot{\phi}$ term in (42) always opposes the motion of the equivalent pendulum means the librations will be damped to zero by the continuing dissipation.

Returning to Eq. (39) we see that within the resonance $\dot{\phi}$ is periodic and $\langle\dot{\phi}\rangle = 0$. Thus, ν_1 continues to decrease until e_{12} has reached a new equilibrium value

$$\bar{e}_{12} = 1/(13D_1)^{1/2} \tag{46}$$

after which $\dot{\nu}_1 = 0$. The current value of e_{12} is 0.0041. If this is the equilibrium value, (46) yields

$$D_1 = 4600, \tag{47}$$

which is consistent with almost certain capture (45) of the system into the Laplace libration. The remaining forced eccentricities are $e_{21} = 0.0026$ and $e_{23} = 0.0075$, up from 0.0014 and 0.0040 respectively at the time of capture into the Laplace resonance.

The damping of the libration amplitude is most conveniently described by the action J.

$$J = \oint \dot{\phi} d\phi, \tag{48}$$

where the integration is over one libration cycle with all slowly varying quantities held constant. Note that $J \to 0$ with the amplitude of libration. The general solutions to the integral in (48) are expressed in terms of complete elliptic integrals (Yoder, 1979a). The action is clearly not an adiabatic invariant here as it decreases with the amplitude of libration from the dissipative term in (42). The reason for choosing the action to describe the damping rather following the amplitude ϕ_m directly is that an elegantly simple equation for $J(t)$ is obtainable which avoids the singularity when ϕ_m nears 180°.

If we multiply (42) by $\dot{\phi}$ ($\delta\dot{\phi} \equiv \dot{\phi}$ in libration) and integrate with respect to time, we obtain

$$E = E_0 + \ddot{\phi}_T\phi - \int^t \sigma\dot{\phi}^2 dt - \int^t n_2^2 \cos\phi\,(dA/dt)dt, \tag{49}$$

where E_0 is a constant, $E = 1/2\,\dot{\phi}^2 - An_2^2 \cos\phi$ is the libration energy. The last term in (49) accounts for the variation in A, since A is a function of the mean ν and ν continues to decrease until $e_{12} = 1/(13D_1)^{1/2}$. In (48) $\dot{\phi} = \dot{\phi}(E,A)$, where E and A would be constant in the absence of the dissipation as would J. Hence,

$$\begin{aligned}\frac{dJ}{dt} &= \left[\frac{dE}{dt}\frac{\partial}{\partial E} + \frac{dA}{dt}\frac{\partial}{\partial A}\right] \oint \dot{\phi} d\phi \\ &= (\ddot{\phi}_T\phi - \sigma\dot{\phi}^2) \oint \dot{\phi}^{-1} d\phi \\ &\quad - n_2^2 \frac{dA}{dt}\left[\oint \dot{\phi}^{-1} \cos\phi\, d\phi - \cos\phi \oint \dot{\phi}^{-1}\, d\phi\right],\end{aligned} \tag{50}$$

where we have replaced dE/dt by $\ddot{\phi}_T\dot{\phi} - \sigma\dot{\phi}^2 - n_2^2 \cos\phi\, dA/dt$ from (49). The time average of dJ/dt over one libration cycle is defined as

$$\left\langle\frac{dJ}{dt}\right\rangle = \frac{1}{T}\oint \frac{dJ}{dt}\,\dot{\phi}^{-1} d\phi, \tag{51}$$

where the libration period $T = \oint \dot{\phi}^{-1} d\phi$. Taking the time average of (50), we find that the terms multiplied by dA/dt and $\ddot{\phi}_T$ respectively vanish, with the result

$$\left\langle\frac{dJ}{dt}\right\rangle = -\sigma \langle J \rangle. \tag{52}$$

The solution to (52)

$$J(t) = J(0) \exp - \int_0^t \sigma\, dt \tag{53}$$

describes the secular variation of J with time.

Since the forced e_{12} increases from $1/(35D_1)^{1/2}$ to $1/(13D_1)^{1/2}$, the coefficient $\sigma \propto \gamma_1^{-3}$ varies from $\sim 250c_1$ up to $1100c_1$. An explicit solution to the integral in (52) can be found if we change the integration variable from t to $z = \gamma_1/\bar{\gamma}_1$ through Eq. (39) ($d\bar{\gamma}_1/dt = d\nu_1/dt$), where $\bar{\gamma}_1$ is the equilibrium value of γ_1 obtained from (27) with $\bar{e}_{12} = 1/(13D_1)^{1/2}$ being the three body equilibrium value of e_{12}. We find

$$dt = \frac{-\bar{\gamma}_1}{0.32c_1n_1}\left(\frac{z^2}{z^2-1}\right) dz, \tag{54}$$

$$\sigma\, dt = \frac{15.2\, dz}{z(1-z^2)}. \tag{55}$$

If we ignore the small variation in c_1n_1 during the damping, integration of (54) and (55) yield

$$t = \frac{\bar{\gamma}_1}{0.32 c_1 n_1} \left\{ z_0 - z + \frac{1}{2} \ln \left[\left(\frac{1+z}{1+z_0} \right) \left(\frac{1-z_0}{1-z} \right) \right] \right\}; \quad (56)$$

$$\exp \left[- \int_0^t \sigma \, dt \right] = \left(\frac{z_0^2(1-z^2)}{z^2(1-z_0^2)} \right)^{7.6}. \quad (57)$$

The Laplacian resonance was established at time $t = 0$ when $z = z_0 = (35/13)^{1/2} = 1.63$. At this time $\phi_m = 180°$ and $\dot{\phi}^2 = 2|An_2^2|(1 - \cos \phi)$. From (48) we find $J(0) = 16|A(0)n_2^2|^{1/2}$. The estimate for the current value of $\phi_m = 0\overset{\circ}{.}066$ (Lieske, 1980), and for small ϕ_m, $J(t) \simeq 4\pi|A(t)n_2^2|^{1/2} \sin^2 \frac{1}{2} \phi_m$. If we substitute these expressions into (53) along with (57), we obtain, for small ϕ_m,

$$\left| \frac{A(t)}{A(0)} \right|^{1/2} \frac{\pi}{4} \sin^2 \frac{\phi_m}{2} = \left[\frac{z_0^2(1-z^2)}{z^2(1-z_0^2)} \right]^{7.6}. \quad (58)$$

From (37) we see $A \propto 1/\gamma_2 = 1/(\gamma_1 - \dot{\omega}_{s1} + \dot{\omega}_{s2})$ and $A(t)/A(0) = [z_0 - (\dot{\omega}_{s1} - \dot{\omega}_{s2})/\bar{\gamma}_1]/[z - (\dot{\omega}_{s1} - \dot{\omega}_{s2})/\bar{\gamma}_1]$ such that (57) can be solved for the value of z corresponding to the current amplitude of libration. We find $z = 1.047$ for $\phi_m = 0\overset{\circ}{.}066$. For a remnant free libration of this amplitude, the equilibrium value for Io's forced eccentricity $\bar{e}_{12} = ze_{12}(t = \text{now})$ or

$$\bar{e}_{12} = 0.0043, \quad (59)$$

and since $D_1 = 1/(13\bar{e}_{12}^2)$

$$D_1 \approx 4200. \quad (60)$$

If the observed value of ϕ_m is bogus or due to a recent impact perturbation, then $z \approx 1.00$. The current eccentricity is the equilibrium value and $D = 4600$ as given by (46). It is seen that whether or not the observed ϕ_m is a remnant free libration has little effect on the value of D since ϕ_m is so small. However, the nature of this libration does have a profound effect on estimates of the age of the resonance. From (56), we find the age of the Laplace resonance to be

$$t = 1600 Q_J \text{ years} \quad (61)$$

where $z = 1.047$ was used. Of course if $\phi_m = 0.066$ is not a remnant-free libration, the age of the Laplace relation cannot be determined in terms of Q_J. There are at least two other possible explanations for the observed ϕ_m.

Consider the impact on one of the three satellites of a mass M_p with velocity $v_p \simeq 12$ km/sec relative to Jupiter ($v_p \simeq v_J$). Gravitational focusing by Jupiter increases v_p to $v_p' = 27.4$, 22.9, and 19.6 km/sec respectively for Io's, Europa's, and Ganymede's distance from Jupiter. The largest momentum impulse is transmitted if $\vec{v}_p'$ is antiparallel to the satellite orbital velocity $\vec{v}$ which is about 17, 14, and 11 km/sec for Io, Europa and Ganymede, respectively. The inelastic, angular momentum impulse $M_p a(v_p' + v)$ will change the mean motion at time $t = 0$ by an amount

$$\delta n(0) \simeq 3 \frac{M_p}{M} \left(\frac{v_p' + v}{v} \right) n \quad (62)$$

if we neglect the effect on $\delta n/n$ of the instantaneous change in e which is of order $e\delta n/n$. Here the symbol δ refers to the *instantaneous* change in n immediately after impact. The impact not only excites the periodic three-body liberation, but also changes the average mean motion of these satellites by an amount Δn_i. The Δn_i must satisfy the condition $\Delta n_1 - 2\Delta n_2 = \Delta n_2 - 2\Delta n_3 = \Delta \nu$. Otherwise the three-body lock is not maintained. The variation in $\delta n_i(t)$ following impact approximately satisfies the set of equations

$$\begin{aligned} \delta n_1(t) &\simeq \Delta n_1 + 0.127 \phi_m \, \omega_L \cos \omega_L t, \\ \delta n_2(t) &= \Delta n_2 - 0.276 \phi_m \, \omega_L \cos \omega_L t, \\ \delta n_3(t) &= \Delta n_3 + 0.023 \phi_m \, \omega_L \cos \omega_L t, \end{aligned} \quad (63)$$

where $\omega_L = 2\pi/2074$ days is the three-body libration frequency. The amplitude of the forced eccentricities is changed by an amount $\sim -(\Delta\nu/\nu)e$(forced). Since $\delta p(0) = \delta p(\text{forced}) + \delta p(\text{free}) \sim 0$, we find that the mean change in ν alters the amplitude of the free eccentricities by $\sim (\Delta\nu/\nu)e$ (forced). The free eccentricities are the source of additional periodic variations in $\delta n_i(t)$ with frequencies $\dot{V} + \dot{\omega}_{si}$ omitted from (63). However, these terms have little effect on the amplitude ϕ_m. If we solve Eq. (63) at time $t = 0$, we find

$$\phi_m = \delta\dot{\phi}(0)\omega_L^{-1}, \tag{64}$$

where the initial $\delta\dot{\phi}(0) = \delta n_1(0) - 3\delta n_2(0) + 2\delta n_3(0)$. The $\delta n_i(0)$ are either zero or are determined by (61).

The minimum-sized body required to excite $\sim 0^\circ\!.1$ libration involves the impact of $\sim$18-km-diameter body on Europa. This estimate is based on an asteroidal density $\rho_p \simeq 2$ g/cm³. The excitation of a $0^\circ\!.1$ libration from impacts on Io and Ganymede require bodies which are three and nine times more massive, respectively than for Europa, given the same initial conditions. An impact frequency of an 18-km asteroid on the Moon with an impact energy of 4×10^{31} ergs is estimated to occur once every 6×10^7 to 1×10^9 years. The range in impact rates reflects the uncertainty in the crater diameter versus energy scaling law (cf. Peale, 1976b). Shoemaker (private communication, 1980) estimates that the impact rate of asteroidal and cometary material on the Galilean satellites is comparable to that on the Moon. It should also be kept in mind that such impact caused increments of $0^\circ\!.1$ for ϕ_m will probably occur even less often than the above global rates for the Moon since we have chosen an optimum direction of approach for the colliding object. The time constant for decay of the small increment in J is $1/\sigma$ from (53) or $2/\sigma$ for ϕ_m since $J \propto \phi_m^2$ for small ϕ_m. Since σ is now approximately $1100c_1$, the time constant for exponential decay of small-amplitude librations is about 130 Q_J years $= 6.5 \times 10^7$ years for $Q_J \approx 5 \times 10^5$. Although damping times are likely to be less than the mean times between collisional excitations of $\phi_m \approx 0^\circ\!.1$, that the currently observed ϕ_m is the result of a recent collision is not entirely out of the question.

Another possibility is that the inferred free-libration amplitude is actually a forced libration. In the expansion of the Sun–satellite gravitational interaction, a forcing term exists with argument $2\lambda_J - \Omega_3 - \psi$ and a period of 2076 days which is very near the libration period of $2{,}074 \pm 10$ days (J. Lieske, personal communication, 1979). Ω_3 refers to the node of Ganymede's orbit and ψ to the node of Jupiter's equator on the Jovian orbit. The amplitude of the forcing term is very sensitive to the difference in these periods. The uncertainty of the free-libration period primarily results from the uncertainty in the masses of Io, Europa, and Ganymede. A reduction in the uncertainty of the satellites' masses from the analysis of Voyager data may resolve this latter possibility.

Once the three-body lock is established, the secular variation of Io's mean motion is described by

$$\frac{dn_1}{dt} = 2.0\frac{d\nu_1}{dt} - 0.23c_1n_1. \tag{65}$$

Based on the interpretation of the observed ϕ_m as a tidally damped remnant, we find from (65) that Io's semimajor axis has increased by less than 0.7% since formation of the Laplace resonance. We find in retrospect that the approximation of holding $c_1n_1 \propto a_1^{-8}$ constant in (56) when estimating the age of the resonance introduces an error of only $\sim$3% into (61).

Even if Lieske's estimate for ϕ_m is only an upper bound of the remnant amplitude, we can use it to obtain an upper bound on Q_J. Since the amplitude ϕ_m must have damped in the age of the solar system, from (56) we find

$$Q_J \lesssim 3.5 \times 10^6. \tag{66}$$

This estimate is of the same order as that based on the minimum tidal heating required to explain the observed surface activity (Yoder, 1979b).

One important consequence of the recent establishment of this resonance lock is that Europa's forced eccentricity was increased from $\sim$0.003 up to $\sim$0.010 in $\sim 5 \times 10^8$ years if we adopt a "plausible" $Q_J \simeq 4 \times 10^5$.

This time scale may be as short as $\sim 1 \times 10^8$ years. The tides of Europa may be responsible for many irregular linear features seen in Voyager images of Europa (Helfenstein and Parmentier, 1980). Also, Europa's surface is apparently quite young ($\sim 10^8$ years) based on the identification of only three to eight craters or crater-like features and estimates of impact rates (Lucchitta and Soderblom, 1981).

5. EFFECTS OF OTHER COMMENSURABILITIES

The purpose of this section is to investigate the possible consequences of other resonance interactions which these satellites may have encountered prior to the establishment of the three-body lock. First, the effects of a second order, Laplace-like resonance is discussed which was almost surely encountered if the tidal scenario developed in the previous section is a correct description. The main consequence of these resonance encounters is the excitation of free eccentricities whose remnants may still persist at least for Europa's orbit.

Next, we describe the possible effects of resonances associated with the two-body 3:1 commensurability on the orbital inclinations of the several satellites. We find that capture into any one of several possible resonance interactions is reasonably likely, leading to rapid growth of the forced inclination. Disruption of these resonances will excite large free inclinations which are incompatible with the long tidal damping time scales and the small free inclinations observed in the present orbits. We infer that these resonance interactions could not have been encountered and that the primordial ratio of semimajor axes of any pair of satellites was $\gtrsim 0.48$.

The Laplacean lock satisfies the condition that the 2:1 frequency $\nu_1 = n_1 - 2n_2$ on average equals $\nu_2 = n_2 - 2n_3$. After the formation of the Io–Europa 2:1 lock and prior to the establishment of the three-body lock, we expect than $\nu_2 > \nu_1$. During this time interval the frequency ν_1 maintains a nearly constant value $\simeq 1\overset{\circ}{.}2$/day, while ν_2 is progressively decreased as the tidal torques pushes the locked pair, Io–Europa, outward. The tidal scenario suggests that the trio of satellites should have evolved through a sequence of Laplacean-like interactions involving the frequencies: $j\nu_1 - \nu_2$: $j = \ldots 3, 2, 1$. The $j = 2$ case corresponds to the last and strongest interaction encountered prior to the establishment of the $j = 1$ lock.

Recall that the Laplacean frequency, $\nu_1 - \nu_2$, in the longitude equation resulted from the substitution of the first-order solution for the 2:1 forced motion of the Europan eccentric variable p_2 into the equations for $\dot{n}_1$, $\dot{n}_2$, and $\dot{n}_3$ which only included terms in the disturbing function R [see Eq. (12)] that were linear in the eccentricities. This suggests that the frequency $2\nu_1 - \nu_2$ may result from the cross coupling of terms which are quadratic in e in R. We should note that the angle variable $S = 2V_1 - V_2 = 2\lambda_1 - 5\lambda_2 - 2\lambda_3$ does not satisfy the condition of rotational invariance and therefore cannot result from the cross coupling of periodic variations derived from R. The set of variables which do satisfy this requirement are:

$$\begin{aligned} \psi_1 &= S + \tilde{\omega}_1, \\ \psi_2 &= S + \tilde{\omega}_2, \\ \psi_3 &= S + \tilde{\omega}_3. \end{aligned} \tag{67}$$

If capture into resonance could occur for any of these interactions such that $\ddot{S}$ vanishes, then the system could not evolve into the observed configuration, given that $\dot{S} \gg 0$ at some time in the past. The existence of such a nearby, stable resonance lock would place severe constraints on the range of tidal evolution of the three satellites prior to formation of the observed three-body lock and severely undermine the plausibility of the scenario for resonance formation developed in Section 4.

We demonstrate in this section that capture into resonance for any of these possible variables in (67) does not halt the tidal acceleration of S. These resonance locks eventually become unstable, cease librating and begin circulating.

Furthermore, transition excites sizeable free eccentricities in the orbits of Io and Europa. These free eccentricities can easily be damped to their observed values, even if formation of the present three-body configuration occurred relatively recently (61). In fact it is somewhat easier to interpret the observed free eccentricity of Europa as a tidally damped remnant of this excitation rather than a tidally damped remnant of a primordial free eccentricity.

From (4)–(6), we find that the exponential damping rate of eccentricity by satellite solid-body friction is $\tau^{-1}(e) = 10.5n (M_J/M)(R/a)^5 kf/Q$. If the tidal change in the semimajor axis is neglected, then the value of e at time t is: $e = e_0 \exp - t/\tau$. Since the orbits may have been considerably closer to Jupiter in the past, an upper bound in the effective $\tau(e)$ over the past 4.6 by is obtained by using the present values for a and n and setting $\mu = 5 \times 10^{11}$ dyne-cm in evaluating k (1).

$$\begin{aligned} \tau_1 &= 6.5 \times 10^4 (Q_1/f_1)\text{year}, \\ \tau_2 &= 2.9 \times 10^6 (Q_2/f_2)\text{year}, \\ \tau_3 &= 1.1 \times 10^7 (Q_3/f_3)\text{year}, \\ \tau_4 &= 6.4 \times 10^8 (Q_4/f_4)\text{year}. \end{aligned} \tag{68}$$

The free eccentricity of Io, if any exists, is likely to be the result of a very recent impact. We see from Table I that the observed value is one-half the uncertainty of the estimate. The remaining satellites have significant free eccentricities that are much too large to be the result of impacts. If the free eccentricities listed in Table I are the remnant amplitude of a primordial free $e < 0.1$, we find that $\tau_2 > 0.7$ by, $\tau_3 > 1.2$ by, and $\tau_4 \gtrsim 1.8$ by. We can transform (68) to *lower bounds* on Q_i/f_i using these lower bounds on τ_i.

$$\begin{aligned} Q_2/f_2 &\gtrsim 230, \\ Q_3/f_3 &\gtrsim 100, \\ Q_4/f_4 &\gtrsim 3. \end{aligned} \tag{69}$$

Given that most other solid, planetary bodies have $Q \lesssim 10^2$, Europa's lower bound is large and suggests that the observed free $e_2(= e_{20})$ may not be a remnant of a primordially established eccentricity. We shall argue later that the observed e_{20} may instead be the remnant of a free eccentricity which was induced in Europa's orbit about $2 \times 10^3\ Q_J$ years ago. This excitation was the result of passage through the three-body resonance which is described above.

Peale *et al.* (1980) in discussing the effect of tidal heating of the satellites of Saturn believe that the rigidity of icy satellites may be closer to $\mu_{\text{ice}} = 4 \times 10^{10}$ dyne-cm^2. The corresponding Q/f must be increased by a factor of 10: $Q_3/f_3 \simeq 10^3$ and $Q_4/f_4 \simeq 30$. Again, we are faced with a lower bound of Q/f for Ganymede which appears to be too high when compared with estimates appropriate to a rocky body. Unless there exists a nearby orbital resonance that was encountered in the recent past which could account for the observed e_3(free) we must conclude that $\mu_3 \gg \mu_{\text{ice}}$.

For the analysis of these earlier resonances, the dynamical equations contain contributions which are second order in the eccentricities. If the tidal contributions are ignored, the equations of motion for n_i are

$$\begin{aligned} \frac{dn_1}{dt} = \frac{3M_2}{M_J} n_2{}^2\alpha^{-2} \{&-1.19e_1 \sin(V_1 + \tilde{\omega}_1) + 0.43e_2 \sin(V_1 + \tilde{\omega}_2) \\ &+ 3.4e_1{}^2 \sin 2(V_1 + \tilde{\omega}_1) - 5.0e_1e_2 \sin(2V_1 + \tilde{\omega}_1 + \tilde{\omega}_2) \\ &+ 7.2e_2{}^2 \sin(2V_1 + 2\tilde{\omega}_2)\}; \end{aligned} \tag{70}$$

$$\begin{aligned} \frac{dn_3}{dt} = -\frac{6M_1}{M_J} n_2{}^2\alpha^3 \{&-1.19e_2 \sin(V_2 + \tilde{\omega}_2) + 0.43e_3 \sin(V_2 + \tilde{\omega}_3) \\ &+ 3.4e_2{}^2 \sin(2V_2 + 2\tilde{\omega}_1) - 5.0e_1e_2 \sin(2V_2 + \tilde{\omega}_2 + \tilde{\omega}_3) \\ &+ 7.2e_3{}^2 \sin(2V_2 + 2\tilde{\omega}_3)\}; \end{aligned} \tag{71}$$

$$\frac{dn_2}{dt} = -2\frac{M_1}{M_2}\alpha^2 \frac{dn_1}{dt} - \frac{1}{2}\frac{M_3}{M_2}\alpha^{-2}\frac{dn_3}{dt}. \tag{72}$$

The dynamical equations for p_i are

$$\frac{dp_1}{dt} + i\dot{\omega}_{s1}p_1 = -in_2\alpha^{-1/2}\frac{M_2}{M_J} \times \{-1.19 \exp iV_1 + 3.4q_1 \exp i2V_1 - 5.0\, q_2 \exp i2V_1\}; \quad (73)$$

$$\frac{dp_2}{dt} + i\dot{\omega}_{s2}p_2 = -\, in_2 \frac{M_1}{M_J} \times \{+0.43 \exp iV_1 + 7.2q_2 \exp i2V_1 - 5.0\, q_1 \exp i2V_i\}; \quad (74)$$

$$- in_2\alpha \frac{M_3}{M_J} \times \{-1.19 \exp iV_2 + 3.4q_2 \exp i2V_2 - 5.0q_3 \exp i2V_1\};$$

$$\frac{dp_3}{dt} + i\dot{\omega}_{s3}p_3 = -in_2\alpha^{3/2}\frac{M_2}{M_J} \times \{+0.43 \exp iV_2 + 7.2q_3 \exp i2V_2 - 5.0q_3 \exp i2V_2\}. \quad (75)$$

The usual technique for solving a set of nonlinear equations such as these, where the resonant interaction lies buried in the coupling of "fast" periodic variations, is based on an ordered expansion of the variables in terms of the relative frequencies. The aim of this expansion is to successively solve for the fast periodic variations and eliminate these terms in the dynamical equations. The result of this process is a set of equations of motion involving only the slow variables. We expect that the lowest-order solution (in e) for the "fast" periodic variation of p_i is still given by (25), (32). Besides this contribution, there is also a significant "fast" periodic variation in the mean longitudes resulting from the circulation of the three-body variable $\phi = V_1 - V_2$. Replace V_i by $V_i^* + \delta V_i$ and p_i by $p_i^* + \delta p_i$ in (70)–(72). Here V_i^* and p_i^* depend on the slow variables ψ_i^*, while δV_i and δp_i correspond to the fast periodic parts. From (35)–(37) the lowest-order solution for the fast periodic variations for δV_1 and δV_2 are

$$\delta V_1 = 0.68\, B \sin \phi^*,$$
$$\delta V_2 = -0.32\, B \sin \phi^*, \quad (76)$$

where $B = A(n_2/\dot{\phi}^*)^2$. The first-order solution (in e) for p_i are (25) (32):

$$\delta p_1 = e_{12} \exp iV_1^*, \quad (77)$$

$$\delta p_2 = -e_{21} \exp iV_1^* + e_{23} \exp iV_2^*, \quad (78)$$

$$\delta p_3 = -e_{32} \exp iV_2^*. \quad (79)$$

The next step is to expand the left-hand side of (70)–(75) through first order in δV_i and δp_i. The equations of motion for the slow variables n_i^* and p_i^* obtained from the appropriate mixing of periodic terms are

$$\frac{dn_1^*}{dt} = 3n_2{}^2\alpha^{-2}\frac{M_2}{M_J}\{-(5.0e_{23} - 0.40B)e_1^* \sin(S^* + \tilde{\omega}_1^*) + (14.4e_{23} - 0.15B)e_2^* \sin(S^* + \tilde{\omega}_2^*)\}, \quad (80)$$

$$\frac{dn_2^*}{dt} = -2\alpha^2 \frac{M_1}{M_2}\frac{dn_1^*}{dt}, \quad (81)$$

$$\frac{dp_1^*}{dt} + i\dot{\omega}_{s1}p_1^* = in_2\alpha^{-1/2}\frac{M_2}{M_J} \times (5.0e_{23} - 0.40B) \exp iS^*, \quad (82)$$

$$\frac{dp_2^*}{dt} + \dot{\omega}_{s2}p_2^* = -in_2\frac{M_1}{M_J} \times (7.2e_{23} - 0.15B) \exp iS^*. \quad (83)$$

We find from inspection of (70)–(75), (76)–(79) that no slow terms affecting n_3^* and p_3^* result from the coupling of the linear contributions δV_i and δp_i with other periodic terms in (70)–(75). Thus neither resonance variable ψ_1^* nor ψ_2^* can transfer angular momentum to Ganymede's orbit and ν_2 continues to tidally decrease whether or not either of these "locks" is established. Prior to the encounter, the frequency $\xi = 2\nu_1^* - \nu_2^*$ is negative and is gradually increased by the tidal torque. Unlike the 2:1 lock on the total pericenter motion we find that the initial resonant response induces a slow prograde motion in the slow components $\tilde{\omega}_1^*$ and $\tilde{\omega}_2^*$. The approximate solution for p_1^* and p_2^* are

$$p_1^* = -e_{12}^*(1 - i\dot{\xi}/(\xi + \dot{\omega}_{s1})^2) \times \exp iS + e_{10} \exp - i\tilde{\omega}_{s1},$$
$$p_1^* = e_{21}^*(1 - i\dot{\xi}/(\xi + \dot{\omega}_{s2})^2) \times \exp iS + e_{20} \exp - i\tilde{\omega}_{s2}, \quad (84)$$

where $\xi = dS/dt$, $\quad \tilde{\omega}_{si} = \dot{\omega}_{si}(t - t_0)$

and

$$e_{12}^* = -\frac{M_2}{M_J}\frac{n_2}{(\xi + \dot{\omega}_{s1})}(5.0e_{23} - 0.40B),$$
$$e_{21}^* = -\frac{M_1}{M_J}\frac{n_2}{(\xi + \dot{\omega}_{s2})}(7.2e_{23} - 0.15B). \tag{85}$$

Here e_{10} and e_{20} are included for the stability analysis. Since $\dot{\omega}_{s1} \gg \dot{\omega}_{s2}$, we find that the frequency $\xi + \dot{\omega}_{s1}$ increases more rapidly than $\xi + \dot{\omega}_{s2}$ if both are at some time negative. From (85) we find that the forced eccentricity e_{12}^* increases more rapidly than e_{21}^* if $\xi + \dot{\omega}_{s1} < 0$. Near the encounters ($\psi_i \rightarrow 0$), we find $e_{23} \simeq 0.0025$, $B \simeq e_{23}$, and $\nu_1^* \simeq \frac{1}{2}\nu_2^* \simeq \dot{\phi}^* \simeq 1^\circ.2$/day. If we substitute these solutions for p_1^* and p_2^* into the second-order equation for S^*, we obtain

$$(1 - K^*)\frac{d^2S^*}{dt^2} = 6\alpha^{-2}n_2^2\frac{M_2}{M_J} \times \left(1 + 5\alpha^2\frac{M_1}{M_2}\right)\{(5.0e_{23} - 0.40B)e_{10}\sin(S^* + \tilde{\omega}_{s1}) - (14.4e_{23} - 0.15B)e_{20}\sin(S^* + \tilde{\omega}_{s2})\}. \tag{86}$$

The parameter K^* is given by

$$K^* \simeq (0.0025/e_{23}) \times ((630e_{12}^*)^3 + (560e_{21}^*)^3) \tag{87}$$

and is positive if either $\xi + \dot{\omega}_{s1} < 0$ or $\xi + \dot{\omega}_{s2} < 0$ and $|\xi + \dot{\omega}_{s2}| \ll |\xi + \dot{\omega}_{s1}|$. We find from (85), (87) that when $e_{12}^* \simeq 1/630$ that $\xi + \dot{\omega}_{s1} = -0^\circ.002$/day while $e_{21}^* \sim 0.01e_{12}^*$. This indicates that the interaction involving the two resonance variables ψ_1^* and ψ_2^* very nearly decouple near each resonance and thus can be analyzed separately.

The solution for the periodic variation δS^* of the secularly increasing S^* is

$$\delta S^* \simeq \frac{K^*}{1 - K^*} \times \{(630e_{10})(630e_{12}^*)^2\sin(S^* + \tilde{\omega}_{s1}) - (560e_{20})(560e_{12}^*)^2\sin(S^* + \tilde{\omega}_{s2})\}, \tag{88}$$

which is adequate if $e_{10} \ll (1 - K^*)/630$ or $e_{20} \ll (1 - K^*)/560$. The existence of nonzero free eccentricities corresponds to a libration of either ψ_1^* or ψ_2^* if either $e_{10} \ll e_{12}^*$ or $e_{20} \ll e_{21}^*$, respectively. The fact that ν_2 and therefore ξ continue to tidally decrease even if either angle variable librates implies that K^* continues to increase. As $1 - K^*$ tends to vanish unimpeded by the tides, this solution eventually breaks down no matter how small we choose e_{10} or e_{20}. This suggests that this interaction is unstable and that once $\delta S^* \sim O(1)$ that the appropriate resonance variable ceases to librate and begins circulating.

We shall use the Hamiltonian formalism developed in Section 3 to derive an estimate of the excited e_{20} associated with transition from libration to cirulation. The one-dimensional Hamiltonian in this case is

$$H = -(\xi + \dot{\omega}_{s2})x^* + \tfrac{1}{2}h_0x^{*2} + h_1e_2^*\cos\psi_2^*, \tag{89}$$

where $h_0 = 12(1 + 5\alpha^2M_2/M_1)/M_1a_2^2$ and $h_1 \simeq 7\,M_1a_2^2\,(M_2/M_J)n_2^2e_{23}$. The action variable x^* canonically conjugate to ψ_2^* is related to n_1^*, n_2^*, and e_{21}^* by the relations

$$n_1^* = n_{10} - 6x^*/M_1a_1^2,$$
$$n_2^* = n_{20} + 15x^*/M_2a_2^2,$$
$$e_2^{*2} = e_{20}^2 - 2x^*/M_2n_2a_2^2. \tag{90}$$

These relations can be verified by comparing the dynamical equations generated by the Hamiltonian (14), (89) with (80)–(83), omitting the contribution due to e_{10}. Again, let us consider the limit where the forced $e_2^* \gg e_{20}$. From a stability analysis, we find that the stationary center at $\psi_2^* = 0$ is unstable for a critical, forced e_2^* which satisfies

$$e_{2c} = (h_1/h_0L_2^2)^{1/3} \simeq 1/560. \tag{91}$$

From (89) we find that two solutions exist if $e_2^* = e_{2c}$ at $\psi_2^* = 0$. The first is the (unstable) stationary solution and the second is a periodic solution where ψ_2^* circulates. Evaluating (89) in the limit where $e_2^* = e_{2c}$ and $e_{20} \ll e_{2c}$ at $\psi_2^* = 0$, we find that $y = x^*/x_c^* \simeq$

$(e_2^*/e_{2c})^2$ obeys the equation

$$H = \tfrac{3}{8}h_0 e_{2c}{}^4 L_2{}^2 = h_0 e_{2c}{}^4 L_2{}^2(-\tfrac{3}{4}y + \tfrac{1}{8}y^2 + y^{1/2}\cos\psi_2^*. \quad (92)$$

The action integral J for this example is

$$J = \oint x^* d\psi_2^*. \quad (93)$$

If we momentarily ignore the tidal damping of the free eccentricity, we can determine the amplitude of the excited e_{20} by evaluating the action integral (92) at both transition and far past transition when $\xi \gg 0$. At start of circulation, $J = 12\pi x_c^*$. In the circulating case, the periodic component of x^* is proportional to the forced e_2^* while the constant part is proportional to e_{20}. Once transition into the circulating phase has occurred, the tidal torque continues to increase the frequency ξ and decrease e_2^*. The action integral (93) asymtotically approaches the time average value of x^* times 2π as $\xi \to \infty$. From (90) we find the excited $e_{20} \simeq (-4\pi \langle x^* \rangle / M_2 n_2 a_2{}^2)^{1/2}$ far past transition. The action integral (93) can be evaluated explicitly at transition in the limit $e_{20} \to 0$ prior to passage (Yoder, 1973, 1979a). We find

$$e_{20}(\text{excited}) = 6^{1/2} e_{2c} = 0.0043. \quad (94)$$

The decrease in the forced e_2^* as ξ increases is more rapid than the tidal damping of e_{20}. In each case, passage through each resonance excites a free eccentricity that is $6^{1/2}$ larger than the critical forced e_c which causes the factor $(1 - K^*)$ to vanish in (88). Io's free eccentricity e_{10}, once Ψ_1^* begins circulating, is $\simeq 0.0039$ and is reduced to less than 1×10^{-5} in $4 \times 10^5\, Q_1/f_1$ years by tidal dissipation in Io (68). Even though this excitation may have occurred only 2×10^3 Q_J years ago, the remnant free e_{10} is well below the observational limit, $\sim 10^{-5}$. On the other hand, Europa's e_{20} is $\simeq 0.0043$ immediately after passage. The observed $e_{20} = 0.00009$ may well be a remnant of this event. We find from (68) that the elapsed time necessary to tidally reduce e_{20} is $\simeq 3.9$ $\tau_2(e_2) = 1.1 \times 10^7\, Q_2/f_2$ years. We can compare this damping time with the upper bound for the time since passage based on the three body libration amplitude. We find $Q_2/f_2 \gtrsim 2 \times 10^{-4}\, Q_J$. This lower bound on Q_2/f_2 seems more plausible than the earlier estimate (69) and is consistent with the hypothesis that the three-body lock is relatively young and the Jovian Q is relatively small ($\sim$few $\times\, 10^5$).

The interaction involving the Ganymedan variable ψ_3^* arises from terms in R which are third order in the eccentricities and second order in δV_2. We estimate that passage in this case excites a free $e_{30} \sim 10^{-4}$. Since the observed e_{30} is considerably larger ($\sim$0.0015), we conclude that the observed Ganymedan eccentricity is most likely the tidal remnant of a primordially established e_{30}. Still, we should point out that these subtle interactions involving the frequency ξ are sufficiently complex that we may have missed some contribution which could account for the observed e_{30}.

Before the sequence of three-body resonances was approached, it is possible that the ratios of the semimajor axes of adjacent orbits were small enough (<0.48) that at least one pair of satellites was driven through a two-body resonance in which $n_i : n_j = 3:1$. Potentially three distinct 3 : 1 encounters may have occurred in the distant past. The first two involve the pairs Io–Europa and Europa–Ganymede. Since the tidal rate of change of the semimajor axis is $\propto a^{-11/2}$ we find that either of these encounters must have happened within the first billion years after planetary formation. The third candidate is Ganymede–Callisto for which the present ratio of semimajor axes is 0.57. Since the tidal expansion of the three-body lock is the only mechanism which can substantially alter the ratio a_3/a_4, this last encounter could only have happened if the three-body lock is very old.

The six resonance variables associated with this commensurability for the Io–Europa pair are $\lambda_1 - 3\lambda_2 + 2\Omega_1$, $\lambda_1 - 3\lambda_2 + \Omega_1 + \Omega_2$, $\lambda_1 - 3\lambda_2 + 2\Omega_2$, $\lambda_1 - 3\lambda_2 + 2\tilde{\omega}_2$, $\lambda_1 - 3\lambda_2 + \tilde{\omega}_1 + \tilde{\omega}_2$, and $\lambda_1 - 3\lambda_2 + 2\tilde{\omega}_1$ with relative strengths $\sin^2 I_1$, $\sin I_1 \sin I_2$, $\sin^2 I_2$,

e_2^2, e_1e_2, and e_1^2 respectively. The corresponding frequencies vanish in the same order as $n_1 - 3n_2$ is decreased by the tides, since the resonance induced motion of both Ω_i and $\tilde{\omega}_i$ are small compared to the frequency spacing (unlike the 2:1 resonance). The reasonably large frequency spacing compared to the libration frequency also ensures that the variables only weakly interact as one of the variables vanishes (undergoes transition into resonance), so the resonances associated with the 3:1 commensurability can be considered one at a time.

Yoder (1973) has shown that capture of any of the two-body resonance variables into a liberating mode from a circulating mode depends on the value of the corresponding free eccentricity or free inclination. For a free eccentricity or inclination below a critical value capture is certain, whereas capture is probabilistic otherwise and depends on the initial conditions as the resonance variable begins its last circulation before reversing its direction. An analogous situation arises in the three-body resonance discussed above if we assume that ξ is initially positive and slowly decreasing as the resonance is approached. That is, the system is initially circulating with some free eccentricity.

The excited free eccentricity given by (94) now assumes the role of the critical eccentricity separating certain from probabilistic capture. If the free eccentricity is smaller than this critical value, capture of the three-body resonance from this approach from positive values of ξ is certain and probabilistic otherwise.

Subsequent to capture of a resonance variable into libration the corresponding eccentricity or inclination is forced to larger values as tides drive $n_i - 3n_j$ closer to exact commensurability (see Section 4). With these ideas we can infer the consequences of encounters with the 3:1 resonances by the Galilean satellites.

Table II contains estimates of the critical free eccentricities and inclination for the

TABLE II

CRITICAL INCLINATIONS I_c AND ECCENTRICITIES e_c FOR THE SIX 3:1 TWO-BODY RESONANCE INTERACTIONS[a]

	Io–Europa	Europa–Ganymede	Ganymede–Callisto
$I_{1c}(I_1^2)$	0.0021	0.0064	0.0036
$I_{1c}(I_1I_2)$	$0.0021\kappa_I$	$0.0061\kappa_I$	$0.0034\kappa_I$
$I_{2c}(I_1I_2)$	$0.0026\kappa_I^{-1}$	$0.0014\kappa_I^{-1}$	$0.0033\kappa_I^{-1}$
$I_{2c}(I_2^2)$	0.0027	0.0014	0.00434
$e_{2c}(e_2^2)$	0.0038	0.0020	0.0048
$e_{2c}(e_1e_2)$	$0.0067\kappa_e^{-1}$	$0.0035\kappa_e^{-1}$	$0.0083\kappa_e^{-1}$
$e_{1c}(e_1e_2)$	$0.0053\kappa_e$	$0.0156\kappa_e$	$0.0087\kappa_e$
$e_{1c}(e_1^2)$	0.0020	0.0060	0.0034

[a] If the free $I_0 \leq I_c$ or free $e_0 \leq e_c$ then the capture probability $P_c = 1$ for the appropriate interaction. The factors $\kappa_I = (I_{20}/I_{10})^{1/2}$ and $\kappa_e = (e_{20}/e_{10})^{1/2}$. The subscripts "1" and "2" refer to the inner and outer member of the pair, respectively.

3:1 resonance variables (Yoder, 1973). Two complementary estimates are obtained for the mixed-e- and mixed-I-type resonances. Here the critical values for, say, I_1 and e_1 are based on the assumption that I_2 and e_2 are relatively large, whereas I_1 and e_1 are assumed large when estimating the critical values of I_2 and e_2. Comparison of Tables I and II shows that the critical e's and I's are of the same order as their present values. But the encounters (if they occurred) happened in the distant past, when the free-orbital eccentricities and inclinations may have been substantially larger. We have seen in Section 4 that tidal friction within the satellites themselves is very efficient in damping the eccentricities of at least Io and Europa but tidal damping of orbital inclinations is given by

$$\tau(I) = \tfrac{7}{4}\tau(e)\,(\sin I/\sin \epsilon)^2, \qquad (95)$$

where ϵ is the relative obliquity of the satellite spin axis to the orbit normal. The factor $(\sin I/\sin \epsilon)^2$ equals ~500, 100, 100, and ~10 for Io, Europa, Ganymede, and Callisto. These estimates were obtained using the hydrostatic value of the satellite's J_2 and C_{22} and the present values of the semimajor axis to determine the obliquities in Cassini state 1 (see Peale, 1977). Except for Io, this eliminates satellite solid friction as a significant mechanism which tidally reduces the orbital inclinations. On the other hand, if any of the satellites had a large fluid core for a significant fraction of the age of the solar system, core–mantle friction might have plausibly reduced the satellite orbital inclinations. If core–mantle friction is not an important process, present values of I are more nearly representative of primordial values, and we shall concentrate on the I-type interactions.

Since the primordial inclination is uncertain, we can adopt a "plausible" upper bound of $I_0 \leq 0.01$ so that we can proceed with estimates of the probability P_c for capture into libration (Yoder, 1973). Figure 1 is a graph of P_c versus the I_0/I_c for two distinct types of interactions. Curve A describes the capture probability for the variable $\lambda_1 - 3\lambda_3 + \Omega_1 + \Omega_2$. The capture probability for the variables $\lambda_1 - 3\lambda_3 + 2\Omega_1$ and $\lambda_1 - 3\lambda_3 + 2\Omega_2$ are obtained from curve B. We find that the probability that the satellite pairs Io-Europa, Europa-Ganymede and Ganymede-Callisto evaded capture into any one of the three possible I-type locks is $\lesssim 0.5$, $\lesssim 0.2$, and $\lesssim 0.4$, respectively, if we assume each I(free) <0.01 during the encounter.

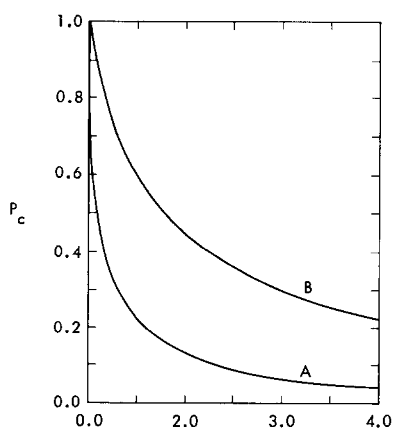

FIG. 1. Probability of capture for order 1 (curve A) and order 2 (curve B) type resonance interactions versus $I_0(\text{free})/I_c$.

It seems probable that *at least one* of the three I-type interactions should have been engaged for each 3 : 1 commensurability. Once engaged, the lock tends to pump up the appropriate orbit inclination unimpeded by tidal friction in either the satellite or planet. It may be that the 3 : 1 I-type locks become unstable for sufficiently large I such that the resonance partners disengage. More study is required to resolve this possibility. Even if this occurs, the disengagement would result in the excitation of a free I of the same order as the forced I. The fact that all I(free) are now small and could not have been much larger in the past is offered as evidence that the ratios of the mean motions n_1/n_2, n_2/n_3, and n_3/n_4 have *always* been less than 3 and the corresponding $\alpha > 0.48$. This limitation on the maximum primordial spacing between the Galilean satellites is consistent with the observed spacing for both the planets and other satellite systems.

6. RELAXATION HYPOTHESIS

The tidal scenario for the origin of the Galilean satellite resonances presumes the existence of a strong tidal torque acting to expand Io's orbit over time. We have argued that the observed configuration was formed and is maintained by the competing forces arising from tidal dissipation in Io and Jupiter and the resonant gravitational interactions. However, there is no compelling observational evidence, as yet, that Io's orbit is expanding or that a mechanism exists which accounts for significant tidal dissipation in Jupiter. The present state of affairs regarding these questions is discussed in the following two sections. If the Jovian tidal torque is indeed small, then the Galilean system may not be in quasi-stationary equilibrium but may be relaxing from a more tightly bound orbital configuration where ν is closer to zero and the forced eccentricities are larger.

A simple estimate of maximum energy

available from the relaxation of the three-body lock can be obtained from the change in the total orbital energy $\Delta E_{\mathrm{T}} = -\sum_i E_i \Delta a_i / a_i$ where $E_i = \frac{1}{2} M_i n_i^2 a_i^2$ and $i = 1, 2, 3$. Since tidal dissipation in any of the satellites conserves the sum of their orbital angular momenta $\mathscr{L}_{\mathrm{T}}$, the tidal change in the semimajor axes can be converted into a variation in the orbital eccentricities and the 2:1 frequency $\nu = n_1 - 2n_2 = n_2 - 2n_3$ using $\Delta\mathscr{L}_{\mathrm{T}} = 0$. The result is

$$\Delta E_{\mathrm{T}} = E_1\{0.56\,(\Delta\nu/n_2) - 0.46\Delta e_1^2 - 0.31\Delta e_2^2 - 1.22\Delta e_3^2\}, \quad (96)$$

where $\Delta e_i^2 = e_i^2\,(\text{initial}) - e_i^2\,(\text{final})$ and $\Delta\nu = \nu\,(\text{initial}) - \nu\,(\text{final})$.

Reasonable upper bounds are e (initial) < 0.1 and $\Delta\nu < 0.^\circ 74$/day. We find $E_{\mathrm{T}} < 0.024\, E_1 \sim 3 \times 10^{36}$ ergs. Even if this energy were uniformly dissipated over time, the upper bound on the tidal heating rate is 2×10^{19} ergs/sec. We shall argue here that a more reasonable upper bound on $E_{\mathrm{T}} \lesssim 0.004\, E_1$.

Recall that the Hamiltonian (12)–(14) is a constant of the motion in the absence of dissipation. We can also verify that the condition that all the angle variables (16) are stationary (i.e., are constant) represents a local energy extreme. This is equivalent to the condition that the free eccentricities vanish. For small eccentricities and positive ν, the stable libration centers are located at $\phi_1 = 0$, $\phi_2 = \pi$, $\phi_3 = 0$, and $\phi_4 = \pi$. The variables ϕ_1 and ϕ_3 correspond to the perturbation of the "inner" by the "outer" satellite. The condition that ϕ_1 and $\phi_3 = 0$ means that pairwise conjunction occurs at "inner" satellites' pericenter. The "outer" satellite variables ϕ_2 and $\phi_4 = \pi$, and mutual conjunction occurs at the outer satellites' apocenter.

In order to evaluate the hypothesis that the resonances are primordial and we are watching them decay due to dissipation in the satellites, the stability must be ascertained at the higher forced eccentricities inferred for the past. The current configuration of the satellites is almost a stationary solution in the sense that ϕ_1, ϕ_2, ϕ_3 are librating respectively about $\phi_1 = 0$, $\phi_2 = \pi$, and $\phi_3 = 0$. Only ϕ_4 is circulating due to the fact that the current free eccentricity of Ganymede exceeds the forced eccentricity. However, our immediate purpose is to test the stability of the resonant configuration at higher forced eccentricities where the forced eccentricity of Ganymede may be sufficiently high to make ϕ_4 a librating variable. A test of the stability of the complete stationary solution is thereby appropriate.

The stability of the stationary solutions can be determined by expanding the dynamical equations about the stationary values of the variables. If we replace x_i by $x_{i0} + \delta x_i$ and ϕ_i by $\phi_{i0} + \delta\phi_i$ in (14) and expand to first order in the small variations δx_i and $\delta\phi_i$, we obtain

$$\begin{aligned} \delta\dot{x}_i &= H^{im}\,\delta\phi_m + H_m{}^i\,\delta x_m, \\ \delta\dot{\phi}_i &= H_i{}^m\,\delta\phi - H_{im}\,\delta x_m, \end{aligned} \quad (97)$$

where $H^{im} = \delta^2 H/\delta\phi_i\delta\phi_m$, $H_{im} = \delta^2 H/\delta x_i \delta x_m$, and $H_m{}^i = \delta^2 H/\delta x_m \delta\phi_i$, evaluated at $\phi_i = \phi_{i0}$. Summation over the repeated indices is understood. The mixed partials $H_m{}^i = H_i{}^m$ vanish if $\phi_{i0} = 0$ or π. The time derivative of the $\delta\phi_i$ equation yields

$$\begin{aligned} \delta\ddot{\phi}_i + M_{im}\,\delta\phi_m &= 0, \\ M_{im} &= H_{il}H^{lm}. \end{aligned} \quad (98)$$

If a solution of the form $\delta\phi = \delta\phi_{\max} e^{i\omega t}$ is assumed, the frequencies ω must satisfy the equation

$$\det\,(\mathbf{M} - \omega^2\mathbf{I}) = 0. \quad (99)$$

The various stationary positions are stable if the appropriate ω^2 as determined by (99) are positive definite.

The evaluation of $H^{il} = R^{il}$ in M_{im} follows easily from (12). However, H_{lm} is considerably more complicated although much simplification of the complete expressions is possible. We can write

$$H_{lm} = \frac{\partial^2 H}{\partial x_l \partial x_m} = \sum_{i,k} \frac{\partial^2 H}{\partial L_k \partial L_i} \frac{\partial L_k}{\partial x_l} \frac{\partial L_i}{\partial x_m} + \frac{\partial^2 H}{\partial \Gamma_k \partial L_i} \frac{\partial \Gamma_k}{\partial x_l} \frac{\partial L_i}{\partial x_m} + \frac{\partial^2 H}{\partial L_k \partial \Gamma_i} \frac{\partial L_k}{\partial x_l} \frac{\partial \Gamma_i}{\partial x_m} + \frac{\partial^2 H}{\partial L_k \partial \Gamma_i} \frac{\partial L_k}{\partial x_l} \frac{\partial \Gamma_i}{\partial x_m}. \quad (100)$$

where from (17) we have, for example,

$$H_{11} = \frac{\partial^2 H}{\partial L_1{}^2} - \frac{4\partial^2 H}{\partial L_1 \partial L_2} + \frac{4\partial^2 H}{\partial L_2{}^2} + \frac{2\partial^2 H}{\partial \Gamma_1 \partial L_1} - \frac{4\partial^2 H}{\partial \Gamma_1 \partial L_2} + \frac{\partial^2 H}{\partial \Gamma_1{}^2}.$$

From (7) the right-hand side of (100) can be expressed in terms of $\partial H/\partial a_i$, $\partial^2 H/\partial a_i \partial a_j$, $\partial H/\partial e_i$, $\partial^2 H/\partial e_i \partial e_j$, and $\partial^2 H/\partial a_i \partial e_j$. The evaluation of these last partial derivatives shows that all terms involving $\partial R/\partial a_i$, $\partial^2 R/\partial a_i \partial a_j$, $\partial^2 R/\partial a_i \partial e_j$ are smaller by factors of at least e_i^2 than $\partial^2 R/\partial \Gamma_i^2$ and smaller by at least a factor of e_i than $\partial^2 R/\partial \Gamma_i \partial \Gamma_j$. As every $\partial/\partial L_i$ contains $\partial/\partial a_i$, we can therefore neglect all $\partial R/\partial L_i$. This elimination of $\partial/\partial a_i$ in fact allowed the simplification used in Section 2, where $R_1{}^2 = R_2{}^1$ was assumed. The fact that $\partial R_1{}^2/\partial a_i \neq \partial R_2{}^1/\partial a_i$ is not relevant since these terms are relatively small. Finally, from the form of H^0 (13), only $\partial^2 H^0/\partial L_i^2$ are nonzero. The only terms which need be kept in the expressions for H_{lm} (100) are $\partial^2 H^0/\partial L_i^2$, $\partial^2 R/\partial \Gamma_i^2$, and $\partial^2 R/\partial \Gamma_i \partial \Gamma_j$.

The forms of M_{lm} are now determined but their numerical values require the stationary values of e_i. These can be determined from $\dot{\phi} = 0$ at $\phi_1, \phi_2, \phi_3, \phi_4 = 0, \pi, 0, \pi$ respectively or perhaps more directly from the condition $\dot{\tilde{\omega}}_1 = \dot{\tilde{\omega}}_2 = \dot{\tilde{\omega}}_3$. The stationary values of the e_i as a function of ν are given in Fig. 2.

For the current values of the forced eccentricities the eigenfrequencies obtained from (99) are $\omega_1 = 0^\circ.899$/day, $\omega_2 = 0^\circ.785$/day, $\omega_3 = 0^\circ.749$/day, and $\omega_4 = 0.179^\circ$/day. The first three are essentially the frequencies of oscillation of the e_i about their stationary values ($\nu + \dot{\tilde{\omega}}_{si}$ with $\nu = 0^\circ.740$/day) and the last is the libration frequency of the Laplace angle $\lambda_1 - 3\lambda_2 + 2\lambda_3$, which corresponds to a period of 2011 days.

The match of these eigenfrequencies to those obtained from theories first order in e_i shows that whether or not ϕ_4 is circulating has little effect. The eigenfrequencies ω_1, ω_2, ω_3 decrease monotonically as ν is decreased (force e_i increased) and ω_4 decreases monotonically after a maximum at $e_1 = 0.0055$. For $e_1 > 0.012$, ($e_2 > 0.035$, $\nu < 0^\circ.14$/day) $\omega_4^2 < 0$ and the Laplace libration is unstable. Hence, for larger values of e, the condition $\nu_1 = \nu_2$ can no longer be assumed and $e_1 = 0.012$ is an upper bound beyond which the current configuration cannot exist. In other words $\nu_1 = 0^\circ.14$/day is as deep into the existing set of resonances that the system could have started if it had had a primordial origin. On the other hand, it may be possible to capture the system into the present configuration as it decays from some higher energy state.

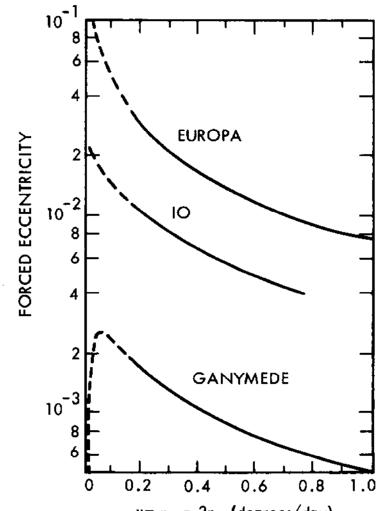

FIG. 2. The 2:1 forced eccentricities of Io, Europa, and Ganymede versus ν for $\nu > 0$. The solid part of each curve corresponds to stable three-body libration while the dashed line indicates instability.

We can determine the time of relaxation from this initial condition to the present configuration by adding the effects of tidal dissipation as a perturbation in the Hamiltonian formalism involving the conjugate variables x_i and ϕ_i. Equations (17) can be solved for the x_i and the tidal contributions to dx_i/dt determined by the tidal variations in L_i and Γ_i which follow from (3)–(7) and the relations $dL/dt = -L(dn/dt)/(3n)$ and $d\Gamma/dt \simeq -L(de^2/dt)/2$. We find

$$\begin{aligned}
\frac{dx_1}{dt} &= \frac{d\Gamma_1}{dt} = \frac{\partial R}{\partial \phi_1} + \frac{d\Gamma_1}{dt_{\mathrm{T}}},\\
\frac{dx_2}{dt} &= \frac{d\Gamma_2}{dt} + \frac{1}{2}\frac{dL_3}{dt} + \frac{d\Gamma_3}{dt} = \frac{\partial R}{\partial \phi_2} + \frac{d\Gamma_2}{dt_{\mathrm{T}}},\\
\frac{dx_3}{dt} &= -\frac{1}{2}\frac{dL_3}{dt} - \frac{d\Gamma_3}{dt} = \frac{\partial R}{\partial \phi_3},\\
\frac{dx_4}{dt} &= \frac{d\Gamma_3}{dt} = \frac{\partial R}{\partial \phi_4},\\
\frac{dx_5}{dt} &= \frac{dL_1}{dt_{\mathrm{T}}} - \frac{d\Gamma_1}{dt_{\mathrm{T}}} - \frac{d\Gamma_2}{dt_{\mathrm{T}}},\\
\frac{dx_6}{dt} &= \frac{dL_2}{dt_{\mathrm{T}}} + \frac{2d\Gamma_1}{dt_{\mathrm{T}}} + \frac{2d\Gamma_2}{dt_{\mathrm{T}}},
\end{aligned} \tag{101}$$

with

$$\begin{aligned}
\frac{dL_i}{dt_{\mathrm{T}}} &= \frac{L_i c_i}{3}[1 - 7D_i e_i^2],\\
\frac{d\Gamma_i}{dt_{\mathrm{T}}} &= \frac{7L_i c_i D_i e_i^2}{3},
\end{aligned} \tag{102}$$

where the subscript T indicates a variation due to tidal dissipation. In (102) we have neglected the constant coefficient of e_i^2 compared with D_i, and in (101) we have neglected the tidal effects on Ganymede as being negligibly small. Note that x_5 and x_6, formerly constants of the motion, are now time variable in the presence of the dissipation.

As the forced value of e_1 (or ν) completely specifies the remaining variables given the constraints of the resonances with damped librations, our goal is to find the tidal variation of either of these parameters. From (13) (7) and (17)

$$\begin{aligned}
\frac{\partial H^\circ}{\partial x_1} &= \frac{\partial H^\circ}{\partial L_1} - \frac{2\partial H^\circ}{\partial L_2} - \dot{\omega}_{s1}\\
&= -n_1 + 2n_2 - \dot{\omega}_{s1}
\end{aligned}$$

and from

$$\frac{d}{dt}(n_1 - 2n_2 + \dot{\omega}_{s1}) = \frac{d\nu_1}{dt} = -\frac{d}{dt}\frac{\partial H^\circ}{\partial x_1}$$

and similar relations, we have

$$\frac{d\nu_i}{dt} = -\sum_l H^\circ_{ml}\frac{dx_l}{dt}, \tag{103}$$

where $i = 1$ for $m = 1$ or 2 and $i = 2$ for $m = 3$ or 4.

If we assume the librations are completely damped or, alternatively, we average over libration periods, we have $d\phi_i/dt = -\partial H/\partial x_i = 0$. In addition,

$$\begin{aligned}
\frac{d^2\phi_i}{dt^2} = 0 &= -\sum_l H_{il}\frac{dx_l}{dt}\\
&= \sum_l -H^\circ_{il}\frac{dx_l}{dt} - R_{il}\frac{dx_l}{dt}.
\end{aligned} \tag{104}$$

From (103) and (104)

$$\frac{d\nu_i}{dt} = \sum_l R_{ml}\frac{dx_l}{dt}, \tag{105}$$

where $i = 1$ for $m = 1$ or 2 and $i = 2$ for $m = 3$ or 4.

From the discussion immediately following (100) it can be shown that R_{15} and R_{16} are negligibly small. Also with $R_{12} = R_{13} = \partial^2R/\partial\Gamma_1\partial\Gamma_2$ and $d\Gamma_i/dt \approx -L_i(de_i^2/dt)/2$ we have from (100) and (101),

$$\begin{aligned}
\frac{d\nu_i}{dt} &= -R_{m1}\frac{L_1}{2}\frac{de_1^2}{dt}\\
&\quad - R_{m2}\frac{L_2}{2}\frac{de_2^2}{dt} - R_{m4}\frac{L_3}{2}\frac{de_3^2}{dt},
\end{aligned} \tag{106}$$

where $i = 1$ for $m = 1$ or 2 and $i = 2$ for $m = 3$ or 4 and e_i^2 are understood to be the forced values. Since $R_{2j} = R_{3j}, j = 1, 2, 4$, the right-hand sides of (105) for $i, m = 1, 2$ and $i, m = 2, 3$ are identical, and $d\nu_1/dt = d\nu_2/dt$, another condition for maintaining the resonance, follows from $d^2\phi_2/dt^2 = 0$ and $d^2\phi_3/dt^2 = 0$. Equating the right-hand sides of the three independent forms of (106) provides two equations from which we can write

$$\frac{de_2^2}{dt} = \chi_{12} \frac{L_1}{L_2} \frac{de_1^2}{dt},$$
$$\frac{de_3^2}{dt} = \chi_{13} \frac{L_1}{L_3} \frac{de_1^2}{dt}, \quad (107)$$

where

$$\chi_{12} = \frac{R_{11}R_{44} - R_{11}R_{24} - R_{12}R_{44}}{R_{22}R_{44} - R_{12}R_{44} + R_{12}R_{24} - R_{24}^2},$$
$$\chi_{13} = \frac{R_{11}R_{22} - R_{11}R_{24} + R_{12}R_{24} - R_{12}^2}{R_{22}R_{44} - R_{12}R_{44} - R_{12}R_{24} - R_{24}^2}, \quad (108)$$

with $R_{14} = R_{41} = 0$ being used.

From (107) we can write

$$\frac{d\nu}{dt} = [R_{11} + \chi_{12}R_{12}] \frac{L_1}{2} \frac{de_1^2}{dt}. \quad (109)$$

In the two independent relations for $d\nu/dt$ in (103), $H^\circ_{12} \neq H^\circ_{13}$, H°_{15} and H°_{16} are not small and dx_5/dt and dx_6/dt appear explicitly. We can eliminate $dx_3/dt = -(dL_3/dt)/2 - d\Gamma_3/dt$ from these two equations and obtain $d\nu/dt$ in terms of de_i^2/dt and the tidal change in x_5 and x_6. Combining this result with (109) together with (101) and (102) yields

$$\frac{de_1^2}{dt} = -X[n_1c_1(1 - 13D_1e_1^2) + 0.094n_2c_2(1 - 94D_2e_2^2)], \quad (110)$$

where

$$X = \frac{0.646}{L_1[R_{11} + \chi_{12}R_{12} + (1.134/M_2a_2^2)(1 + \chi_{12} + \chi_{13})]}. \quad (111)$$

In (110) and (111) functions of H°_{ij} have been evaluated explicitly using $\partial^2H^\circ/\partial L_i^2 = 3/(M_ia_i^2)$ in (100) with $\alpha = 0.63$ and M_i/M_J taken from Table I. Equation (110) reduces essentially to the equivalent of (39) (with $\langle d\phi/dt\rangle = 0$) when we restrict R to first-order terms in e_i, with de_1^2/dt being expressed in terms of $d\nu_1/dt$ from (26).

Equation (110) can be integrated numerically for the case where the torque from Jupiter vanishes ($Q_J \to \infty$) in order to find the minimum time required for the system to relax from $e_1 = 0.012$ to the current $e_1 = 0.0041$. Dissipation in Europa has only a modest effect since $c_1D_1 \approx 0.03c_2D_2$ and $e_2/e_1 < 3$, so we find for the relaxation time ($k_1 = 0.035$)

$$\Delta t \approx 3.4 \times 10^6\, Q_1/f_1 \text{ years}. \quad (112)$$

With $\Delta t \approx 4.6 \times 10^9$ years $Q_1f_1 \approx 1400$. This bound is comparable to that obtained by Peale and Greenberg (1980) even though the latter analysis incorrectly extended the theory linear in e_i to values of e_i where it is invalid. The reason for the near agreement is that the system spends most of the time near the smaller e_i where the first- and third-order analyses converge.

We can determine the total energy dissipated from (96) since $\Delta\nu$ and Δe_i^2 are known.

$$\Delta E_T \lesssim 4.1 \times 10^{-3}E_1 = 5.5 \times 10^{35} \text{ ergs}, \quad (113)$$

with nearly all of the energy being deposited in Io. On the other hand, the observation of Io's elevated interior temperatures (Morabito *et al.*, 1979) and inferences that a considerable fraction may be melted (Peale *et al.*, 1979a) means that tidal dissipation has surely exceeded the total energy input due to radioactive decay in Io over the 4.6 $\times$ 10^9-year time span. Only in this way could we account for the drastic difference between the thermal state of Io and that of, say, the Moon. Radioactive element distributions from lunar samples and current heat flow are folded into a model of lunar thermal history which allows estimates of primordial and current heating by radioactive decay (Cassen *et al.*, 1979). The current average heating rate estimated in this model is 6.9×10^{-8} erg g^{-1} sec^{-1} with 2.6×10^{-7} erg g^{-1} sec^{-1} being the initial rate 4.6×10^9 years ago (P. Cassen, private communication, 1980). The same distribution of radioactivity in Io leads to a total heat input of 1.6×10^{36} ergs over solar system history. About the same total heating would result if Io had the radioactive element content of carbonaceous chondrites and 2.1×10^{36} ergs would result from ordinary chondritic composition. These values exceed that in (113) by a factor of 3 or 4, and we therefore infer that the tidal dissipation in Io has been considerably larger than that necessary to relax the system from the most extreme primordial values of the parameters in the resonance to the current values in the absence of any torques supplied by Jupiter.

The only way this dissipation can be accommodated is for Jupiter to supply torque to retard the relaxation, otherwise the system would have relaxed to smaller eccentricities (larger ν) than we now observe. This same conclusion was reached by Peale and Greenberg (1980). Moreover, this latter analysis is valid for values of e_1 approaching the current value. Since the total relaxation time from a hypothesized deep primordial resonance is about the same for the two analyses involving first- or up to third-order terms in the eccentricity, we conclude with Peale and Greenberg *that the current eccentricity of Io's orbit is very close to the equilibrium value, where $d\nu/dt$ vanishes whether or not the resonance configuration was primordial.* The upper bound $Q_J \lesssim 3.5 \times 10^6$ (66) obtained earlier still applies.

Therefore, even though the primordial origin of the existing resonance is not ruled out, the system cannot be stored deeper in this resonance for the age of the solar system in hopes of accommodating a larger Q_J as suggested by Greenberg (1981). The observed high dissipation rate in Io requires that Jupiter supply a significant torque to prevent the system from decaying too far in the age of the solar system.

There is, however, another stable stationary solution to (12)–(14) which offers the possibility of longer storage times. This case occurs when $\nu \ll 0$ with $\phi_1 = \phi_2 = \phi_3 = \pi$ and $\phi_4 = 0$ (Sinclair, 1975). Forced eccentricities as a function of negative ν are shown in Fig. 3. The libration about ϕ_{10} becomes unstable when $e_1 > 0.012$ or $\nu \gtrsim -0°.50$/day. If we use data from Fig. 3 in (96) we find that less negative ν and larger e_i corresponds to a state of lower total energy. This means that dissipation within the satellites tends to drive the system toward the resonance whereas a torque from Jupiter drives ν more negative and hence away from the resonance. If the Jupiter torque is negligibly small while the system is locked in resonance with $\nu \ll 0$, dissipation in the satellites would cause the system to fall deeper into the resonance. However, unlike the case for $\nu > 0$, the dissipation and hence the rate of evolution could be initially small since the e_i would be small. If the system is to reach the current configuration from this initial state it must pass through

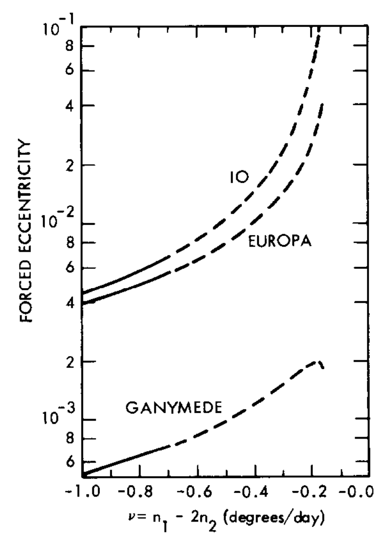

FIG. 3. The 2:1 forced eccentricities of Io, Europa and Ganymede as a function of ν, where $\nu < 0$.

the instability which occurs when $e_1 >$ 0.012 and reassemble itself on the other side of the resonance with $\nu > 0.^\circ 14$/day and $e_1 < 0.012$. Whether or not this scenario is even possible requires a detailed analysis which is beyond the scope of this paper. We must regard the scheme as highly improbable in any case since the system must be precisely started with eccentricities large enough to ensure the equality of $\dot{\tilde{\omega}}_1$ and $\dot{\tilde{\omega}}_2$ and to guarantee that dissipation in the satellites is sufficient to dominate the evolution of ν, but still as small as possible so the evolution does not accelerate too fast. This constraint suggests that the minimum initial value of $\nu \sim -3°$/day based on a maximum effective $Q_J \approx 1 \times 10^7$ and $Q_1/f_1 \approx 100$ appropriate to an initially cold Io. The upper bound on Q_J is the equivalent for the present electromagnetic torque on Io. As e_1 increases, tidal heating of Io is expected to lead to core formation, thereby increasing f_1 and accelerating the increase in ν. From (96), the maximum available energy for this process is $\sim 0.02E_1 = 3 \times 10^{36}$ ergs or about twice that available from the decay of radioactive material within Io over a 4.6×10^9-year time span.

Passage through the instabilities for ν near zero will also excite sizable free eccentricities of order 0.01 or greater in their orbits (Section 5). The relaxation time of the system to the present configuration from the instability point (112) would likely be no greater than 10^8 years for reasonable values of Q_1/f_1. But the damping times of free eccentricities shown in (68) (particularly for Europa) are too large to be compatible with the relaxation time if it is as short as a few tens of millions of years.

If current estimates of the flux of energy from Io of 1000–2000 ergs/cm² sec (Matson *et al.*, 1981; Sinton, 1981; Morrison and Telesco, 1980) are representative of the current rate of tidal dissipation, the effective $(Q_1/f_1) \approx 3$ to 6 leading to a decay time from (112) of only 9.4×10^6 to 1.9×10^7 years. The resonance would thus have just jumped the instability gap and settled into the current configuration if in fact its origin were primordial. Moreover, the rate of decay would mean $\dot{n}_1 = 0.9$ to 1.8×10^{-22} rad/sec² (= 185 arcsec/Cy² to 370 arcsec/Cy²) and $\dot{\nu}_1 = 4.5$ to 9×10^{-23} rad/sec². The contraction of Io's orbit would lead to eclipses of Io being 25–50 sec too soon after 100 years. In Section 8 we show that careful analysis of the old eclipse data should produce a tighter constaint on Σ_1 which should allows us to discern whether the system is indeed relaxing or in a nearly equilibrium configuration.

Given the improbability of the sequence of events necessary for long storage of the three satellites in the Laplace resonance with $\nu < 0$ before the current configuration appeared, even if this sequence is physically possible, it seems that it should not have occurred. Storage with $\nu > 0$ is incompatible with the observed high dissipation in Io if there are no significant Jupiter torques. There appear therefore to be no compelling arguments showing that a primordial origin of the orbital resonances can reduce the torques from Jupiter from those values which allow the assembly of the resonances and their subsequent evolution from initially random orbits.

7. DISSIPATION IN JUPITER

Tidal dissipation in Jupiter can be described by the following "solid-body" formula

$$\frac{dE_J}{dt} = \frac{3}{2}\frac{k_J}{Q_J}\frac{M^2}{M_J}n^2a^2\left(\frac{R_J}{a}\right)^5(\Omega_J - n). \quad (114)$$

Here, k_J is the Jovian second harmonic potential Love number, Q the dissipation function, and Ω_J the Jovian rotation rate. M, n, and a refer to the mass, mean angular velocity or mean motion, and semimajor axis of the tide-raising satellite. The torque N_J on the satellite orbit equals

$$N_J = \frac{dE_J}{dt}(\Omega_J - n)^{-1}. \quad (115)$$

The planetary Q is related to the geometric phase lag δ of the tide by $Q^{-1} = \sin 2\delta$.

Also, Q is frequently defined as the ratio of the tidal strain energy divided by 2π times the energy dissipated per cycle. We see from (114) that the energy dissipation rate depends on both k_J and Q_J. It happens that k_J has varied in different models of Jovian dissipation. The Love number k equals $\frac{3}{2}$ for a homogeneous, incompressible liquid and this value was adopted by Goldreich and Soter (1966) in their study of dissipation. Either rigidity or radial density structure can substantially reduce this upper limit to k. Peale *et al.* (1979a) find $k_J = 0.5$ by equating k to the "secular" Love number k_s and using the formula $J_2 = k_s\Omega^2R^3/3\,GM$ which relates the second harmonic gravity coefficient to rotational distortion (cf. Munk and MacDonald, 1960). Gavrilov and Zharkov (1977) find that the bulk $k_J = 0.38$ from an explicit calculation of the effect of tides on Jupiter and this is the value adopted in this paper. The explicit equation relating dissipation and Q_J is therefore

$$dE_J/dt = 1.34 \times 10^{26} Q_J^{-1} \text{ ergs/sec.} \quad (116)$$

for the tide raised by Io. We can obtain lower bounds on Q_J based on the tidal evolution of the three resonantly locked satellites using the expression for the total orbital angular momentum

$$\mathscr{L}_T = \sum_{i=1}^{3} M_i n_i a_i^2 (1 - e_i^2)^{1/2} \quad (117)$$

and the Jovian tidal torque (115). If we assume that the three-body resonance configuration has been maintained for 4.6 by (i.e., $a_1/a_2 = a_2/a_3 = 0.63$ and the eccentricities are constant), we find that the required Q_J to tidally evolve Io from just above synchronous orbit at $a_1 \sim 2.4R_J$ is $\sim 6.6 \times 10^4$. If the three-body lock is recent but the Io–Europa lock has similarly evolved, then Q_J is $\sim 1.7 \times 10^5$.

The Jovian tidal torque is not the only mechanism acting to expand Io's orbit. An alternative mechanism involves the electromagnetic interaction of Io with Jupiter's magnetic field $\vec{B}_J$ which not only expands Io's orbit but also can potentially heat its interior. The relative motion of Io with respect to Jupiter's rotating magnetic field generates a voltage drop of ~400 kV across Io's diameter. In the models of Piddington and Drake (1968) and Goldreich and Lynden-Bell (1969), current flows in two loops through or around Io, along opposite sides of the tube of force containing Io to close in Jupiter's ionosphere in the northern and southern hemispheres. The resulting torque from the $\vec{J} \times \vec{B}$ force acting on Io drives Io outward, $\vec{J}$ being current density in Io. The counterbalancing torque decreases Jupiter's spin rate. However, because of Jupiter's much greater spin angular momentum as compared to Io's orbital angular momentum, this latter deceleration is minute. Goldreich and Lynden-Bell find that the electromagnetic torque N_{EMF} acting on Io is $\simeq \pi I R_J B_J a$, where B_J at Io's distance $\simeq$ 0.02 G. The original estimate from Voyager data on the current I flowing through the northern loop of 5×10^6 A (Ness *et al.*, 1979) has since been revised downward to $\sim 2.8 \times 10^6$ A (Acuna *et al.*, 1980). If the current flows through Io, the ohmic dissipation in the interior must be less than $dE/dt_{EMF} = (\Omega_J - n)\, N_{EMF} \sim 2 \times 10^{19}$ ergs/sec. For comparison, a lower bound on tidal dissipation would be comparable to this upper bound on ohmic dissipation for a homogeneous Io ($f_1 = 1$) of the highest reasonable rigidity ($k_2 \approx 0.025$) and $Q_1 = 100$. Current radiogenic heating is estimated to be about 5×10^{18} ergs/sec (Cassen *et al.*, 1981). Colburn (1980) argues that electrical heating must be considerably smaller than this upper bound due to the relatively high conductivity of Io's ionosphere and low conductivity of the surface layers. Even with good conductivity through Io's surface layers, only a fraction of the total power could be dissipated within Io. The recent models of Goertz (1980), Neubauer (1980), and Southwood *et al.* (1980) also lead to relatively little electrical dissipation within Io.

The Jovian magnetic dipole is tilted ~10°

with respect to its spin axis giving rise to a periodic component in the magnetic field as seen by Io. Colburn also estimated the resulting electrical heating within Io associated with this periodic component of B_J and found it to be negligible.

Finally, the electromagnetic torque $\vec{N}_{EMF}$ expands Io's orbit by only ~0.3% in 4.6 by while the lower bound on the orbital expansion set by the upper bound on the remnant libration amplitude is 0.7%. Thus the electromagnetic interaction plays at most a minor role in either heating Io's interior or tidally evolving the observed resonance locks as compared to the Jovian tidal torque. However, there seems to be some skepticism that tidal friction in Jupiter is significant based in part on Goldreich and Nicholson's (1977) estimate of the effects of eddy viscosity.

Goldreich and Soter (1966) originally suggested that dissipation in the major planets is confined to the upper atmosphere of depth $\zeta \sim 10^3$ km. Their model assumed that the major source of dissipation involved turbulent skin friction generated by the differential tidal velocity $\delta\vec{v}_t$ of the light atmosphere overlaying a heavier liquid interior. The turbulent stress is $\sim 0.002\,\rho\,|\delta\vec{v}_t|\,\delta\vec{v}_t$ and is independent of the molecular viscosity if the Reynold's number is sufficiently large. They estimated that $\delta v_t \sim (\Omega - n)R_J\eta/\zeta$, where η is the equilibrium tide height $\simeq \frac{3}{4}(M/M_J)(R_J{}^4/a^3)$. The actual tidal velocity depends on the differential equilibrium tide height between $R_J - \zeta$ and R_J (Houben, private communication, 1980). If the atmosphere is relatively light compared to the interior then $\eta \sim \frac{3}{4}(M/M_J)(R_J{}^4 - (R_J - \zeta)^4)/a^3$ and the dissipation rate $\propto|\delta\vec{v}_t|\delta\vec{v}_t{}^2$ is both much weaker than Goldreich and Soter's original estimate and is also insensitive to the exact depth ζ.

Hubbard (1968) argued that a solid–liquid interface, if it exists, occurs at a depth of $\sim 1.5 \times 10^4$ km and would involve the transformation of hydrogen to its metallic phase. Hubbard calculated the shear associated with a quasi-laminar tidal flow imbedded in a convecting fluid whose turbulent motions are presumably driven by internal heat sources or gravitational contraction. Dissipation is proportional to a turbulent "eddy viscosity" related to the internal convection. Hubbard obtained $Q_J \sim 5 \times 10^6$. Goldreich and Nicholson (1977) reconsidered Hubbard's calculations and argued that he overestimated the turbulent viscosity by a factor of 10^7.

It may be that Goldreich and Soter's original model in which tidal energy is dissipated in a turbulent boundary layer may be closer to the truth if Jupiter is sufficiently stratified. The thermal energy radiated per unit area by Jupiter and which drives internal convection is $\sim 10^4$ ergs/cm^2-sec or $\sim 5 \times 10^{24}$ ergs/sec overall (Chase *et al.*, 1974). It is possible that the organized convection that is obvious in the upper Jovian atmosphere is also operating at one or more internal boundary layers. If the typical convective boundary layer shear velocity is $\delta\vec{v}_c$ and $\delta\vec{v} \gg \delta\vec{v}_t$, then the tidal stress is $\propto|\delta\vec{v}_c|\delta\vec{v}_t$. Dissipation is $\propto|\delta\vec{v}_c|\delta\vec{v}_t{}^2$ and would require $\delta\vec{v}_c \sim 10$ m/sec in a 1.5×10^4-km-deep atmosphere to dissipate $\sim 10^{20}$ ergs/sec if $\rho \sim 1$ g/cm^3 at depth ζ. Of course, rigorous analysis may reveal that this estimate, like that of Goldreich and Soter, is grossly inflated. However, there exist other models which predict as much dissipation.

Houben and Gierasch (unpublished) have estimated the amount of tidal energy dissipated in an isothermal Jovian atmosphere. They find that the tide raised by Io can dissipate $\sim 10^{18}$ ergs/sec through the excitation of inertial gravity waves which propagate radially upward. Dermott (1979) argues that significant tidal dissipation can occur in the small rocky cores of the major planet if their cores are solid. Dermott is unable to resolve whether Jupiter's core is solid given our uncertain knowledge of the melting point at core pressure. Assuming that it is solid, he estimates $\sim 2 \times 10^{20}$ ergs/sec could be dissipated within the Jovian core three times the Earth's volume given a plausible $Q \sim 30$ within the core. This core Q is equivalent to a bulk Q_J of $\sim 7 \times 10^5$ which lies between the bounds on Q_J established by the dynamical analysis in Sections 4 and 6. Finally Stevenson (1980) has proposed that tides raised on Jupiter by Io induce a phase transition at the molecular–metallic hydrogen phase boundary. A hysteresis depending on thermal diffusivity during the periodic phase adjustments can lead to sufficient dissipation of tidal energy for an effective Q_J below the upper bound established in Sections 4 and 6 provided the phase transition is first order.

None of the models yielding relatively high dissipation of tidal energy (low Q_J) can account for a dissipation in Jupiter as high as 3×10^{21} ergs/sec inferred from the Ionian heat flow estimates of Matson *et al.*, 1981, Sinton (1981), and Morrison and Telesco (1980) and the condition that e_1 is the equilibrium value. It thus appears that theoretical estimates of the dissipation of tidal energy in Jupiter may be sufficient to accommodate the assembly of the resonances by differential tidal expansion of the orbits, although none of the estimates is very secure. The much higher dissipation in Jupiter implied by the high heat flux measurements for Io remains an enigma unless the measured values are not representatives of the time average.

8. HEAT FLOW AND $\dot{n}_{\mathrm{Io}}$

The plume volcanoes are the most obvious manifestations of internal activity on Io, but the thermal anomalies or "hotspots" appear to be radiating more energy. Matson *et al.* (1981) estimate that Io's heat flow is 48 ± 24 μcal/cm^2/sec. This implies that the net heat flow is $\sim 8 \times 10^{20}$ erg/sec and from (2), $Q_1 \sim 3f_1$. Their estimate is based on the cooling and heating curves obtained from measurements of the infrared brightness of Io at wavelengths 8.4, 10.6, and 20 μm as Io moves in and out of Jupiter's shadow. Their model assumes that most of the radiation is emitted from hotspots at a typical temperature of 200°K and covering ~1–2% of Io's surface. Sinton (1981) obtains similar results for the heat flow ($\sim 1800 \pm 600$ ergs/cm^2) using similar data which also includes recent measurements at 2.2, 3.8, and 4.8 μm. Sinton's model differs from Matson et al in that he introduces two sets of hot spots at temperatures of 600 and 300°K with fractional areas of 2×10^{-5} and 4×10^{-3}, respectively. The measurements at different wavelengths used by Matson *et al.* and Sinton were not taken simultaneously and are not necessarily from the same hemisphere. This means that the temperature distribution over the surface was most likely unique for each observation given the observed high variability of thermal activity (Smith *et al.*, 1979). These derivations of heat flow may therefore be revised by more appropriate data sets. However, Pearl and Sinton (1981) obtain a similar estimate for the Ionian heat flow based on a preliminary analysis of the Voyager flyby IRIS data, and Morrison and Telesco (1980) find a heat flow of 1500 ± 300 ergs/cm^2-sec by obtaining a complete spectrum from 3 to 30 μm, while Io was in eclipse thereby eliminating most of the ambiguities of the earlier techniques.

If we now assume this last measurement of the heat flux to represent the current rate of tidal dissipation in Io and that the system orbits are expanding in a quasi-stationary configuration ($e_1 \approx$ equilibrium value), $Q_J \approx 4 \times 10^4$ and Io's orbit is expanding at a rate of 2.4 cm/year. This value of Q_J is almost a factor of 2 below the lower bound imposed by Io's proximity to Jupiter after 4.6×10^9 years of tidal evolution in the Laplace resonance, so it cannot represent a time-averaged value.

The implied secular acceleration of Io is $\dot{n}_1 \cong 220$ arcsec/Cy^{-2} or about 10 times that of the Moon. This rate of expansion implies that Io's orbital position falls behind by $a_1\Delta\gamma_1 = \frac{1}{2}\, a_1\dot{n}_1\Delta t^2 \simeq 230\,\mathrm{km}\,(\Delta t/\mathrm{Cy})^2$. From Io's orbital velocity $\simeq 17.3$ km/sec, we can deduce that the predicted time discrepancy $\Delta\tau = \Delta\lambda_1/n_1$ of eclipse times separated by time Δt is $\simeq 13.3$ sec $(\Delta t/\mathrm{Cy})^2$. Goldstein

(1975) finds $\Delta\tau(\Delta t = 2.4\,\text{Cy}) = 15 \pm 39$ sec based on a comparison of elipse timings which span ~2.4 Cy. The earliest observations were made by the 17th century astronomers Picard and Roemer during the years 1668–1690. The individual timings appear to be accurate to 30–60 sec. Goldstein compared this data with Innes' observations made during the period 1905–1925. Goldstein placed an upper bound on the tidal change in the timing of Io's eclipses of ~ 100 sec over $2\frac{1}{2}$ Cy which is marginally consistent with the acceleration inferred from estimates of Io's heat flow. Goldstein's upper bound is ~$2\frac{1}{2}$ times the formal uncertainty of his estimate of the tidal change in eclipse timings and in part reflects the widely disparate estimates of the lunar acceleration that were then current in the literature.

It happens that the uncertainty in the lunar acceleration can significantly affect clock corrections which must be applied to the old data. The clocks used by the 17th century astronomers were apparently periodically reset using high noon as a benchmark. Thus their timings are based on Earth rotation and hence measure solar or *Universal Time* (i.e., UT1). The Earth is far from a perfectly uniform clock. The Earth rotation rate is affected by aperiodic sources such as weather, ocean currents, convection in the fluid core, etc. in addition to the uniform tidal deceleration. The conversion from UT1 to a more uniform clock based on the orbital motion of the Moon involves an additional data set of timings such as lunar occultations taken during the same time span as the Galilean satellite observations.

Goldstein uses Brouwer's (1952) estimate of the clock correction ΔUT1 which must be added to the old observations. Brouwer relies on Spencer-Jones' (1939) estimate of lunar acceleration: $\dot{n}_{☾} = -22''\!.4/\text{Cy}^2$. There has been some controversy over the last decade concerning this value. Some "recent" determinations have been as high as $\sim -40''/\text{Cy}^2$ (e.g., Oesterwinter and Cohen, 1972). The present consensus is that the Spencer-Jones measurement is actually closer to the actual value although it may be a little low. Williams *et al.* (1979) find $\dot{n}_{☾} = -23.8 \pm 4''/\text{Cy}^2$ based on an analysis of lunar laser ranging data. Morrison and Ward (1975) find $\dot{n}_{☾} = -26.0 \pm 2''/\text{Cy}^2$ from classical astronomical observations of the moon and planets. The moon moves $0''\!.54$ in longitude in 1 sec of time. Therefore a decrease of $\sim 4''/\text{Cy}^2$ in lunar acceleration results in a change in UT1 ~30 sec over 3 Cy.

The quality of these 3-Cy-old timings is reflected in the fact that the error in the lunar tidal acceleration is a major source of error in estimating the secular acceleration of Io and the other Galilean satellites. Lieske (private communication, 1980) has reexamined Picard and Roemer's observations by comparing the observed timings with those obtained from his semianalytic theory (Lieske, 1980). This theory does not presently allow for the tidal variation of the orbital periods of these satellites. Hence, any discrepancy in the observed minus calculated timings may be the result of the orbital expansion. His preliminary results confirm Goldstein's findings. Lieske believes that a more realistic upper bound on the differential change in timing of Io's eclipses over 3 Cy is $\sim \pm 50$ sec. This corresponds to $|\dot{n}_1| \lesssim 90$ arcsec/Cy^2 and $Q_1/f_1 \gtrsim 10$ for an equilibrium configuration. If Lieske is correct, then either the heat flow measurements are in error or the Ionian heat flow is episodic on a time scale $\gg 10$ years. Furthermore, the same bound implies that $Q_1/f_1 \gtrsim 30$ if the system is still relaxing, unimpeded by either the Jovian tidal or electromagnetic torques. As mentioned earlier, the heat flow estimates appear to be inconsistent with a constant rate of tidal dissipation within Jupiter over the past 4.6 by. That is, if we integrate the Jovian system backwards in time, assume that the three-body lock is primordial and Q_J = constant, we find that Io is just above synchronous orbit ~2 by ago. A similar time scale problem arises when we attempt to integrate the lunar orbit backward in time assuming that the relative rate of tidal dissipation in the Earth is constant. The conventional wisdom is that since the oceans are the major source of tidal dissipation, oceanic dissipation could have been considerably smaller in the past due to a different configuration of the continents (Brosche and Sundermann, 1977). It is not clear that an increase in the primordial Q_J of Jupiter can be similarly invoked to accommodate the current heat flux measurements for Io.

Our estimate (61) of age of formation of the three-body lock was based on the questionable interpretation of the observed libration amplitude $\phi_m = 0\overset{\circ}{.}066$ (Lieske, 1980) as a tidally reduced remnant. A more reliable estimate can be deduced from the secular acceleration of the 2:1 frequency $\dot{\nu}_1 = \dot{n}_1 - 2\dot{n}_2$. If $\phi_m = 0\overset{\circ}{.}066$ and $e_{12} = 0.95\,\bar{e}_{12}$ we find from (37), (66) that $\dot{\nu}_1 = 0.14\dot{n}_1$. This value may be too small to be observed, given the accuracy of the data. Still, the clock corrections for nearly simultaneous astrometric observations of both satellites tend to cancel, improving the relative accuracy of this measurement of $\dot{n}_1$. The relaxation hypothesis predicts that $\dot{\nu}_1 = 0.5\,\dot{n}_1$ and constraints on $\dot{\nu}_1$ may be a more powerful test of whether the system is still relaxing or is nearly in equilibrium.

9. CONCLUDING REMARKS

We have described in considerable detail the effect of dissipative tides in Jupiter and its satellites on the orbital evolution of the Galilean system. The accurate determination of Io's heat flow and secular acceleration will pose important constraints on both the scale of tidal expansion of the three orbits and the total heat input which has marked Io as the most geologically active satellite in the solar system. The tidal scenario predicts a certain relationship between these two measurements if both are constant in time.

We have considered two different models of resonance formation. The most plausible sequence of events is in four stages. (1) All three satellites are in orbits far from either the 2:1 commensurabilities or the three-body lock. The tide raised on Io quickly damps down the free eccentricity on a time scale $<10^5\,Q_1$ year. Only modest tidal heating in Io occurs. (2) The dissipative tide raised on Jupiter by Io is dominant and causes Io's orbit to spiral outward. No significant tidal heating occurs. Io approaches the 2:1 commensurability with Europa in stage (3), where Io's forced eccentricity rapidly increases to the critical value $\sim 1/(35\,D)^{1/2} \sim 0.0026$. Thereafter the resonant interaction forces the orbits of Io and Europa to expand together such that $a_1/a_2 \simeq 0.63$ is maintained. Tidal heating probably leads to the formation of a fluid core. Finally, (4) Europa approaches the 2:1 commensurability with Ganymede but instead of dissipation in Europa repelling Ganymede we find that Io must work even harder using the three-body resonance to transfer angular momentum from Europa to Ganymede's orbit. We again rapidly reach a steady state as e_1 approaches $1/(13\,D_1)^{1/2}$. The high dissipation of tidal energy in Io is critical for the rapid evolution within the resonances. The assumption that the orbital spacings in the semimajor axes ratio α were initially less than 0.63 (but greater than 0.48) implies that the Jovian torque must be sufficiently strong to push these satellites into the resonance locks. If the original value of α were say 10% less than the current value, the upper bound on Q_J required to drive the satellites to their present configuration is $\sim 4 \times 10^5$. No existing model of Jovian dissipation suggests that Q_J is as small as 4×10^4 as suggested by the heat flow measurements of Matson *et al.* (1981), Sinton (1981) and Morrison and Telesco (1980).

The second model assumed that establishment of the resonance locks was contemporaneous with satellite formation. We considered here the possibility that the original orbital configuration was initially more tightly bound and then relaxed due to dissipation in Io only. We found that if the system is still relaxing, $Q_1 \gtrsim 1400$, and that the tidal heating of Io would be less than one-half the expected radiogenic rate of ~6 × 10^{18} ergs/sec. The conclusion is that either an electrical or tidal torque from Jupiter must prevent the system from relaxing unless the resonance "jumped the gap" from negative to positive ν within the last 10^7–10^8 years. The estimated torque from $\vec{J} \times \vec{B}$ forces is equivalent to a tidal torque with $Q_J \sim 7 \times 10^6$ and $Q_1 \sim 600$ if the system is in equilibrium. The heat flow measurements suggest that $Q_1 \sim 3$ and although this may be in error, estimates of the energy associated with other processes such as the volcanic activity and the resurfacing rate require power $>10^{19}$ ergs/sec or $Q_1 < 200$ to drive them. Primordial resonance formation is attractive in that it maximizes the tidal energy budget for Io. On the other hand, we found no compelling arguments showing that such an origin would allow Q_J to be larger than the upper bound which allows the resonance assembly from initially random orbits.

Determination of the secular acceleration of Io's mean motion would yield Q_J and determine whether or not the current heat flux from Io is episodic.

APPENDIX: SYMBOLS

a_i	Semimajor axis of ith satellite
A_i	Three-body resonance coefficients
$b_{1/2}{}^j(\alpha)$	Laplace coefficients
B_J	Jovian magnetic field
C_{nmhk}	Coefficients in expansion of $R_1{}^2$
D_i	Ratio of satellite to Jupiter tidal strength
e_i	Eccentricity
e_{i_0}	Free eccentricity
e_{ij}	Forced e of ith due to jth satellite
$\bar{e}_{21}$	Equilibrium value of e_{21}
E	Energy
f_i	Dissipation enhancement factor
g	Surface gravity
G	Gravitational constant
h_0, h_1	Constants defined following (89)
H°	Zero-order Hamiltonian
H	Hamiltonian
I	Inclination, current
J	Action, current density
k_i	Satellite Love number
K_i	Constants defined in (31), (38), (87)
L, Γ, Z	Poincaré action variables
$\mathscr{L}$	Orbital angular momentum
M_J	Jupiter mass
M_i	Satellite mass
$\mathbf{M}$	Matrix defined in (98)
n_i	Satellite mean motion
$\dot{n}_{☾}$	Lunar tidal acceleration
N_J	Jovian tidal torque
N_{EMF}	Jovian electromagnetic torque
p, q	Poincaré eccentric variables
P_c	Capture probability
Q_J	Jupiter dissipation factor
Q_i	Satellite dissipation factor
r_i	Satellite orbital distance
R_J	Jupiter radius
R_i	Satellite radius
R_i^j	Disturbing function
S	$2\lambda_1 - 5\lambda_2 + 2\lambda_3$
v	True anomaly
v	Velocity
δv_t	Differential tidal velocity
δv_c	Shear velocity
V_1	$\lambda_1 - 2\lambda_2$
V_2	$\lambda_2 - 2\lambda_3$
x, ϕ	Generalized canonical variables
X	Function defined in (111)
z	$\bar{e}_{21}/e_{21}$
α	a_i/a_j
γ_i	$n_1 - 2n_2 + \dot{\tilde{\omega}}_{si}$
δ_i	Phase lag
Δ	Distance of separation
ϵ	Satellite obliquity
ζ	Jovian atmospheric depth
η	Equilibrium tide height
κ_I	$(I_{20}/I_{10})^{1/2}$
κ_e	$(e_{20}/e_{10})^{1/2}$
λ_i	Satellite mean longitude
μ	Rigidity
ν_1	$n_1 - 2n_2$
ν_2	$n_2 - 2n_3$

ξ	$2\nu_1 - \nu_2$
ρ	density
σ	Coefficient of $\dot{\phi}$ term in (40)
$\tau_t(e)$	Tidal damping time scale for e
$\tau_t(I)$	Tidal damping time scale for I
ϕ	Laplace resonance variable
$\ddot{\phi}_T$	Tidal acceleration of ϕ
ϕ_m	Libration amplitude of ϕ
χ_{ij}	Coefficients defined in (108)
ψ_i	$S + \tilde{\omega}_i$
ω_L	Three-body libration frequency
$\tilde{\omega}_i$	Orbital pericenter longitude
$\dot{\tilde{\omega}}_{si}$	Secular motion of pericenter
Ω	Longitude of ascending node
Ω_J	Jovian rotation rate

ACKNOWLEDGMENTS

We would like to thank an unknown referee for an exhaustive review and many suggestions for improving the manuscript. This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NAS 7-100, sponsored by the National Aeronautics and Space Administration. S.J.P. is supported in part by NASA Grant NGR 05-010-062 and by Ames Research Center NASA, under interchange NCA-2-OR680-805.

REFERENCES

ACUNA, M. H., NEUBAUER, F. M., AND NESS, N. F. (1981). Standing Alfven wave current system at Io: Voyager 1 observations. *J. Geophys. Res.*, in press.

BROSCHE, P., AND SUNDERMAN, J. (1977). Effects of ocean tides on the rotation of the Earth. In *Scientific Applications of Lunar Ranging* (J. D. Mulholland, Ed.), pp. 133–142. Reidel, Dordrecht.

BROUWER, D. (1952). A study of the changes in the rate of rotation of the Earth. *Astron. J.* **57**, 125.

BROUWER, D., AND CLEMENCE, G. M. (1961). *Methods of Celestial Mechanics.* Academic Press, New York.

BULLEN, K. E. (1975). *The Earth's Density*, p. 360. Wiley, New York.

CASSEN, P., REYNOLDS, R. T., GRAZIANI, F., SUMMERS, A. L., MCNELLIS, J., AND BLALOCK, L. (1979a). Convection and lunar thermal history. *Phys. Earth Planet. Interiors* **19**, 183–196.

CASSEN, P., PEALE, S. J., AND REYNOLDS, R. T. (1979b). Is there liquid water on Europa? *Geophys. Res. Lett.* **6**, 731–734.

CASSEN, P., PEALE, S. J., AND REYNOLDS, R. T. (1980). Dissipation in Europa: A correction. *Geophys. Res. Lett.* **6**, 731–734.

CASSEN, P., PEALE, S. J., AND REYNOLDS, R. T. (1981). Structure and thermal evolution of the Galilean satellites. In *Satellites of Jupiter* (D. Morrison, Ed.). Univ. of Arizona Press, Tucson.

CHASE, S. C., RUIZ, R. D., MÜNCH, G., NEUGEBAUER, G., SCHROEDER, M., AND TRAFTON, L. M. (1974). Pioneer 10 infrared radiometer experiment: Preliminary results. *Science* **183**, 315–317.

CHAO, C. (1976). *A General Perturbation Method and Its Application to the Motions of the Four Massive Satellites of Jupiter.* Ph.D. thesis, University of California, Los Angeles.

CHENG, C. H., AND TOKSOZ, M. N. (1978). Tidal stresses in the Moon. *J. Geophys. Res.* **83**, 845–853.

COLBURN, D. S. (1980). Electromagnetic heating of Io. *J. Geophys. Res.* **85**, 7257–7261.

DAVIES, M. E., KATAYAMA, F. Y. (1981). *Coordinates of Features on the Galilean Satellites.* Rand Report N-1617-JPL/NASA. *J. Geophys. Res.*, in press.

DERMOTT, S. F. (1979). Tidal dissipation in the solid cores of the major planets. *Icarus* **37**, 310–321.

GAVRILOV, S. V., AND ZHARKOV, V. N. (1977). Love numbers of the giant planets. *Icarus* **32**, 443–449.

GOERTZ, C. K. (1980). Io's Interaction with the Plasma Torus. *J. Geophys. Res.* **85**(A6), 2949–2956.

GOLDREICH, P., AND LYNDON-BELL, D. (1969). Io, a Jovian unipolar inductor. *Astrophys. J.* **156**, 59–78.

GOLDREICH, P., AND PEALE, S. J. (1966). Spin orbit coupling in the solar system. *Astron. J.* **71**, 425–438.

GOLDREICH, P., AND NICHOLSON, P. D. (1977). Turbulent viscosity and Jupiter's tidal Q. *Icarus* **30**, 301–304.

GOLDREICH, P., AND SOTER, S. (1966). *Q* in the solar system. *Icarus* **5**, 375–389.

GOLDSTEIN, S. J., JR., (1975). On the secular change in the period of Io, 1668–1926, *Astron. J.* **80**, 532–539.

GREENBERG, R. (1981). Orbital evolution of the Galilean satellites. In *Satellites of Jupiter* (D. Morrison, Ed.). Univ. of Arizona Press, Tucson.

HELFENSTEIN, P., AND PARMENTIER, E. M. (1980). Fractures on Europa: Possible response of an ice crust to tidal deformation. *Proc. Lunar Planet. Sci. Conf. 11th*, 1987–1998.

HAGIHARA, Y. (1972). *Celestial Mechanics*, Vol. 2, Part 1, pp. 328–337. MIT Press, Cambridge, Mass.

HUBBARD, W. B. (1968). Tides in the giant planets. *Icarus* **23**, 42–50.

KAULA, W. M. (1964). Tidal dissipation by solid friction and the resulting orbital evolution. *Rev. Geophys.* **2**, 467–539.

LAPLACE, P. S. (1805). *Mechanique Celeste* Vol. 4. Courcier, Paris. [Translation by N. Bowditch, reprinted 1966. Chelsea New York.]

LIESKE, J. H. (1980). Improved ephemerides of the Galilean satellites, *Astron. Astrophys.* **82**, 340–348.

LIN, D. N. C., AND PAPALOIZOU, J. (1979). On the structure of circumbinary accretion disks and the tidal evolution of commensurable satellites. *Mon. Not. Roy. Astron. Soc.* **188**, 191–201.

LUCHITTA, B. K., AND L. A. SODERBLOM (1981). Geology of Europa. In *Satellites of Jupiter* (D. Morrison, Ed.) Univ. of Arizona Press, Tucson.

MORABITO, J., SYNNOTT, S. P., KUPFERMAN, P. N., AND COLLINS, S. A. (1979). Discovery of currently active extraterrestial volcanism. *Science* **204**, 972.

MATSON, D. L., RANSFORD, G. A. AND JOHNSON, T. V. (1980). Heat flow from Io, *J. Geophys. Res.* **86**, 1664–1672.

MORRISON, D. AND TELESCO, C. M. (1980). Io: Observational constraints on internal energy and thermophysics of the surface. *Icarus* **44**, 226–233.

MORRISON, L. V., AND WARD, C. G. (1975). The analysis of the transits of Mercury: 1677–1973. *Mon. Not. Roy. Astron. Soc.* **173**, 183–206.

MUNK, W. AND MACDONALD, G. F. (1960). *The Rotation of the Earth.* Cambridge Univ. Press, New York.

NAKAMURA, Y., LATHAM, G. V., AND DORMAN, H. J. (1976). Seismic structure of the Moon. *Lunar Sci. VII*, 602–603.

NESS, N. F., ACUNA, M. H., LEPPING, R. P., BURLAGA, L. F., BEHANNON, K. W., AND NEUBAUER, F. M. (1979). Magnetic field studies at Jupiter by Voyager 1: Preliminary results. *Science* **204**, 982.

NEUBAUER, F. M. (1980). Nonlinear standing Alfven wave current system at Io: Theory. *J. Geophys. Res.* **85**, 1171.

NEWCOMB, S. (1895). A development of the perturbative function. *Astron. Papers, Naut. Almanac* **5**, 1–48.

NULL, G. W. (1976). Gravity field of Jupiter and its satellites from Pioneer 10 and Pioneer 11 tracking data. *Astron. Journal* **81**, 1142–1161.

OESTERWINTER, C. AND COHEN, C. J. (1972). New orbital elements for Moon and Planets. *Celestial Mech.* **5**, 317–395.

PEALE, S. J. (1976a). Orbital resonances in the Solar System. *Annu. Rev. Astron. Astrophys.* **14**, 215–246.

PEALE, S. J. (1976b). Excitation and relaxation of the wobble, precession and libration of the Moon. *J. Geophys. Res.* **8**, 1813–1826.

PEALE, S. J. (1977). Rotation histories of the natural satellites. In *Planetary Satellites* (J. A. Burns, Ed.), pp. 87–112, Univ. Of Arizona Press, Tucson.

PEALE, S. J., AND GREENBERG, R., (1980). On the Q of Jupiter. *Lunar Planet. Sci. XI*, 871–873.

PEALE, S. J., CASSEN, P., AND REYNOLDS, R. T., (1979). Melting of Io by tidal dissipation. *Science* **203**, 892–894.

PEALE, S. J., CASSEN, P., AND REYNOLDS, R. T. (1980). Tidal Dissipation, orbital evolution and the nature of Saturn's inner satellites. *Icarus* **43**, 65–72.

PEARL, J. C. AND SINTON, W. M. (1981). Hotspots of Io. In *Satellites of Jupiter*, (D. Morrison, Ed.), Univ. of Arizona Press, Tucson.

PIDDINGTON, J. H., AND DRAKE, J. F. (1968). Electromagnetic effects of Jupiter's satellite Io. *Nature* **217**, 935.

POLLACK, J. B. (1977). Phobos and Deimos. In *Planetary Satellites* (J. A. Burns, Ed.), pp. 319–345. Univ. of Arizona Press, Tucson.

SHOEMAKER, G. AND WOLFE, R. J. (1981). Cratering time scales for the Galilean satellites. In *Satellites of Jupiter* (D. Morrison, Ed.), Univ. of Arizona Press, Tucson.

SINCLAIR, A. (1975). The orbital resonance amongst the Galilean satellites of Jupiter. *Mon. Not. Roy. Astron. Soc.* **171**, 59–72.

SINTON, W. M. (1981). The thermal emission spectrum of Io and a determination of the heat flux from its hot spots. *J. Geophys. Res.*, in press.

SMITH, B. A., SODERBLOM, L. A., BEEBE, R., BOYCE, J., BRIGGS, G., CARR, M., COLLINS, S. A., COOK, A. F., II, DANIELSON, G. E., DAVIES, M. E., HUNT, G. E., INGERSOLL, A., JOHNSON, T. V., MASURSKY, H., MCCAULEY, J., MORRISON, D., OWEN, T., SAGAN, C., SHOEMAKER, E. M., STROM, R., SUOMI, V. E., VEVERKA, J. (1979). The Galilean satellites and Jupiter: Voyager 2 imaging science results. *Science* **206**, 927–950.

SMITH, J. C., AND BORN, G. H. (1976). Secular accelerations of Phobos and Q of Mars. *Icarus* **27**, 51–53.

STEVENSON, D. J. (1980). Dissipative processes and tidal Q for the major planets. (A) *Bull. Amer. Astron. Soc.* **12**, 696.

WILLIAMS, J. G., SINCLAIR, W. S., AND YODER, C. F. (1978). Tidal acceleration of the Moon. *Geophys. Res. Lett.* **5**, 943.

YODER, C. F. (1973). *On the Establishment and Evolution of Orbit–Orbit Resonances.* Thesis, University of California, Santa Barbara.

YODER, C. F. (1979a). Diagrammatic theory of transition of pendulumlike systems. *Celestial Mechanics* **19**, 3–29.

YODER, C. F. (1979b). How tidal heating in Io drives the Galilean orbital resonance locks. *Nature* **279**, 747–770.

YODER, C. F. (1979c). Consequences of Joule heating versus tidal heating of Io. *Bull. Amer. Astron. Soc.* **11**, 599.