Pohyb gyračního středu

- korektní řešení pohybové rovnice
  ↳ korekce vůči normální gyraci
- pomalu proměnná pole

$$\frac{r_L}{L} = \frac{v_\perp}{\omega_c L} \ll 1 \quad \wedge \quad \frac{1}{\omega_c \tau} \ll 1$$

↳ char. délka prostorové změny $\vec{E}$ a $\vec{B}$ polí

↖ char. časová perioda, na níž se $\vec{E}$ a $\vec{B}$ pole mění

$\omega_c^{-1} \propto \underbrace{\frac{m}{q}}_{\text{je malé}} \equiv \epsilon$ parametr malosti

potom $r_L \propto \epsilon L$
$\tau^{-1} \propto \epsilon \omega_c$

řešíme rovnice

$$\dot{\vec{x}} = \vec{v}$$
$$\dot{\vec{v}} = \frac{q}{m}\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

budeme expandovat v ϵ

užitečná transformace souřadnic

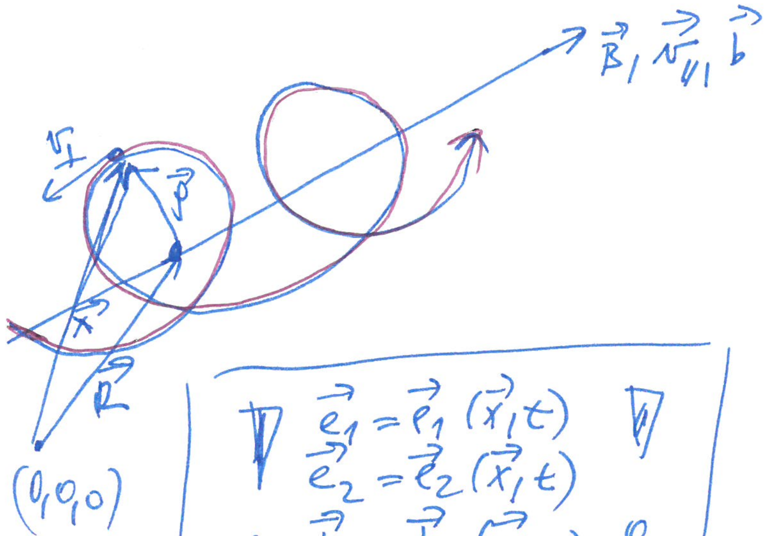
$$(\vec{x}, \vec{v}) \longrightarrow (\vec{R}, \vec{w})$$

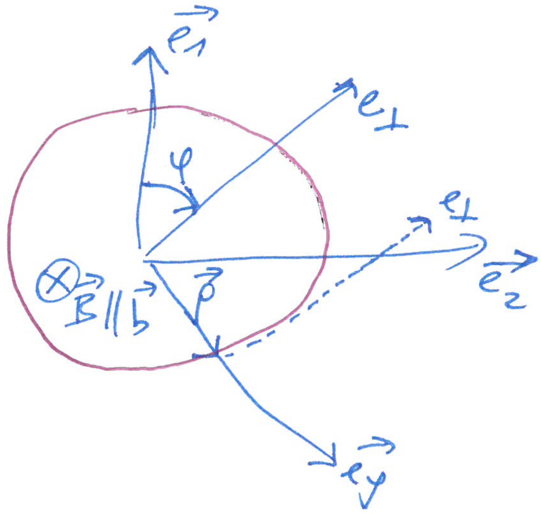
$\vec{R}(t) = \vec{x}(t) - \vec{\rho}(t)$; $\vec{w} = \vec{v} - \vec{v}_E$ ... $E \times B$ drift

okamžitá pozice gyr. středu

ok. pozice částice

pozice částice vůči gyr. středu v rovině $\perp \vec{B}(\vec{x}, t)$





$$\vec{e}_1 = \vec{e}_1(\vec{x}, t)$$
$$\vec{e}_2 = \vec{e}_2(\vec{x}, t)$$
$$\text{a} \quad \vec{b} = \vec{b}(\vec{x}, t)$$

takže: $\vec{\rho} = r_L \vec{e}_\rho = \frac{\epsilon}{B} \vec{b} \times \vec{w}$

$\vec{b} = \frac{\vec{B}}{|\vec{B}|}$ $\quad w = v - v_E$ $\quad v_E = \frac{\vec{E} \times \vec{B}}{B^2}$

$\omega_c(x,t) \equiv \frac{\vec{B}(x,t)}{\epsilon}$ ~ okamžitá gyrační frekvence

válcové souřadnice:

$$\vec{w} = v_\parallel \vec{b} + w_\perp \vec{e}_\perp$$
$$\vec{e}_\perp \equiv \vec{e}_1 \cos\varphi + \vec{e}_2 \sin\varphi \quad ; \quad \varphi = \varphi_0 - \omega_c t$$
$$\vec{e}_\rho \equiv \vec{e}_2 \cos\varphi - \vec{e}_1 \sin\varphi$$

užitečné vztahy:
$$\vec{x} = \vec{R} + \vec{\rho}$$
$$\vec{\rho} = -\frac{m}{qB^2} \vec{v} \times \vec{B} = |\rho| \operatorname{sgn} q \, \vec{e}_\rho$$
$$\vec{v}_\perp = \frac{d\vec{\rho}}{dt} = w_\perp \vec{e}_\perp$$
$$\vec{v} = v_\parallel \vec{b} + w_\perp \vec{e}_\perp + \vec{v}_E$$

přepisujeme rovnice z $(x,t) \Rightarrow (R, w)$

$w = v - v_E$

$R = x - \rho \quad ; \quad \rho = \frac{\epsilon}{B} b \times w$

$$\dot{R} = \dot{x} - \dot{\rho} = v - \frac{d}{dt}\left(\frac{\epsilon}{B} b \times w\right) =$$
$$= v_\parallel b + w_\perp e_\perp + v_E + \epsilon w \times \frac{d}{dt}\left(\frac{b}{B}\right) - \frac{\epsilon}{B} b \times \frac{dw}{dt} =$$
$$= v_\parallel b + w_\perp e_\perp + v_E + \epsilon w \times \frac{d}{dt}\left(\frac{b}{B}\right) + \frac{\epsilon}{B} b \times \frac{dv_E}{dt} -$$
$$- \frac{\epsilon}{B} b \times \dot{v}$$

$$-\frac{\epsilon}{B}\, b\times\dot{v} = -\frac{\epsilon}{B}\, b\times\left[\frac{1}{\epsilon}(E+v\times B)\right] = -\frac{1}{B}\, b\times(E+v\times B) =$$

$$= -\frac{1}{B}\, b\times E - \frac{1}{B}(v\, b\cdot B - B\, b\cdot v) =$$

$$= -\frac{1}{B^2}\, B\times E - \frac{1}{B}\, v\,\frac{B\cdot B}{B} - \frac{\vec{B}}{B}\, v_\parallel =$$

$$= +v_E - v + b\, v_\parallel = -(-v_E + v - v_\parallel b) = -\vec{e}_\perp\, w_\perp$$

tedy

$$\boxed{\dot{\vec{R}} = v_\parallel \vec{b} + \vec{v}_E + \epsilon\, \vec{w} \times \frac{d}{dt}\left(\frac{\vec{b}}{B}\right) + \frac{\epsilon}{B}\,\vec{b}\times\frac{d\vec{v}_E}{dt}}$$

$\dot{x} = v$ v nových souřadnicích

poh. rce

$$\dot{v} = \frac{q}{m}(E+v\times B) \to \frac{1}{\epsilon}(E+v\times B)$$

$$v = w + v_E = v_\parallel b + w_\perp e_\perp + v_E$$

pak

$$\frac{d}{dt}(v_\parallel b) + \frac{d}{dt}(w_\perp e_\perp) + \frac{d v_E}{dt} = \frac{1}{\epsilon}\left[E + (v_\parallel b + w_\perp e_\perp + v_E)\times B\right] =$$

$$= \frac{1}{\epsilon}\left[\vec{E} + \cancel{v_\parallel \vec{b}\times\vec{B}} + w_\perp \vec{e}_\perp\times\vec{B} + \vec{v}_E\times\vec{B}\right] =$$

$$= \frac{1}{\epsilon}\left[E + w_\perp e_\perp\times B + \frac{E\times B}{B^2}\times B\right] =$$

$$E_\parallel$$

$$= \frac{1}{\epsilon}\left[\cancel{\vec{E}} + w_\perp \vec{e}_\perp\times\vec{B} - \cancel{\vec{E}\,\frac{\vec{B}\cdot\vec{B}}{B^2}} + \vec{B}\,\frac{\vec{E}\cdot\vec{B}}{B^2}\right] =$$

$$= \frac{1}{\epsilon}\left[\vec{E}_\parallel + w_\perp\, \vec{e}_\perp\times\vec{B}\right] = \frac{d}{dt}(v_\parallel\vec{b}) + \frac{d}{dt}(w_\perp\vec{e}_\perp) + \frac{d\vec{v}_E}{dt}$$

→ počítáme projekce do směrů $b, e_\perp, e_\varphi$

1) $e_\perp$: $\frac{1}{\epsilon}\left[\underbrace{E_\parallel\cdot e_\varphi}_{=0} + w_\perp \underbrace{(e_\perp\times B)\cdot e_\perp}_{\propto (e_\perp\times e_\perp)\cdot B = 0}\right] = \frac{d v_\parallel b}{dt}\cdot e_\perp + \frac{d w_\perp e_\perp}{dt}\cdot e_\perp + \frac{d v_E}{dt}\cdot e_\perp$

$$0 = e_\perp\cdot v_\parallel\frac{db}{dt} + \underbrace{(e_\perp\cdot b)}_{e_\perp\perp b\to 0}\frac{dv_\parallel}{dt} + \underbrace{(e_\perp\cdot e_\perp)}_{e_\perp^2=1}\frac{dw_\perp}{dt} + w_\perp\, e_\perp\cdot\frac{de_\perp}{dt} + e_\perp\cdot\frac{dv_E}{dt}$$

$$e_\perp\cdot\frac{de_\perp}{dt} = \frac{1}{2}\frac{d}{dt}(e_\perp\cdot e_\perp) = \frac{1}{2}\frac{de_\perp^2}{dt} = \frac{1}{2}\frac{d\,1}{dt} = 0$$

$$\Rightarrow \boxed{\dot{w}_\perp = -\vec{e}_\perp\cdot\left(v_\parallel\frac{d\vec{b}}{dt} + \frac{d\vec{v}_E}{dt}\right)}$$

2) $\vec{b}$: $\frac{1}{\epsilon}\left(E_\parallel \cdot b + \cancel{w_\perp (e_\perp \times B)\cdot b}\right) = \frac{d v_\parallel b}{dt}\cdot b + \frac{d w_\perp e_\perp}{dt}\cdot b + \frac{d v_E}{dt}\cdot b$

$\alpha - (B\times e_\perp)\cdot b =$

$E_\parallel$ $= -(b\times B)\cdot e_\perp = 0$

$$\frac{E_\parallel}{\epsilon} = \underbrace{b\cdot b}_{=1}\frac{dv_\parallel}{dt} + v_\parallel \underbrace{\frac{db}{dt}\cdot b}_{=0} + \underbrace{b\cdot e_\perp}_{=0}\frac{dw_\perp}{dt} + w_\perp\, b\cdot\frac{de_\perp}{dt} + \frac{d}{dt}(v_E\cdot b) - v_E\cdot\frac{db}{dt}$$

$$\frac{E_\parallel}{\epsilon} = \dot{v}_\parallel + w_\perp \underbrace{\frac{d(e_\perp\cdot b)}{dt}}_{=0} - w_\perp e_\perp\cdot\frac{db}{dt} - v_E\cdot\frac{db}{dt}$$

$$\boxed{\dot{v}_\parallel = \frac{E_\parallel}{\epsilon} + (w_\perp \vec{e}_\perp + \vec{v}_E)\cdot\frac{d\vec{b}}{dt}}$$

3) e_φ: $\frac{1}{\epsilon}\left[\underbrace{E_\parallel\cdot e_\varphi}_{=0} + w_\perp (e_\perp\times B)\cdot e_\varphi\right] = e_\varphi\cdot\frac{dv_\parallel b}{dt} + e_\varphi\cdot\frac{dw_\perp e_\perp}{dt} + e_\varphi\cdot\frac{dv_E}{dt}$

$$\frac{1}{\epsilon}w_\perp (e_\perp\times B)\cdot e_\varphi = e_\varphi\cdot v_\parallel\frac{db}{dt} + e_\varphi\cdot b\frac{dv_\parallel}{dt} + e_\varphi\cdot\frac{dw_\perp e_\perp}{dt} + e_\varphi\cdot\frac{dv_E}{dt}$$

$$\frac{1}{\epsilon}w_\perp (e_\varphi\times e_\perp)\cdot B = \frac{w_\perp}{\epsilon}\left[\begin{pmatrix}-\sin\varphi\\ \cos\varphi\\ 0\end{pmatrix}\times\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\right]\cdot B = \frac{w_\perp}{\epsilon}\begin{pmatrix}0\\0\\-1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\B\end{pmatrix} = -\frac{w_\perp}{\epsilon}B$$

$$e_\varphi\cdot\frac{dw_\perp e_\perp}{dt} = \underbrace{e_\varphi\cdot e_\perp}_{=0}\frac{dw_\perp}{dt} + w_\perp(e_2\cos\varphi - e_1\sin\varphi)\cdot\frac{d}{dt}(e_1\cos\varphi + e_2\sin\varphi) =$$

$$= w_\perp\left[(e_2\cos\varphi - e_1\sin\varphi)\cdot(\cos\varphi\,\dot{e}_1 - e_1\sin\varphi\,\dot\varphi + \sin\varphi\,\dot{e}_2 + e_2\cos\varphi\,\dot\varphi)\right] =$$

$$= w_\perp\Big[\cos^2\varphi\, e_2\cdot\dot{e}_1 - \sin\varphi\cos\varphi\underbrace{e_2\cdot e_1}_{=0}\dot\varphi - \sin\varphi\cos\varphi\,\underbrace{e_1\cdot\dot{e}_1}_{=0} + \underbrace{e_1\cdot e_1}_{=1}\sin^2\varphi\,\dot\varphi +$$
$$+ \cos\varphi\sin\varphi\,\underbrace{e_2\cdot\dot{e}_2}_{=0} - \sin^2\varphi\, e_1\cdot\dot{e}_2 + \underbrace{e_2\cdot e_2}_{=1}\cos^2\varphi\,\dot\varphi + \underbrace{e_1\cdot e_2}_{=0}\sin\varphi\cos\varphi\,\dot\varphi\Big] =$$

$$= w_\perp\left[\sin^2\varphi\,\dot\varphi + \cos^2\varphi\,\dot\varphi + \cos^2\varphi\, e_2\cdot\dot{e}_1 - \sin^2\varphi\, e_1\cdot\dot{e}_2\right] =$$

$$+\sin^2\varphi\, e_2\cdot\dot{e}_1 - \sin^2\varphi\, e_2\cdot\dot{e}_1$$

$$= w_\perp\left[\dot\varphi + \cos^2\varphi\, e_2\cdot\dot{e}_1 + \sin^2\varphi\, e_2\cdot\dot{e}_1 - \sin^2\varphi\, e_1\cdot\dot{e}_2 - \sin^2\varphi\, e_2\cdot\dot{e}_1\right] =$$

$$= w_\perp\left[\dot\varphi + e_2\cdot\frac{de_1}{dt} - \sin^2\varphi\left(e_1\cdot\dot{e}_2 + e_2\cdot\frac{de_1}{dt}\right)\right] =$$

$$= w_\perp\left[\dot\varphi + e_2\cdot\frac{de_1}{dt} - \sin^2\varphi\left(\frac{d}{dt}\underbrace{(e_1\cdot e_2)}_{=0} - \cancel{e_2\cdot\dot{e}_1} + \cancel{e_2\cdot\dot{e}_1}\right)\right] =$$

$$= w_\perp\left[\dot\varphi + e_2\cdot\frac{de_1}{dt}\right]$$

celkově 3)

$$-\frac{w_\perp}{\epsilon}B = e_\varphi\cdot\left(v_\parallel\frac{db}{dt}+\frac{dv_E}{dt}\right)+w_\perp\dot\varphi+w_\perp e_2'\frac{de_1}{dt}$$

$$\Rightarrow \boxed{\dot\varphi = -\frac{B}{\epsilon}-\vec e_2\cdot\frac{d\vec e_1}{dt}-\frac{1}{w_\perp}\vec e_\varphi\cdot\left(v_\parallel\frac{d\vec b}{dt}+\frac{d\vec v_E}{dt}\right)}$$

→ máme novou sadu 6 dif. rovnic v modifikovaných proměnných. Argumenty jsou ale stále $(\vec x, t)$

⇒ vložíme úplnou časovou derivaci:

$$\frac{d}{dt}\bullet = \left(\frac{\partial}{\partial t}+\vec v\cdot\nabla\right)\bullet = \left[\frac{\partial}{\partial t}+(v_\parallel\vec b+\vec v_E+w_\perp\vec e_\perp)\cdot\nabla\right]\bullet$$

definujeme derivaci podél gyračního středu

$$D_t\bullet = \left[\frac{\partial}{\partial t}+(v_\parallel\vec b+\vec v_E)\cdot\nabla\right]\bullet$$

pouze do $\dot R$

$$\dot R = v_\parallel b+v_E+\epsilon w\times\underbrace{\frac{d}{dt}\left(\frac{b}{B}\right)}_{b\frac{d}{dt}\left(\frac{1}{B}\right)+\frac{1}{B}\frac{db}{dt}}+\frac{\epsilon}{B}b\times\frac{dv_E}{dt}$$

$w = v_\parallel b+e_\perp w_\perp$

$$\dot R = v_\parallel b+v_E+\underbrace{\epsilon(v_\parallel b+e_\perp w_\perp)\times\frac{1}{B}\left[\frac{\partial}{\partial t}+(v_\parallel b+v_E+w_\perp e_\perp)\cdot\nabla\right]b}_{A}+$$

$$+\underbrace{\epsilon(v_\parallel b+e_\perp w_\perp)\times b\left[\frac{\partial}{\partial t}+(v_\parallel b+v_E+w_\perp e_\perp)\cdot\nabla\right]\left(\frac{1}{B}\right)}_{B}+$$

$$+\frac{\epsilon}{B}b\times\left[\frac{\partial}{\partial t}+(v_\parallel b+v_E)\cdot\nabla\right]v_E+\frac{\epsilon}{B}b\times[(w_\perp e_\perp)\cdot\nabla]v_E+$$

$$+\frac{\epsilon}{B}b\times b\left[\frac{\partial}{\partial t}+(v_\parallel b+v_E)\cdot\nabla\right]v_\parallel =$$

přidáme nulový člen

$$\frac{\epsilon}{B}b\times b\,D_t v_\parallel = \frac{\epsilon}{B}b\times D_t(v_\parallel b)-\frac{\epsilon}{B}b\times v_\parallel D_t b$$

$$= v_\parallel b+v_E+\underbrace{\frac{\epsilon}{B}v_\parallel B\times D_t b}_{A_1}+\underbrace{\frac{\epsilon}{B}v_\parallel b\times(w_\perp e_\perp\cdot\nabla)b}_{A_2}+\underbrace{\frac{\epsilon}{B}e_\perp w_\perp\times\left[\frac{\partial}{\partial t}+(v_\parallel b+v_E+w_\perp e_\perp)\cdot\nabla\right]b}_{A_3}+$$

$$+\underbrace{\epsilon v_\parallel \underbrace{b\times b}_{=0}\frac{d}{dt}\frac{1}{B}}_{B_1}+\underbrace{\epsilon e_\perp w_\perp\times b\left[\frac{\partial}{\partial t}+(v_\parallel b+v_E+w_\perp e_\perp)\cdot\nabla\right]\left(\frac{1}{B}\right)}_{B_2}+$$

$$+\frac{\epsilon}{B}b\times D_t v_E+\frac{\epsilon}{B}b\times[w_\perp e_\perp\cdot\nabla]v_E+\frac{\epsilon}{B}b\times D_t v_\parallel b-\frac{\epsilon}{B}v_\parallel b\times D_t b =$$

$$= v_E + v_\parallel b + \frac{\epsilon}{B} b \times D_t (v_E + v_\parallel b) + \underbrace{\frac{\epsilon}{B} v_\parallel b \times (w_\perp e_\perp \cdot \nabla) b}_{A_2} +$$

$$+ \underbrace{\frac{\epsilon w_\perp}{B} e_\perp \times \left[\frac{\partial}{\partial t} + (v_\parallel b + v_E + w_\perp e_\perp)\cdot\nabla\right] b}_{A_3} -$$

$$- \underbrace{\frac{\epsilon w_\perp}{B^2} e_\perp \times b \left[\frac{\partial}{\partial t} + (v_\parallel b + v_E + w_\perp e_\perp)\cdot\nabla\right] B}_{B_2} + \frac{\epsilon}{B} b \times [w_\perp e_\perp \cdot \nabla] v_E$$

$$= \dot{R}$$

→ zbavíme se rychlého pohybu → středujeme přes Larmorovu rotaci.

$$\langle \bullet \rangle = \frac{1}{2\pi} \int_0^{2\pi} \bullet \, d\varphi \quad \Rightarrow \quad \langle e_\perp \rangle = 0, \quad \langle e_\rho \rangle = 0, \quad \langle e_\perp \cdot e_\rho \rangle = ?$$

matematika:

máme soustavu rovnic symbolicky

$$\frac{dz}{dt} = f_z(z) \quad ; \quad z = \{R, v_\parallel, v_\perp, \varphi\}$$

↑ rychlá proměnná

rozdělíme na střední a oscilující

$$\frac{dz}{dt} = \langle f_z(z) \rangle + \tilde{f}_z \quad \Leftarrow \text{ můžu vždycky!}$$

hledáme transformaci, která to zařídí

$$\bar{z} = z + \frac{\epsilon}{B} \int_0^\varphi \tilde{f}_z(\varphi')\, d\varphi' \quad \text{by byla dobrá!}$$

dokazujeme, že je to to, co chceme

z rovnic (pohybová do směru e_ρ)

$$\dot{\varphi} = -\frac{B}{\epsilon} - e_2 \cdot \frac{de_1}{dt} - \frac{1}{w_\perp} e_\rho \cdot \left(v_\parallel \frac{db}{dt} + \frac{dv_E}{dt} \right) =$$

$$= -\frac{B}{\epsilon} + \mathcal{O}(\epsilon^0) \quad \Leftarrow \text{ jen člen nejvyššího řádu}$$

totální časový diferenciál v nových proměnných:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{dR}{dt}\frac{\partial}{\partial R} + \frac{dv_\parallel}{dt}\frac{\partial}{\partial v_\parallel} + \frac{dw_\perp}{dt}\frac{\partial}{\partial w_\perp} + \frac{d\varphi}{dt}\frac{\partial}{\partial \varphi}$$

máme v rovnicích, členy řádu ϵ^0, až na $\dot{\varphi} = -\frac{B}{\epsilon} + \mathcal{O}(\epsilon^0)$ a $\dot{v}_\parallel = \frac{E_\parallel}{\epsilon} + \mathcal{O}(\epsilon^0)$

pokud $E_\parallel = \mathcal{O}(\epsilon)$, pak $\frac{d}{dt} = -\frac{B}{\epsilon}\frac{\partial}{\partial \varphi} + \mathcal{O}(\epsilon^0)$

$$\hookrightarrow \frac{dZ}{dt} = \frac{dz}{dt} - \frac{B}{\epsilon}\frac{\partial}{\partial\varphi}\int_0^{\varphi}\tilde{f}_z(\varphi')d\varphi' + \sigma(\epsilon) =$$

$$= \frac{dz}{dt} - \tilde{f}_z = \langle f_z(z)\rangle + \tilde{f}_z - \tilde{f}_z + \sigma(\epsilon) =$$

$$= \langle f_z(z)\rangle + \sigma(\epsilon)$$

tedy ve vystředované formě rovnice též platí s chybou $\sigma(\epsilon)$.

středujeme rovnici pro $\boxed{\dot{R}}$

$$\langle\dot{R}\rangle = \underbrace{\langle v_n b\rangle}_{①} + \underbrace{\langle v_E\rangle}_{②} + \underbrace{\left\langle \frac{\epsilon}{B} b \times D_t\,(v_n b + v_E)\right\rangle}_{③} +$$

$$+ \underbrace{\left\langle \frac{\epsilon w_\perp}{B} e_\perp \times (w_\perp e_\perp\cdot\nabla) b\right\rangle}_{④} - \underbrace{\left\langle \frac{\epsilon w_\perp}{B^2} e_\perp \times b\,[w_\perp e_\perp\cdot\nabla] B\right\rangle}_{⑤}$$

①, ② a ③ lze odstranit $\langle\,\rangle$, nezmění se

$$⑤: \langle (e_\perp\times b)\,(e_\perp\cdot\nabla)B\rangle = \left\langle \left[\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\times\begin{pmatrix}0\\0\\b\end{pmatrix}\right]\left[\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\cdot\begin{pmatrix}\nabla_1 B\\ \nabla_2 B\\ \nabla_3 B\end{pmatrix}\right]\right\rangle =$$

$$= \left\langle \begin{pmatrix} b\sin\varphi\\ -b\cos\varphi\\ 0\end{pmatrix}\left(\cos\varphi\,\nabla_1 B + \sin\varphi\,\nabla_2 B\right)\right\rangle =$$

$$= \left\langle \begin{pmatrix} \cancel{b\sin\varphi\cos\varphi\,\nabla_1 B} + b\sin^2\varphi\,\nabla_2 B\\ -b\cos^2\varphi\,\nabla_1 B - \cancel{b\sin\varphi\cos\varphi\,\nabla_2 B}\\ 0\end{pmatrix}\right\rangle =$$

$$= -\frac{1}{2}\begin{pmatrix}-b\nabla_2 B\\ b\nabla_1 B\\ 0\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}0\\0\\b\end{pmatrix}\times\begin{pmatrix}\nabla_1 B\\ \nabla_2 B\\ \nabla_3 B\end{pmatrix} = -\frac{1}{2}\,b\times\nabla B$$

$$⑥: \langle e_\perp \times \underbrace{[e_\perp\cdot\nabla] b}_{e_\perp\cdot(\nabla b)}\rangle = \left\langle \begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\times\left[\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\cdot\begin{pmatrix}\nabla_1 \vec{b}\\ \nabla_2 \vec{b}\\ \nabla_3 \vec{b}\end{pmatrix}\right]\right\rangle =$$

$$\hookrightarrow \nabla_i b_j e_i e_j$$

$$= \left\langle \begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\times\left[\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\cdot\begin{pmatrix}\nabla_1 b_1 & \nabla_1 b_2 & \nabla_1 b_3\\ \nabla_2 b_1 & \nabla_2 b_2 & \nabla_2 b_3\\ \nabla_3 b_1 & \nabla_3 b_2 & \nabla_3 b_3\end{pmatrix}\right]\right\rangle =$$

$$= \left\langle \begin{pmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{pmatrix}\times(\cos\varphi, \sin\varphi, 0)\begin{pmatrix}\nabla_1 b_1 & \nabla_1 b_2 & \nabla_1 b_3\\ \nabla_2 b_1 & \nabla_2 b_2 & \nabla_2 b_3\\ \nabla_3 b_1 & \nabla_3 b_2 & \nabla_3 b_3\end{pmatrix}\right]\right\rangle =$$

$\Rightarrow$

$$= \left\langle \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix} \times \begin{pmatrix} \cos\varphi\, \nabla_1 b_1 + \sin\varphi\, \nabla_2 b_1 \\ \cos\varphi\, \nabla_1 b_1 + \sin\varphi\, \nabla_2 b_2 \\ \cos\varphi\, \nabla_1 b_3 + \sin\varphi\, \nabla_2 b_3 \end{pmatrix} \right\rangle =$$

$$= \left\langle \begin{pmatrix} \cancel{\sin\varphi\cos\varphi\, \nabla_1 b_3} + \sin^2\varphi\, \nabla_2 b_3 \\ -\cos^2\varphi\, \nabla_1 b_3 - \cancel{\cos\varphi\sin\varphi\, \nabla_2 b_3} \\ -\cancel{\sin\varphi\cos\varphi\, \nabla_1 b_1} - \sin^2\varphi\, \nabla_2 b_1 + \cos^2\varphi\, \nabla_1 b_2 + \cancel{\cos\varphi\sin\varphi\, \nabla_2 b_2} \end{pmatrix} \right\rangle =$$

$$= \begin{pmatrix} \frac{1}{2} \nabla_2 b_3 \\ -\frac{1}{2} \nabla_1 b_3 \\ -\frac{1}{2} \nabla_2 b_1 + \frac{1}{2} \nabla_1 b_2 \end{pmatrix} = \frac{1}{2} (\vec{e}_1 \nabla_2 - \vec{e}_2 \nabla_1) b_3 + \frac{1}{2} \vec{b} (\nabla_1 b_2 - \nabla_2 b_1) =$$

$$= \underbrace{\frac{1}{2} (\vec{e}_1 \nabla_2 - \vec{e}_2 \nabla_1) b_3}_{\nabla_i b_3 = \mathcal{O}(\nabla_i b^2_{[1,2]})} + \frac{1}{2} \vec{b} (\nabla \times b)_3 = \frac{1}{2} \vec{b} [\vec{b} \cdot (\nabla \times \vec{b})]$$

$\rightarrow$ lze zanedbat vůči dalšímu členu $\rightarrow$ důsledek normalizace

rozvoj

$$b' = b(R + \delta R) = b(R) + \delta R \cdot \nabla b(R) + \mathcal{O}(\delta R^2) =$$

$$= b(R) + \sum_{i=1}^{3} \underbrace{\delta R \cdot \nabla b_i}_{\delta b_i} e_i$$

$$= b + \delta b_1 e_1 + \delta b_2 e_2 + \delta b_3 e_3$$

$b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, b i b' normalizované

$$\Rightarrow |b|^2 = \boxed{1 = |b'|^2 = \left| \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{matrix} \delta b_1 \\ \delta b_2 \\ \delta b_3 \end{matrix} \right|^2 = \delta b_1^2 + \delta b_2^2 + (1 + \delta b_3)^2 = \delta b_1^2 + \delta b_2^2 + \delta b_3^2 + 2\delta b_3 + 1}$$

$$\Rightarrow \underbrace{2\delta b_3}_{\text{malé}} = -\delta b_1^2 - \delta b_2^2 - \delta b_3^2 \sim \mathcal{O}(\delta b^2_{[1,2]})$$

Tedy celkem

$$\langle \dot{\vec{R}} \rangle = \underset{1}{v_\parallel \vec{b}} + \underset{2}{\vec{v}_E} + \underset{3}{\frac{\epsilon}{B} \vec{b} \times D_t (v_\parallel \vec{b} + \vec{v}_E)} +$$

$$+ \underset{4}{\frac{\epsilon w_\perp^2}{2B} (\vec{b} \cdot \nabla \times \vec{b}) \vec{b}} + \underset{5}{\frac{\epsilon w_\perp^2}{2B^2} \vec{b} \times \nabla \vec{B}} + \mathcal{O}(\epsilon)$$

1. $\sim v_z = \text{konst}$, pohyb podél siločáry
2. $E \times B$ drift
3. drift v zakřiveném poli, polarizační drift a drift v nehomogenním E poli
4. korekce na $j_\parallel \sim \nabla \times b$
5. ∇B drift

podobně pro ostatní rovnice

$$\left\langle \frac{dv_\parallel}{dt} \right\rangle = \left\langle \frac{E_\parallel}{\epsilon} \right\rangle - \left\langle \frac{w_\perp^2}{2B} \nabla_\parallel \vec{B} \right\rangle + \left\langle \vec{v}_E \cdot D_t \vec{b} \right\rangle$$

$$\langle \dot{w}_\perp \rangle = \frac{v_\parallel w_\perp}{2B} \nabla_\parallel B - \frac{w_\perp}{2} (\nabla \cdot v_E - b \cdot \nabla_\parallel v_E)$$

$$\langle \dot{\varphi} \rangle = -\frac{B}{\epsilon} - e_2 \cdot D_t e_1 - \frac{v_\parallel}{2} b \cdot \nabla \times (v_\parallel b + v_E)$$

pozor! argumenty stále (x, v)

řešení: rozvoj

$$b(x,t) = b(R,t) + \rho \cdot \nabla b(R,t) + \ldots =$$

$$= b(R,t) + \epsilon \frac{w_\perp}{B} e_\rho \cdot \nabla b(R,t) + \ldots$$

$$\Rightarrow \langle b(x,t) \rangle = b(R,t) + \mathcal{O}(\epsilon)$$

$\Rightarrow$ rovnice mají stejný tvar i v argumentech (R, t) a chyba je stále $\mathcal{O}(\epsilon^0)$ malá