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On the role of duplicity in the Be phenomenon

1. General considerations and the first attempt at a 3-D gas-dynamical modelling of gas outflow from hot and rapidly rotating OB stars in binaries

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Abstract. This paper begins a new series of studies devoted to a critical re-examination of the role of duplicity for the Be phenomenon and for the variability patterns observed for many Be stars.
Based on both dynamical and energy considerations and a numerical gas-dynamical modelling, a new hypothesis of the formation of Be envelopes in binaries, via an outflow from a rapidly rotating B star in a detached binary, is outlined. It is shown that such an outflow is facilitated by the presence of a companion to the B star and leads to the formation of an envelope but not to any significant mass exchange between the binary components.

Key words. Stars: emission-line, Be – Stars: binaries: close – Stars: binaries: spectroscopic – Stars: fundamental parameters

1. A brief critical evaluation of the existing models of Be stars

1.1. The Be phenomenon

More that one tenth of all B stars exhibit time-variable emissions in their Balmer (and some other) line profiles. These stars, called Be stars, represent perhaps the most variable objects among massive stars. Their continuum and line spectra vary on several time scales, ranging from minutes to decades or more. Actually, the longest time scale of their variability is not known and it is conceivable that every B star may become a Be star for an interval during its evolution. The phenomenon also partly extends to hotter Oe stars and cooler Ae stars.

In spite of all the effort of several generations of stellar astronomers, neither the nature of the Be phenomenon nor the physical causes of variations are well understood at present. There is general agreement that the emission lines are formed in extended gaseous envelopes around these stars. Such envelopes have dimensions of one or even two orders of magnitude larger than the stars themselves and re-radiate the stellar radiation in all directions. Evidence has been accumulated to confirm Struve's (1931) original suggestion that the Be envelopes are rotationally flattened disks. An ultimate proof of this statement came from spectro-interferometric and spectro-polarimetric observations of several Be stars which allowed a partial spatial resolution of their envelopes – see Quirrenbach et al. (1993, 1997).

However – as far as the understanding the very complex Be phenomenon is concerned – the current situation with many partial models, explaining often only one aspect of it, is not very satisfactory.

1.2. Attempts to explain the Be phenomenon

Concerning the very origin of the Be envelopes, the number of hypotheses has been growing steadily and there is no general agreement on a single one. These hypotheses are as follows:

1. *Rotational hypothesis* by Struve (1931) explains the formation of Be envelopes by rotational instability of underlying, rapidly rotating stars. Its main argument is the observed correlation between the width of the observed Balmer emission lines and $v \sin i$ of the respective stars (which has been confirmed by several later studies – c.f., e.g., Hanuschik et al. 1988). Struve himself was aware of the fact that the rotational model does not explain variability, namely the long-term V/R variations, observed for a number Be stars at certain epochs. He ingeniously suggested a slow spatial

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revolution of temporarily elongated envelopes as an explanation. Critics of the rotational model argue that one observes a number of similarly rapid rotators, called Bn stars, which were never observed to have any emission lines (e.g. Baade 1992). Many of the investigators are also convinced that the Be stars are rotating *below* their break-up speeds at the equator (cf., e.g., Porter 1996). Boyarchuk (1958) pointed out that the rapid rotation itself cannot account for the Be phenomenon and all later quantitative models led to the same conclusion (see, e.g., Limber & Marlborough 1968). Note also that the rotational hypothesis itself does not offer a clue to the observed variations on several time scales.

2. *Outflow models* Several attempts were made to explain the origin of Be envelopes as a spheroidal outlow of gaseous material from the underlying star: the stellar-wind model by Gerasimovič (1934, 1935), the variable mass flux model by Doazan &Thomas (1987) and Doazan (1987), the 'bi-stable' or 'axi-symmetric' radiation-driven wind model by Lamers & Pauldrach (1991), Araújo et al. (1994) and Stee & Araújo (1994) and the wind-compressed disk model by Bjorkman & Cassinelli (1993) (see also the review by Bjorkman 2000). The latest one was considered as a very promising idea several years ago but the radiation-hydrodynamics simulations by Owocki et al. (1996) showed that the small non-radial components in line forces inhibit formation of an equatorial disk by the wind-compressed model. Also, this model could have problems providing strong enough stellar winds in Be stars of later spectral subclasses. More generally – outflow models do not offer an explanation for the variability of Be stars.
3. *Non-radial pulsations* of Be stars were first suggested to explain the observed rapid light and line-profile variations (Baade 1979, 1982, Bolton 1982) but the idea has been developed in a complex attempt to explain the Be phenomenon by Baade and his students. In particular, Rivinius et al. (1998a) argued that constructive interference of several pulsational modes can lead to release of a new Be envelope and claimed agreement with the observed emission-line episodes for μ Cen. They did not present any energy balance considerations of whether such a mechanism could really work, however. It is certainly true that the series of line profiles obtained for a few Be stars are remarkably similar to theoretical line profiles, based on non-radial pulsation modelling. It is conceivable, however, that the description of line-profile variations in terms of spherical harmonic functions would also work to model line-profile variations due to co-rotating structures, located slightly above the stellar surface, which has been advocated, for instance, by Harmanec (1989), Smith et al. (1998) or Balona (2000). Note also that the attempts to derive the macroscopic stellar properties from the modelling of line-profile changes, interpreted as non-radial pulsations, led to too small values of stellar radii, contradicting the estimates based on Hipparcos parallaxes (compare the Maintz et al. 2002 radius of 3.4 $R_\odot$ of the B2e star μ Cen from the line-profile modelling to Harmanec's 2000 estimate of 5.2-5.4 $R_\odot$ from the V-band brightness outside emission-line episodes and the Hipparcos parallax). Aerts (2000) also warned that the interpretation of Be stars in terms of non-radial pulsations needs further careful tests. Harmanec (2001) pointed out that the formal description of radial-velocity variations of μ Cen by six non-radial modes did not lead to the gradual decrease of the rms error of the fit, as would be expected for real multiperiodicity. It is fair to say, however, that until such a modelling as was carried out for the non-radial pulsation model will also be carried out for alternative models, the model of non-radial pulsations represents the best available description of the observed rapid line-profile changes of Be stars.
4. *The helmet-type magnetic field model* was presented by Underhill (1983, 1987) which assumes the presence of organized magnetic fields in Be stars and interprets their envelopes as helmet-type co-rotating structures. This model has been largely ignored by the community (perhaps due to the fact that there was no observational technique allowing the detection of the putative magnetic fields). Given the observational evidence in favour of co-rotating structures slightly above the stellar photospheres (e.g. Harmanec 1989, 1999, Harmanec & Tarasov 1990, Smith et al. (1998) or Balona 2000), and the improving sensitivity to detect even weaker magnetic fields in hot stars, this hypothesis probably deserves further critical examination.
5. *Binary models* The last class of models are models that assume that the Be phenomenon is somehow related to the duplicity of the Be stars. They are discussed in detail in the next section.

2. The binary models and the role of duplicity for the Be phenomenon

Kříž & Harmanec (1975) and Harmanec & Kříž(1976) formulated a general hypothesis of the binary nature of Be stars, explaining the Be star envelope as an accretion disk created from gas flowing to the Be star via a Roche-lobe overflow from its unrecognized binary companion. They pointed out that with increasing orbital period one observes either a typical Algol interacting binary or a Be binary or a symbiotic star. Since a given B star occupies less and less space when placed in a binary system with longer and longer orbital period, the space available for the formation of an extended accretion disk also gets larger with increasing orbital period. In systems with shorter orbital periods, the Roche-lobe filling and less massive secondaries are usually much less luminous than their B-type primaries. Only for very long orbital periods do the absolute dimensions of the Roche lobe around the secondary become so large that even a very cool secondary has an optical luminosity comparable to, or even larger than the B star and one observes a symbiotic binary. Kříž & Harmanec (1975) were also able to explain rapid rotation of Be stars as a consequence of tangential accretion from the disk which brings some extra angular momentum. Furthermore, they explained at least some of the observed types of variations of Be stars, for instance the phase-locked V/R, RV, line-width and luminosity changes or long-term E/C and V/R variations and offered some ideas as to how rapid changes could be related to co-rotating structures in the accretion disks, caused by resonances.

While their hypothesis certainly represents a serious attempt to address the Be phenomenon in its complexity and very probably explains the nature of *some* of the actually observed Be binaries (for example AX Mon, RX Cas, SX Cas, KX And, V360 Lac, β Lyr etc. – see Harmanec 2001 for a catalogue of known emission-line binaries), it is now well proven that it cannot be accepted as a universal explanation for the origin of Be envelopes. Already Plavec (1976) pointed out that if all Be stars were binaries with Roche-lobe filling secondaries, one should observe more eclipsing binaries among them than what is actually observed. While Harmanec (1987) slightly weakened this objection, there is a stronger one: Detailed studies of several known Be binaries (φ Per = HD 10516: Poeckert 1981, Gies et al. 1998; V839 Her = 4 Her: Koubský et al. 1997, for instance) clearly demonstrated that the secondaries in those binaries *are not* Roche-lobe filling objects but very small stars. The same is also true of binaries composed of a Be star and a compact, X-ray companion. Harmanec (1985) came up with the provocative suggestion that even in massive X-ray binaries the mass is flowing from the X-ray star towards the Be primary and presented some observational facts to support such a view. Recalling an earlier suggestion by Kříž (1982), he also argued that the contraction of the originally mass-losing star had to lead to its rotational instability near the equator, leading to another phase of mass transfer from this star to its (now more massive) counterpart. He called it a case PB of mass transfer and mentioned φ Per as a system being possibly in such a mass transfer stage. It is obvious, however, that unless somebody can show how to excite X-ray emission from compact stars without allowing them to accrete mass from their optical companions, Harmanec's (1985) idea is not tenable.

Obviously unaware of Kříž's (1982) and Harmanec's (1985) studies, Pols et al. (1991) also investigated the possibility of formation of Be stars as products of case B mass exchange in binaries. Their approach was different, however. They accepted the idea of Kříž & Harmanec (1975) that some Be stars are case B mass-exchanging binaries but argued that the majority of Be stars are remnants of case B mass exchange in intermediate-mass close binaries after the termination of mass transfer. In other words, they postulated that the Be phenomenon occurs due to some still unknown physical mechanism which is only operational in rapidly rotating stars. The role of the mass exchange in their hypothesis is to rejuvenate and spin-up the original secondaries in binaries. They argued that Be stars in mass-exchanging binaries represent only a small fraction of all Be stars. Estimating the lifetimes of different evolutionary stages, they concluded that more than 80 % of post mass-transfer Be stars should have a helium-star companion and that there should be 10 times more Be stars with a white-dwarf companion than those with a neutron-star secondary (observable as an X-ray source). They predicted that many new helium-star and white-dwarf companions should be detectable in the XUV spectral region.

The role of duplicity was critically examined by Baade (1992). Using high-S/N IR spectra near 880 nm, he carried out a search for late-type companions of 35 southern Be stars with a completely negative result. He also expressed some doubts about the existence of many binaries with hot compact companions and his conclusion was that the cause of the Be phenomenon cannot be related to their duplicity.

For the following reasons we believe, however, that the role of duplicity was not still investigated well enough and that Baade's view cannot be accepted as the final word:

1. Perhaps most importantly, the number of known binaries among emission-line stars has been growing steadily – see, e.g., Gies (2000) or Harmanec (2001). As pointed out by Harmanec (2001), duplicity of Be stars can actually serve two different roles: (i) to explain the formation of Be envelopes via some kind of binary interaction, and (ii) to explain some of the variability patterns of Be stars.
2. Studies by Slettebak (1987) or Harmanec (2000) demostrated that Be stars are observed among stars of clearly different evolutionary ages. This may indicate that the cause of the Be phenomenon has to be sought in some external mechanism, not primarily in a physical mechanism related to the stars themselves.
3. Such a view can be also supported by another similar argument. B stars span a huge range of stellar masses and it is well known from the theory of stellar evolution that the time scales of all processes depend strongly on the stellar mass. However, as pointed by Horn et al. (1982) the time scale of the formation of a new Be envelope was found to be very similar for three Be stars of spectral classes B0, B6 and B8. This again seems to indicate an external mechanism for the formation of such envelopes.

It is obvious that a new generation of powerful optical interferometers will soon be able to resolve many close binaries, so far detectable only by spectroscopy. This should allow new stringent tests for various binary scenarios of the Be phenomenon.

3. Physical Model

Let us consider a single, rapidly rotating star. When the velocity of rotation on equator $V_{\rm rot}$ will reach the Keplerian (also called break-up) velocity $V_{\rm br} = \sqrt{GM/R}$, the centrifugal and gravitational attractive forces will compensate each other:

$$F_{\rm cfg} \equiv \frac{V_{\rm rot}^2}{R} = \frac{GM}{R^2} \equiv F_{\rm grav}.$$

In such situations, the presence of a pressure gradient (not counterbalanced by other forces) permits the matter to outflow from the equatorial belt (called the Roche limit). A particle with specific kinetic energy $\simeq V_{\rm br}^2/2$ rotates along closed trajectories and forms an envelope. To escape to infinity the particle would need to aquire additional energy and reach the parabolic velocity $V = \sqrt{2}V_{\rm br}$.

The situation changes dramatically if the same star is a component of a binary system. Let us consider a binary with a spin-orbit synchronization (the velocities of angular rotation of both components are equal to the velocity of angular revolution of the system) and let us use a Cartesian coordinate system

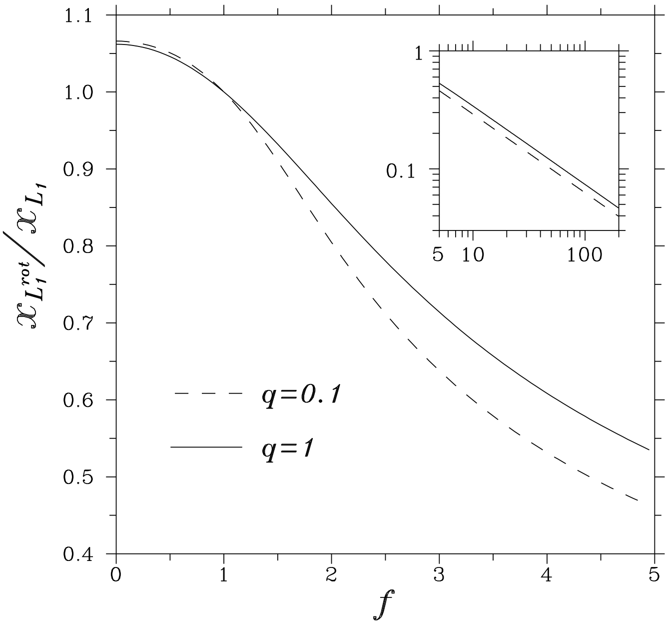

Fig. 1. Relative change in the position of the inner Lagrangian point for the systems with non-synchronous rotation of components $x_{L_1^{\rm rot}}/x_{L_1}$ as a function of the degree of non-synchronous rotation $f = \Omega_\star/\Omega$ for two values of the mass ratio: $q = 0.1$ and $q = 1$. The inserted panel shows the dependence $x_{L_1^{\rm rot}}/x_{L_1}$ vs. q for large values of f

that rotates with an angular velocity Ω and has its origin in the centre of star 1. The X axis is directed toward star 2, Z axis is parallel to the vector of orbital revolution, and Y axis is so oriented to define a right-hand coordinate system. There are several forces acting on a test particle located between the binary components: gravitational attraction of the two binary components, the pressure gradient, and two forces related to the co-rotating frame used: the centrifugal and Coriolis force. The law of motion of such a test particle was first investigated by Roche (1848, 1851) in a ballistic approach (i.e. ignoring the pressure force) as a solution of a restricted three-body problem (assuming the mass to be concentrated into two point masses); for a different formulation, see also Hill (1905). The force field (without Coriolis force and the pressure gradient) in cases of a spin-orbit synchronism can be described by the standard Roche potential Φ:

$$\Phi = -\frac{GM_1}{\sqrt{x^2+y^2+z^2}} - \frac{GM_2}{\sqrt{(x-A)^2+y^2+z^2}} - {}^1\!/_2\Omega^2\left(\left(x - A\frac{M_2}{M_1+M_2}\right)^2 + y^2\right) , \quad (1)$$

where M_1, M_2 are the two point masses, A is binary separation, Ω is the angular velocity of orbital motion, and x, y, z are Cartesian coordinates in the adopted frame.[1]

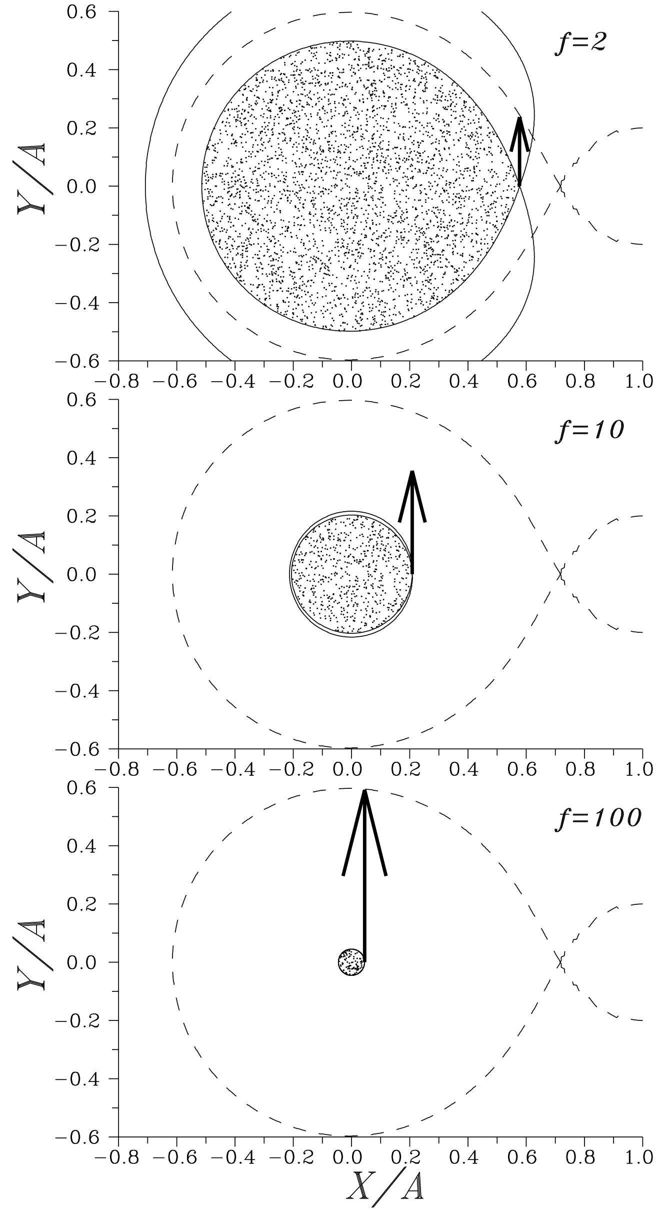

Fig. 2. Roche equipotentials (dashed lines) and equipotentials of 'asynchronous' potential Ψ (solid lines) for $q = 0.1$ and different values of f. An arrow shows the linear velocity in $L_1^{\rm rot}$ point which amounts to $0.58A\Omega$ for the upper panel, $1.88A\Omega$ for the middle one, and $4.45A\Omega$ for the bottom panel (the scale of the absolute value of velocity is different for each panel)

The presence of additional forces (absent in the case of a single star) results in violation of equilibrium in the inner Lagrangian point L_1. In particular, the pressure gradient cannot be counterbalanced there by the gradient of the Roche potential. Hence, as soon as the star expands and fills the cricital Roche lobe, matter begins to flow towards the binary companion in the vicinity of L_1 point *but not from the entire equatorial*

[1] Note that the Roche model, based on two point masses, represents a very realistic description of equipotentials of real binaries. This is due to the fact that the mass concentration toward the centre is usually very high in real stars. Moreover, the inner equipotential surfaces below the critical Roche lobe are nearly circular and it is known that the outer potential of a mass sphere is identical to that of a mass point. The mass of the outer parts of the stars, violating the spherical symmetry, is negligible with respect to the total mass of the star in all practical applications.

zone. The position of inner Lagrangian point L_1 can be derived from equation $\nabla\Phi = 0$, which can be rewritten after Kopal 1959):

$$\left(\frac{x_{L_1}}{A}\right)^{-2} - \frac{x_{L_1}}{A} = q\left(\left(1-\frac{x_{L_1}}{A}\right)^{-2} - \left(1-\frac{x_{L_1}}{A}\right)\right), \quad (2)$$

where $q = M_2/M_1$ denotes the mass ratio.

In case of asynchronous rotation of star 1,[2] we should also include centropedal acceleration and Coriolis force into the equilibrium conditions. The presence of these two terms is related to the motion of the stellar matter in the adopted co-rotating frame. In such a case, the force field at the stellar surface is given by asynchronous Roche potential (see, e.g., Plavec 1958, Kruszewski 1963, or Limber 1963):

$$\Psi = \Phi - {}^1/_2(\Omega_\star^2 - \Omega^2)(x^2+y^2) \quad (3)$$

where $\Omega_\star$ is the angular velocity of rotation of the star in question. Similar to the synchronous case it is possible to introduce a concept of the inner Lagrangian point for asynchronous rotation, $L_1^{\rm rot}$, i.e. a point where the pressure gradient ceases to be counterbalanced by other forces and where the matter begins to flow towards the companion when reaching this point. The position of this point is given by the condition $\nabla\Psi = 0$ which leads to the following equation (see, e.g., Pratt & Strittmatter 1976)

$$\left(\frac{x_{L_1^{\rm rot}}}{A}\right)^{-2} - f^2\frac{x_{L_1^{\rm rot}}}{A} = q\left(\left(1-\frac{x_{L_1^{\rm rot}}}{A}\right)^{-2} - \left(1-f^2\frac{x_{L_1^{\rm rot}}}{A}\right)\right), \quad (4)$$

which is similar to Eq. (2) valid for synchronous rotation. In this equation $f = \Omega_\star/\Omega$ and the sign of f does not affect the solution, i.e., the sense of the stellar rotation in the laboratory coordinate system does not affect the position of $L_1^{\rm rot}$. Therefore – without loss of generality – we consider only the cases of $f \geq 0$, i.e. cases when the directions of stellar rotation and binary revolution are the same. The ratio $x_{L_1^{\rm rot}}/x_{L_1}$ as a function of q and f is shown in Fig. 1. It is obvious that for stellar rotation rates slower than the orbital revolution ($f < 1$), the 'non-synchronous' Roche lobe is larger than the standard Roche lobe, achieving maximum for $f = 0$. When the star rotates faster than the binary revolves, the 'non-synchronous' Roche lobe becomes smaller than the standard one. Note that formally $x_{L_1^{\rm rot}}/x_{L_1} \to 0$ as $f \to \infty$. In real binaries, however, the position of $x_{L_1^{\rm rot}}$ is actually limited by the break-up rotation velocity of the star in question, so that $x_{L_1^{\rm rot}}/A$ cannot be smaller than $(\Omega_{\rm br}/\Omega)^{-2/3}(q+1)^{-1/3}$.

This is illustrated in Fig. 2 which shows the equatorial plane of a binary with $q = 1$. The isoline of the Roche potential Φ passing through L_1 (i.e. the XY projection of the classical Roche limit) is shown by a dashed line, while the isoline

[2] Hereafter, we will call the rotation of star 1 asynchronous if the absolute values of the vector of axial rotation of star 1 and vector of the orbital revolution are not equal, i.e. $|\boldsymbol{\Omega}_\star| \neq |\boldsymbol{\Omega}|$, but they are parallel to each other: $\boldsymbol{\Omega}_\star \parallel \boldsymbol{\Omega}$.

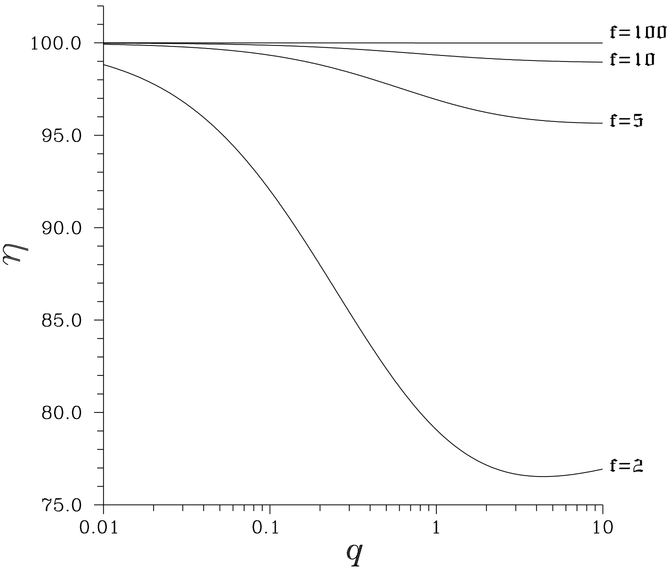

Fig. 3. The ratio of linear velocity in $L_1^{\rm rot}$ point (in observer's coordinate system) to critical velocity $\eta = \Omega_\star \cdot x_{L_1^{\rm rot}}/\sqrt{GM_1/x_{L_1^{\rm rot}}} \cdot 100\%$ vs. q for different values of asynchronicity parameter $f = \Omega_\star/\Omega$.

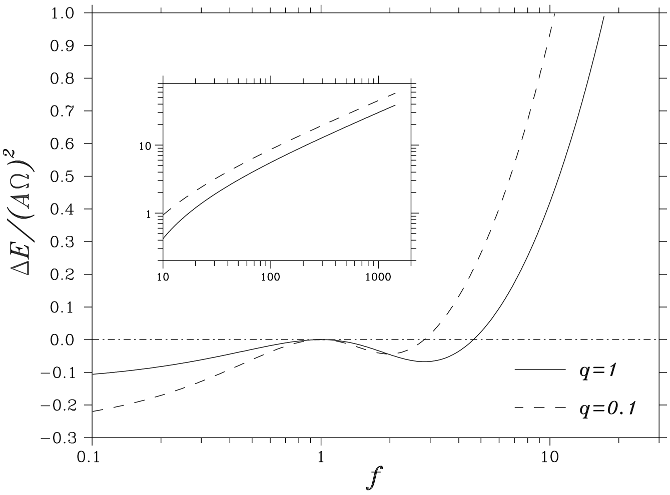

Fig. 4. Extra energy ΔE (in units of $A^2\Omega^2$) needed for a particle leaving $L_1^{\rm rot}$ to reach the L_1 point, plotted vs. asynchronicity parameter $f = \Omega_\star/\Omega$ for values of binary mass ratio of $q = 0.1$ and $q = 1$. Dash-pointed line shows value $\Delta E = 0$. The inserted panel shows the dependence of ΔE vs. f for large values of f.

of potential Ψ, passing through $L_1^{\rm rot}$, is drawn by a solid line. The 'asynchronous Roche lobes' are shown for three values of 'asynchronicity' parameter: $f = 2$, $f = 10$, and $f = 100$ (see panels a), b), and c) of Fig. 2, respectively). The vectors of linear velocity of $L_1^{\rm rot}$ point ($\boldsymbol{V}_{\rm rot} = (\boldsymbol{\Omega}_\star - \boldsymbol{\Omega}) \times \boldsymbol{r}_{L_1^{\rm rot}}$) in the adopted coordinate system are also shown.

As discussed above, the matter can outflow from the stellar surface as it reaches $L_1^{\rm rot}$ point. This fact changes the limiting value of the break-up velocity when the outflow begins. In Fig. 3, the values of the linear velocity at $L_1^{\rm rot}$ point are plotted as a function of binary mass ratio q for different values of asynchronicity parameter f. All velocities are expressed in the units of the critical velocity $V_{\rm br}$, derived for a single star of

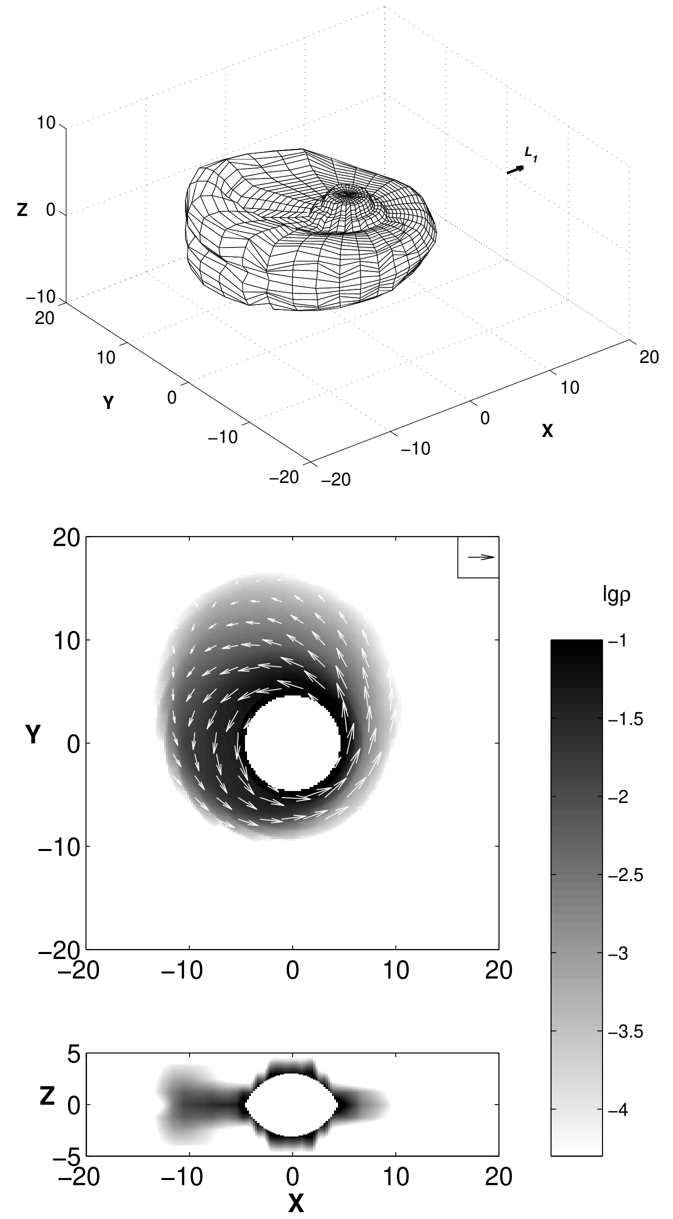

Fig. 5a. *Top panel:* Bird's-eye view of density isosurfaces on level $\rho = 5 \times 10^{-5} \cdot \rho(L_1^{\rm rot})$ for the moment of time $t = 0.514P_{orb}$. *Middle panel:* The slice of formed envelope by XY plane (density distribution and velocity vectors in equatorial plane). Vector in top right corner corresponds to velocity 500 km s^{-1}. *Bottom panel:* The slice of the envelope by XZ (density distribution in frontal plane). Coordinates in all three panels are expressed in $R_\odot$.

the same properties. The results presented in Fig. 3 show that in a number of cases even a small additional increase in the rotational velocity can lead to an outflow of matter in the vicinity of $L_1^{\rm rot}$ point. However, for values typical for Be stars, say $q \sim 0.1$ and $f \sim 100$, the outflow occurs for rotational velocities only slightly smaller than the break-up velocity of the respective star. It is important to realize, however, that when $V_{\rm rot}$ gets close to $V_{\rm br}$ for Be stars, which are members of binary systems, the outflow of matter occurs *only in the vicinity of $L_1^{\rm rot}$, not from the the whole equatorial belt of the star (the Roche limit for single stars).*

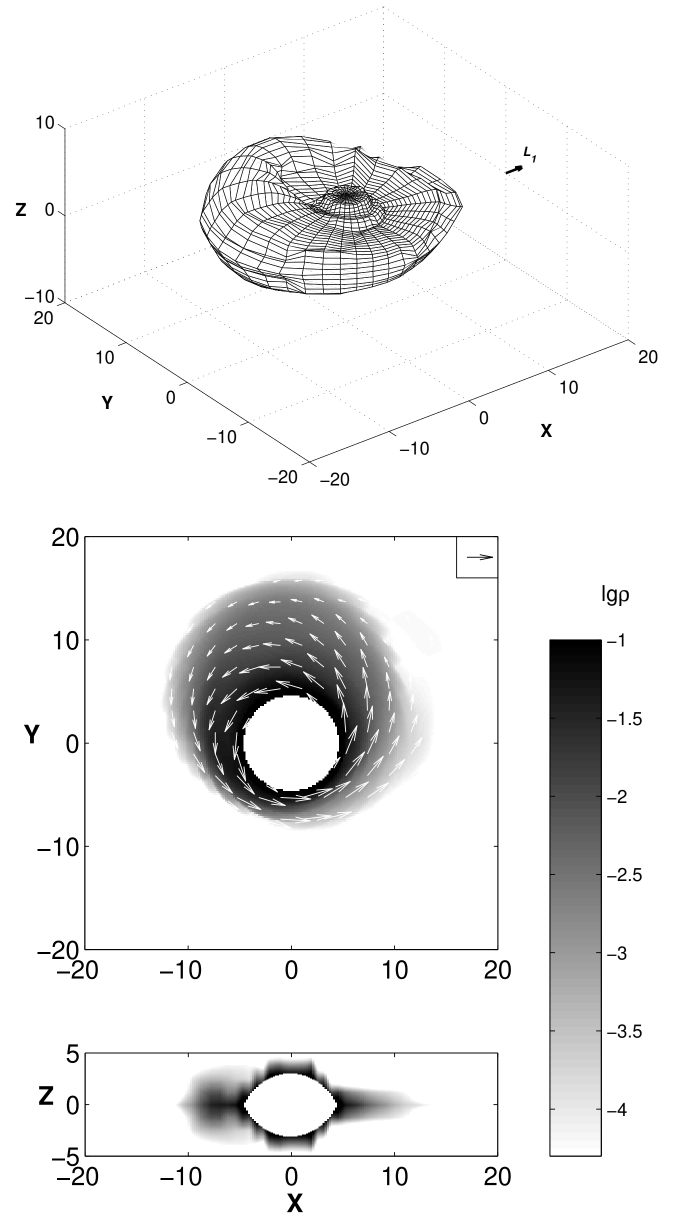

Fig. 5b. The same as Fig. 5a but for the time $t = 0.545P_{orb}$.

The presence of a companion to the Be star results in another notable change in the mechanism of the outflow from the surface of a rotating star. For a single star, a particle can escape to infinity if it has the parabolic velocity. For a binary star, the particle leaving via $L_1^{\rm rot}$ point can escape from the system if its energy is large enough to reach the L_1 point (this energy is smaller than that needed to reach the escape velocity from a single star) since after it reaches L_1, it can be captured by the gravitational field of the companion. Particles with energies insufficient to reach the L_1 point will move along closed trajectories around the star from which they escaped. The extra energy ΔE, needed to get a particle with velocity $V_{\rm rot} = (\Omega_\star - \Omega) \cdot x_{L_1^{\rm rot}}$ to the vicinity of L_1 point, is plotted in Figure 4 as a function of f for two mass ratios, $q = 1$ and $q = 0.1$, and is expressed in the units of the characteristic energy of the system $E_{sys} = A^2\Omega^2$. It is obvious that the sum of potential energy of a particle in

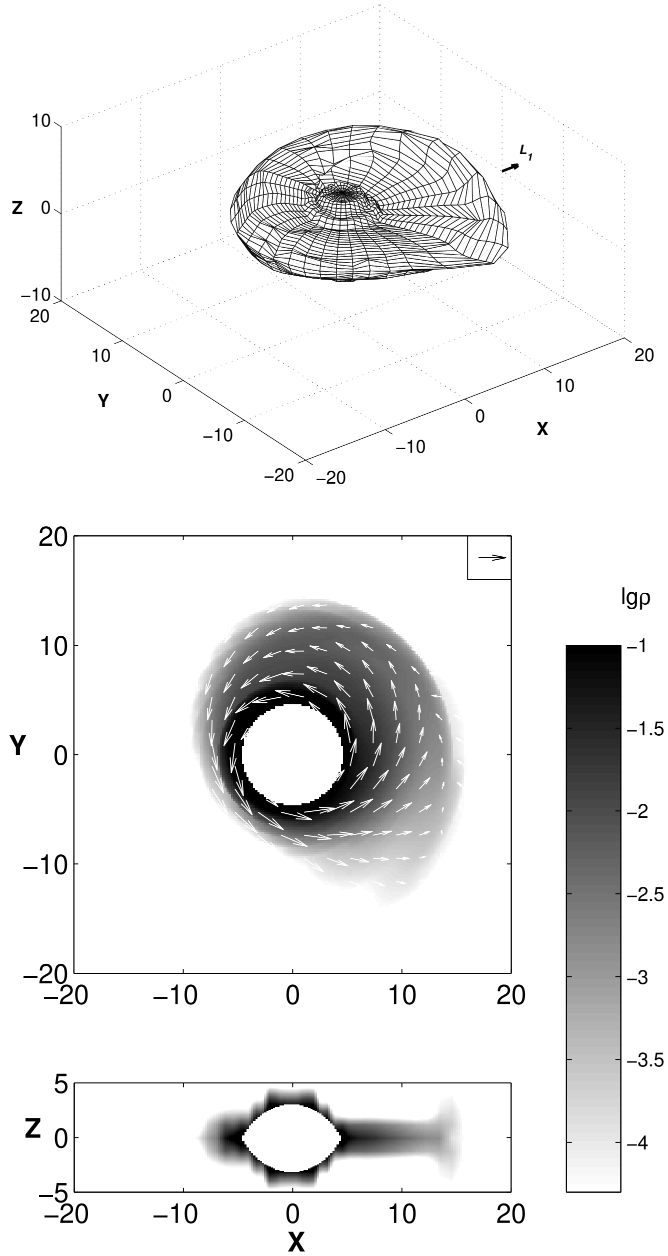

Fig. 5c. The same as Fig. 5a but for the time $t = 0.586 P_{orb}$.

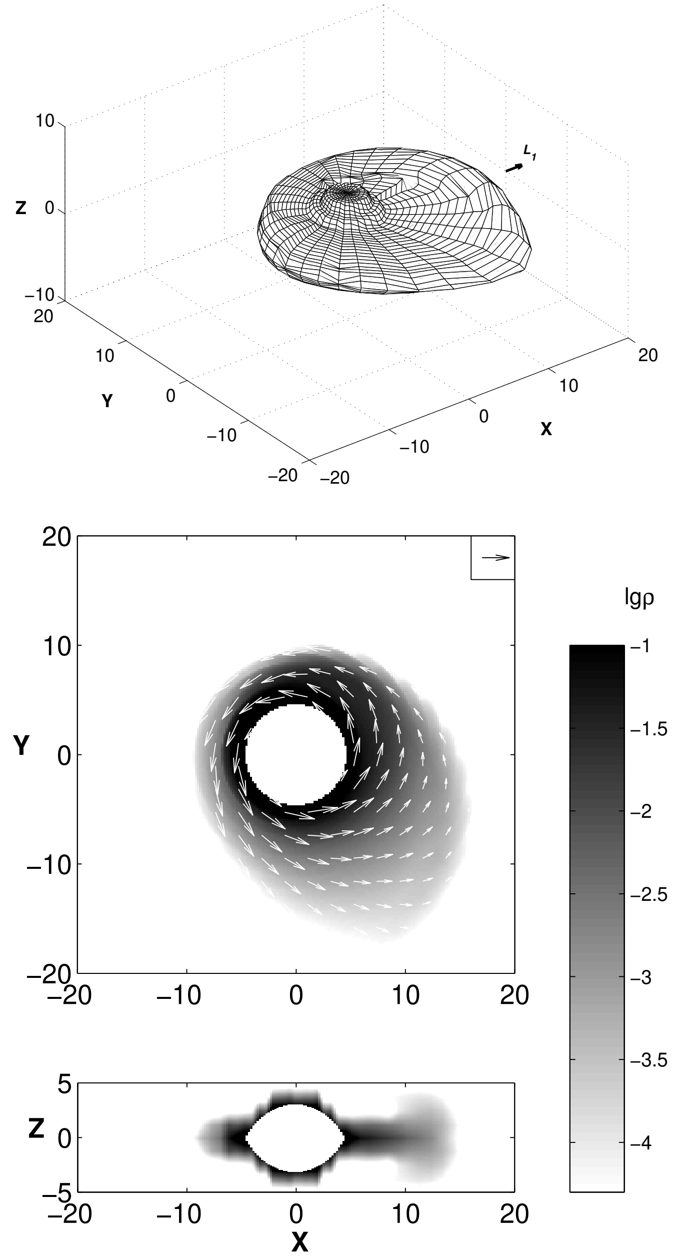

Fig. 5d. The same as Fig. 5a but for the time $t = 0.608 P_{orb}$.

$L_1^{\rm rot}$ point plus its kinetic energy given by the stellar rotation is much smaller than the potential energy in L_1 point

$$\Phi(L_1^{\rm rot}) + {}^1\!/_2(\Omega_\star - \Omega)^2 \cdot x^2_{L_1^{\rm rot}} < \Phi(L_1).$$

The value of extra energy needed to reach L_1 is a few orders of magnitude larger than the characteristic energy of the system $E_{\rm sys}$. Considering that the effective temperatures of B stars range roughly from 10 000 to 30 000 K, it is clear that the thermal energy cannot change the overall energy balance significantly. One is, therefore, led to the conclusion that the outlow from rapidly rotating B stars in binaries via the $L_1^{\rm rot}$ point should lead to the formation of roughly Keplerian equatorial disks around such stars but not to a significant mass transfer towards their companions.

It is necessary to point out, however, that the above analysis of the energy balance is not exhaustive. For instance, the numerical investigations carried out by Narita, Kiguchi & Hayashi (1994) show that if the rotational velocity is close to $V_{\rm br}$ and viscosity is considered, a disk in a binary system may evolve from an accretion disk to an outflowing one. At the same time, their numerical simulations showed that the transfer of the angular momentum is far more pronounced than the mass loss and that the mass loss by the disk is significant only on the evolutionary time scale (i.e. $\sim 10^6$ years). Unfortunately, it is impossible to carry out 3-D gas-dynamical simulations for such long time intervals with present-day computers. One only has to expect that a viscous smearing will not significantly influence the solution obtained only over a time interval comparable to the orbital period of the binary (say, less than a year).

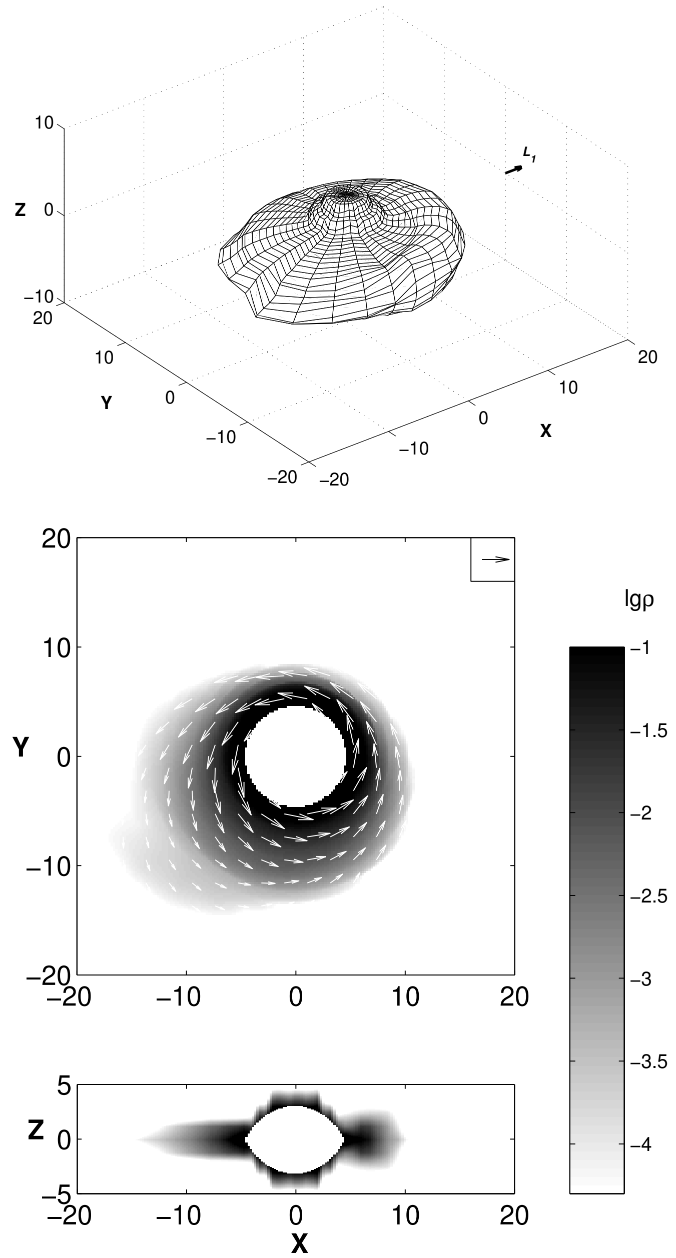

Fig. 5e. The same as Fig. 5a but for the time $t = 0.639P_{orb}$.

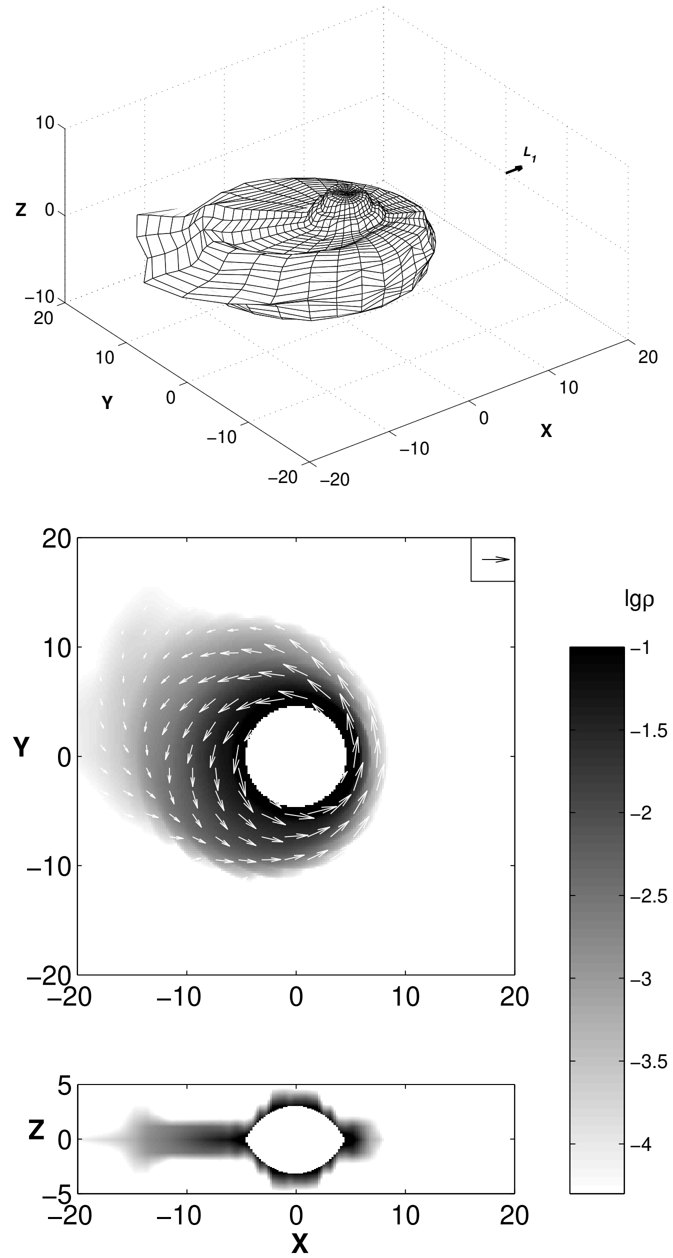

Fig. 5f. The same as Fig. 5a but for the time $t = 0.660P_{orb}$.

It summary, the outlow of matter via the $L_1^{\rm rot}$ point seems to represent the most probable scenario of the formation of the Be envelope for a rapidly rotating B star which is a member of a binary system.

4. The numerical Model

4.1. Parameters of binary

For a numerical investigation, we have chosen a binary with parameters that are typical for binary Be stars. Since Be stars are most abundant around the spectral class B2, we have chosen parameters corresponding to a normal main-sequence B2 star according to the empirical calibration by Harmanec (1988) and adopted a mass ratio of 0.1 which is also typical for known Be binaries. In particular, we used the mass of Be star (star 1) $M_1 = 8.6M_\odot$, mass of star 2 $M_2 = 0.86M_\odot$, binary mass ratio $q = M_2/M_1 = 0.1$, binary separation $A = 121R_\odot$, and orbital period $P = 50^{\rm d}$. Inner Lagrangian point is then located at the distance of $x_{L_1} = 87R_\odot$ from the centre of star 1. The equatorial radius of the Be star $R_1 = 4.6R_\odot$ was chosen. In accordance with the above considerations we assumed the Be component to be large enough to reach the 'asynchronous' Roche lobe, i.e. that $x_{L_1^{\rm rot}} = R_1$. Using the adopted values of $x_{L_1^{\rm rot}}/A$ and q, one gets the asynchronicity parameter of Be star $f = \Omega_\star/\Omega$ via Eq. (4). In this case, f is equal to 128.6 (implying that the period of rotation of the Be star is $0^{\rm d}\!.39$). The value of linear velocity at $L_1^{\rm rot}$ point in the adopted coordinate system is equal to $V_{\rm rot} = (\Omega_\star - \Omega) \cdot x_{L_1^{\rm rot}} = 594$ km s^{-1}. Note that the value of the critical velocity of a single star with the same mass