1993A&A...279..131H

Astron. Astrophys. 279, 131–147 (1993)

ASTRONOMY
AND
ASTROPHYSICS

Orbital elements of β Lyrae after the first 100 years of investigation

Petr Harmanec[1] **and Gerhard Scholz**[2]

[1] Astronomical Institute, Academy of Sciences, 251 65 Ondřejov, Bohemia, Czech Republic
[2] Astrophysical Institute, Telegrafenberg, O-1561, Potsdam, Germany

Received February 11, accepted April 1, 1993

Abstract. A detailed analysis of 1532 new and published radial velocities of the B6-8II component (accumulated over a century of spectroscopic observations), and 33 published velocities of the Be component of β Lyr led to the following results: (a) A new accurate quadratic ephemeris, based solely on radial velocities, was derived which predicts correctly even the very first photometric minima observed as early as 1784. (b) No convincing evidence of significant deviations of the observed times of minima from the new ephemeris was found; there is a suspicion of possible slight γ-velocity variation which, if confirmed, may be due to long-term variations in the circumstellar matter within the system rather than to the presence of a third star in the system. (c) An attempt at the very first direct test of the theory of large-scale mass exchange between the binary components has been carried out. Although not conclusive with the data at hand, this result *does not contradict* the idea that an essentially conservative mass transfer from the B6-8II star towards the Be component is presently going on in the system; (d) Values of $M_1 \sin^3 i$ and $M_2 \sin^3 i$ of unprecedented accuracy (probably of 1% and 3% for the B6-8II and Be star, respectively) were derived from the combined orbital solutions for only the high-dispersion velocities. For a plausible range of the orbital inclination (80–90°), this determination leads to a consistent model in which the B6-8II star fills its Roche lobe and corotates with the orbital motion.

Key words: stars: binaries: eclipsing – stars: binaries: spectroscopic – stars: emission-line, Be – stars: fundamental parameters – stars: individual β Lyr

1. Introduction

β Lyrae (10 Lyr, HR 7106, HD 174638, BD+33°3223, ADS 11745A) is the brightest member of an optical system of six stars. It seems that only components B (B7V) and F (A8-9V) are physically associated with β Lyr A (Abt et al. 1962; Abt & Levy 1976). At the same time, β Lyr is one of the most challenging spectroscopic and eclipsing binaries in the sky. Its optical spectrum is dominated by the absorption lines of the larger but less massive B6-8II star, which fills or nearly fills its critical Roche lobe. In spite of the enormous effort by many investigators, no truly photospheric absorption lines belonging to the more massive component have ever been observed in any part of the electromagnetic spectrum. The observed spectra do contain, however, many emission and nearly stationary blueshifted absorption lines with additional (red- and blueshifted) satellite absorption lines seen immediately prior and after the eclipse of the B6-8II star. Another important feature is the steady increase of the 12.9-day orbital period of the binary at a rate of 19 $\mathrm{s\,yr^{-1}}$.

Very many astronomers have investigated β Lyr, but our present knowledge and the observational data at hand would certainly not be so complete without the enthusiastic interest of Dr. Otto Struve. He undertook a respectable number of studies of this puzzling binary and encouraged and inspired many who followed in his footsteps (cf. e.g. the review by Sahade 1980).

While reviewing a very thorough spectroscopic study of β Lyr by Sahade et al. (1959), Heard (1959) wrote: "In Beta Lyrae, according to Dr. Struve's belief, we can actually watch a star undergoing change, and he has repeatedly re-emphasized that we must not miss the opportunity to build up a mass of observational data not only to compare with data from a century ago, but for future astronomers to compare with their observations made centuries hence."

In reality, the story of the astronomical investigation of β Lyr is also a tale of discovering, forgetting (or mistrusting) and re-discovering the same facts again during the past century. This is true, for instance, for the long-term variations of the continuous and line spectrum and the shape of the light curve (Blagg 1924, 1925, 1928; Maury 1935; Guthnick 1945; Abt 1962; Alduseva 1969, 1974; Larsson-Leander 1970; Batten & Sahade 1973; Bahýl' & Kreiner 1981; Guinan 1989). In recent years, there has

---

Send offprint requests to: P. Harmanec been an unfortunate tendency to underestimate the quality of old observations, which is not always justified (as we shall demonstrate below).

It is our belief after a preliminary inspection of the existing data for β Lyr that a careful synthesis of this large set of observations could be at least as important in deciphering the star's puzzles as the accumulation of new, more accurate data. This study represents a specific attempt in that direction.

2. Why re-analyse the radial-velocity data?

According to the currently accepted view, β Lyr is a system observed in a phase of relatively rapid large-scale mass exchange between its binary components. This idea comes originally from Kuiper (1941) and is indirectly supported by the observed systematic increase of the orbital period, $\dot{P}$, and by rich spectroscopic evidence for circumstellar matter (briefly mentioned above). According to this theory, the B6-8II component is losing mass via the Roche-lobe overflow towards its companion. The mass-gaining star is almost completely hidden in an accretion disk created by the infalling matter. A part of the material outflows from the system and creates extended gaseous structures around the binary.

The very first radial-velocity (hereafter RV) curve of the B6-8II component of β Lyr was obtained by Bélopolsky (1893a, b) in Pulkowa in 1892, just one hundred years ago. Since then, over 1400 RV observations of this star have been accumulated and published by various observers. These data are rather uniformly distributed in time. Since the sharpness of the spectral lines of the B6-8II star ($v \sin i = 40$–65 km s^{-1}) facilitates accurate RV measurements and since the total range of the orbital RV variations amounts to 370 km s^{-1}, the whole body of RV observations accumulated over nearly 100 years represents a unique set of data. Some investigators (e.g. Sahade et al. 1959) re-calculated the orbital elements for some individual previously published data sets, and there were also several attempts to check on the secular constancy of the systemic velocity of β Lyr A (Rossiter 1933; Sahade et al. 1959; Abt 1962; Skulskij 1970, 1971, 1972; Batten & Fletcher 1975). However, in none of these studies was an attempt made to analyse all the data together. Such an approach is attractive for several reasons:

(i) A solution based on a large body of RV data spanning one century can provide the orbital elements of unprecedented accuracy. This is certainly important for a system whose basic physical elements are still uncertain within rather broad limits.

(ii) A consistent analysis of all RV data can lead to the determination of the orbital period, P, and rate of its secular increase, $\dot{P}$, with an accuracy probably superior to the ephemerides derived so far from photometry (cf. Klimek & Kreiner 1973, 1975; Mandushev 1988 and references therein). The light curve suffers from various kinds of cycle-to-cycle variations (cf. e.g. Blagg 1924; Guthnick 1945; Alduseva 1969; Bahýl' & Kreiner 1981; Guinan 1989) and accurate determination of the times of minima is seriously hampered by the long (12.9-day) orbital period. In contrast to this, the RV curve of the B6-8II star is very nearly sinusoidal (with the exception of small characteristic distortions occurring symmetrically around the phases of the primary eclipse) and its steep linear parts ($\mathrm{d(RV)}/\mathrm{d}t = 90$ km s^{-1} d^{-1} on the rising and declining branches of the velocity curve) permit very accurate determination of the times of both minima.

(iii) The existing RV data set is also well suited to a search for possible variations on various timescales different from the orbital period.

(iv) Finally, β Lyr is probably the only binary system in the sky for which the number and accuracy of RV data justify the first serious attempt at *a direct test* of the theory of large-scale mass transfer. (For details of the theory, cf. e.g. Paczyński's 1971 review).

What we mean is that *if* the B6-8II star (hereafter star 2) is indeed losing mass and a substantial part of this mass is accreted by its mysterious more massive companion (hereafter star 1), one should observe a secular change of the semiamplitude of the RV curve of star 2, K_2, caused by two effects: (i) an increase of the separation A of the centres of both stars due to the increase of the orbital period, and (ii) a decrease of the mass of star 2 due to mass loss. As is often the case nature conspires against us and the two effects tend to cancel each other. However, the decrease in K_2 due to increasing separation is faster than increase due to decreasing mass and should, in principle, be detectable after some time.

3. Observations and reductions

Guided by the bibliography of Be stars by Merrill & Burwell (1933, 1943, 1949), bibliography of spectroscopic binaries (Korytnikov et al. 1963), bibliography of the IAU Commission 42, bibliography provided by the Strasbourg Center for Astronomical Data and by references in the original papers, we compiled what we hope is a rather complete list of RV measurements of star 2 in β Lyr *from optical spectra.* (Although there are some useful RV data obtained from satellite UV spectra, we decided not to use them in the present analysis since their accuracy is inevitably lower than that for the optical spectra and they rely on different spectral lines.) To avoid the known complications due to the presence of circumstellar matter, we only compiled the metallic line velocities in cases when the velocities of different ions were published separately in the original papers.

Additionally, we contributed two important data sets ourselves. One of us, G.S., measured RVs on 24 wavelength-calibrated old Potsdam spectrograms of β Lyr obtained by Hartmann and Eberhardt between 1900 and 1909 with two different telescopes. (The spectrographs had

Table 1. Radial-velocity observations of the B6-8II component of β Lyr

Source	Epoch [JD − 2400000]	No. of RVs	No. of spg.	Instrument/ Dispersion	Lines measured
Bélopolsky (1893a, b)	12335–12429	11	1	A/53	Mg II
	12335–12429	13	1		Fe II
Bélopolsky (1897a, b)	14096–14139	26	2	B/14	Mg II
Curtiss (1911)	17730–17812	60	3	C/40	Mean
	17730–17812	58	3		Mg II
Rossiter (1933)	19181–20025	193	4	D/38	Mean
	20235–22955	249	5		Mean
Baxandall &	14431–25551	55	15	E/44	Mean
Stratton (1930)	17730–17819	25	3	C/40	Mean
Baker (1937)	27599–27611	19	12	F/40	Si II, Ca II, Mean
Sherman (1941)	26764–29248	121	9	G/26	Mean metallic
Sahade et al. (1959)	35202–35357	192	11	H/10	Si II
	35203–35357	189	11		Mg II
	35203–35357	191	11		Fe II
	35203–35357	178	11		Ca II
	35202–35357	192	11		Mean
Struve (1958)	36378–36393	108	11	H/10	Si II
Struve et al. (1960)	36375–36396	50	10	I/9	Mg II, Fe II, mean
	36378–36393	108	11	H/10	Mg II, Fe II, mean
Abt (1962)	36793–36807	15	13	J/8.5	Si II
	36809–36823	3	13	K/5.0	Si II
Skulskij (1971)	39237–40571	87	8	L/14	Si II
	39237–40564	63	8		Mg II
	39237–40571	79	8		Fe II
	39237–40567	71	8		Ca II
Skulskij (1973)	41372–41427	26	8	L/14	Si II
	41372–41427	22	8		Fe II
	41372–41427	25	8		Ca II
Batten & Fletcher (1975)	39335–41906	62	17	M/10	Si II
	41152–42306	25	7	M/2.4	Si II
Flora & Hack (1975)	41150–41165	23	6	N/7.0	Si II blue
	41150–41165	23	6		Mg II
	41150–41165	19	6		Ca II
	41150–41165	23	6		Fe II
	41151–41165	12	6	N/12.4	Mg II
	41151–41165	19	16		Si II red
Skulskij (1992)	48139–48563	42	14	R/6.0	Si II red
This paper	15184–18573	18	18	P/27	Si II
	16298–18573	17	18		Mg II
	15184–18573	17	18		Fe II
	16298–18573	17	18		Ca II
	16246–16298	6	19	Q/17	Mg II
	16290–16298	4	19		Fe II
	48449–48521	59	20	O/4.2	Si II
	48449–48521	59	20		Mg II
	48449–48521	59	20		Fe II
	48449–48521	36	20		Ca II

Notes: Following abbreviations are used in column "Instrument": A... 0.75-m Pulkowa refractor, 1-prism spg.; B... 0.75-m Pulkowa refractor, a new large collimator, 2-prism spg.; C... Allegheny, the Keeler Memorial 0.79-m reflector, 1-prism Mellon spg.; D... Ann Arbor 0.95-m reflector, 2-prism spg.; E... Cambridge, the Newall 0.63-m refractor, 2-prism spg.; F... Forcalquier 0.81-m reflector, 4-prism spg.; G... Yerkes 1.02-m reflector, 1-prism Bruce spg.; H... Mt Wilson 2.54-m reflector, coudé grating spg.; I... Lick 0.91-m refractor, Mills prism spg.; J... McDonald 2.08-m reflector, coudé grating spg.; K... DAO Victoria 1.83-m reflector, Cassegrain grating spg.; L... Crimea 1.25-m reflector, Nasmyth ASP-11 grating spg.; M... DAO Victoria 1.22-m reflector, coudé grating spg.; N... OHP 1.52-m reflector, coudé grating spg.; O... Ondřejov 2.00-m reflector, coudé grating spg. P... Potsdam 0.80-m refractor, prism spg.; Q... Potsdam 0.325-m refractor, prism spg.; R... Crimea 2.60-m reflector, coudé grating CCD spg.

prismatic dispersions of 27 Å mm^{-1} and 17 Å mm^{-1} at Hγ). To extend the observations into the present, P.H. measured RVs on a recent series of 59 Ondřejov photographic spectrograms obtained in July–September 1991: 99 years after the Bélopolsky's first observations. These had a linear dispersion of 4.2 Å mm^{-1}. Basic information about the individual data files can be found in Table 1.

To avoid possible errors, a computer code was used to convert the dates of the mid-exposures of all spectrograms into heliocentric Julian dates. In a few cases, we detected obvious misprints in the original tabulations. For convenience of future investigators, these cases are listed in Table 2.

The following remarks about some of the data files used may be useful:

(i) Bélopolsky's very first (1893a, b) observations have a mean dispersion of 53 Å mm^{-1}. He did not use the Hartmann formula in these pioneering times of RV work. Instead, he used formulas which defined wavelengths as quadratic functions of the revolution of the screw of his comparator individually in the neighbourhood of each comparison line. The comparison spectrum contained several Fe I and Na I lines, Hβ and Hγ. Comparing Belopolsky's wavelengths for the comparison lines, we found they are systematically more positive by 0.2–0.4 Å relative to the present-day wavelengths. The mean difference is 0.246 Å, which is close to the wavelength difference we found for the two Balmer comparison lines. Bélopolsky (1893a, b, 1894) himself only studied the radial-velocity curve of the Hβ absorption and emission. His Mg II 4481 and Fe II 4583 velocities were later analysed by Tikhoff (1897). However, Bélopolsky's (1893a) paper also contains the observed wavelengths of several He I and Fe II lines. We could not use the helium velocities since they are clearly affected by the contribution from the circumstellar matter and do not define the orbital motion of star 2. However, the Mg II 4481 and Fe II 5316, 5169, 4629, 4583, 4555, 4522 and 4508 velocities turned out to define the RV curve quite well.

To re-construct the heliocentric radial velocities from Bélopolsky's data (since he did not publish the comparator readings for the individual comparison and stellar lines) we proceeded in the following way:

We used the laboratory wavelength of 4481.6 Å recommended by Bélopolsky (1897a) to calculate the geocentric RV of the Mg II line. We adopted the observed Mg II wavelengths from Tikhoff's (1897) paper since his data contain one correction and one extension of the original Mg II wavelengths published by Bélopolsky (1893a). To calculate the velocities of the Fe II lines, we increased the laboratory wavelengths tabulated by Moore (1945) for the mean wavelength difference of 0.246 Å we found from the comparison lines. We subtracted 2.0219 h from the Pulkowo mean time to obtain the Greenwich mean time and calculated the heliocentric Julian dates and heliocentric RV corrections for all spectra. Our reconstruction of Bélopolsky's 1892 velocities is summarised in Table 3.

(ii) Bélopolsky's (1897a, b) Mg II velocities were measured by a special "differential" technique relative to a solar spectrogram obtained with the same spectrograph. We checked the heliocentric RV corrections and adopted the original velocities from Bélopolsky (1897a), converting the mean Pulkowa times into heliocentric Julian dates as for the 1892 data.

(iii) A selection of Curtiss' 1911 Allegheny spectrograms was later re-measured by Baxandall & Stratton (1930) who used a different set of spectral lines. In neither case were velocities of individual ions (besides Mg II) published. We used Curtiss' original velocities when combining all RV data since they are available for all Allegheny

Table 2. Misprints found in the tables of radial-velocity measurements of β Lyr in the published papers. Columns 3 and 4 contain the originally published, and columns 4 and 5 the corrected values. Misprints and their corrections are printed in italics. Note also that Sherman (1941) used the US date but U.T. hours and minutes for the tabulated times of mid-exposures. The identification of misprints in Skulskij's papers was kindly verified by the author at our request

Source	Date	RV (km s^{-1})	Date	RV (km s^{-1})
Rossiter (1933)	1911 June 12.751	*−44.5*	1911 June 12.751	*+44.5*
	1916 Apr. 27.779	*−162.9*	1916 Apr. 27.779	*+162.9*
	1920 Mar. 5.932	*−165.1*	1920 Mar. 5.932	*+165.1*
	1920 July 25.611	*−163.2*	1920 July 25.611	*+163.2*
Sherman (1941)	1934 June *26* 2:48	−206.8	1934 June *28* 2:48	−206.8
Skulskij (1971)	1966 July *12* 22:41	−60.4	1966 July *21* 22:41	−60.4
	1966 Aug. *16* 17:52	−216.5	1966 Aug. *6* 17:52	−216.5
	1966 June 18 21:50	*41.9*	1966 June 18 21:50	*−41.9*
Skulskij (1973)	1972 Mar. 13 *2:50*	−110.4	1972 Mar. 13 *2:38*	−110.4
	1972 Mar. 13 *2:38*	−124.4	1972 Mar. 13 *2:50*	−124.4
Skulskij (1992)	JD 2448139.387	*−48.0*	JD 2448139.387	*+48.0*
	JD 2448562.212	45.0	*JD 2448563.212*	45.0

Table 3. Our reconstruction of the radial-velocity observations of β Lyr by Bélopolsky (1893a, b); see also Tikhoff (1897).

HJD −2400000	RV Fe II	No. of lines	RV Mg II	RV mean	No. of lines
12335.335	−1.6	3	−31.9	−9.2	4
12336.335	65.7	2	–	65.7	2
12372.250	−171.0	4	−177.1	−172.2	5
12374.249	−34.4	4	−36.7	−34.8	5
12375.249	59.7	5	56.9	59.2	6
12379.249	98.1	2	130.4	108.8	3
12383.249	−166.2	1	–	−166.2	1
12391.207	140.3	4	164.1	145.0	5
12392.207	84.3	3	130.7	95.9	4
12394.207	−50.2	2	−36.5	−45.7	3
12398.207	−174.8	6	−170.0	−174.1	7
12428.122	151.3	4	175.7	156.2	5
12429.122	169.0	4	182.6	171.7	5

spectrograms (with the exception of the last one for which we had to adopt the value obtained by Baxandall & Stratton). However, we also considered Baxandall & Stratton's re-measurement of Allegheny spectra when discussing the problem of the systemic velocity of β Lyr .

(iv) Rossiter (1933) found a systematic difference between velocities obtained before and after 1914 when a new prism had been installed in the spectrograph. Consequently, we handled the relevant subsets as data obtained with two different spectrographs.

(v) Abt's (1962) velocities are based on 15 McDonald and only 3 Victoria spectrograms. We have verified that velocities from both spectrographs are on very nearly the same system. This is also evidenced by the excellent agreement of the mean interstellar Ca II velocities from both data sets: -17.05 ± 1.11 km s^{-1} and -16.97 ± 0.76 km s^{-1}, respectively. We therefore treated all 18 Abt's RVs as originating from one spectrograph.

(vi) We calculated and used mean Si II, Fe II and mean metallic velocities from Skulskij's (1971, 1973) data.

(vii) We treated the 10 Å mm^{-1} and 2.4 Å mm^{-1} Victoria velocities of Batten & Fletcher (1975) as data originating from two different spectrographs since the authors reported a systematic γ-velocity difference between these two data sets.

(viii) Similarly, we also handled blue and red mean Si II velocities published by Flora & Hack (1975) as originating from two different spectrographs since the authors mention a systematic velocity difference between them.

(ix) The Ondřejov spectrograms were obtained at the coudé focus of the Ondřejov 2.0-m reflector. They have a linear dispersion of 4.2 Å mm^{-1}. Some of the spectrograms cover the wavelength region of approximately 3600–4600 Å, the rest 4000–5000 Å (those for which the measurement of the sharp He I 3888 line is missing in Table 4). The spectra were digitized with the Ondřejov 5-channel microdensitometer which records separately the stellar spectrum, both sets of comparison spectra and background on both sides of the stellar spectrum. They were wavelength-calibrated, transformed into intensities and rectified and the radial velocities were derived from an intercomparison of direct and reverse images of each line profile displayed on a computer monitor. All these reductions were carried out with the help of software developed by Dr. J. Horn.

The radial velocities measured on the Ondřejov spectrograms are presented in detail in Table 4. The Si II velocities are the mean velocities of lines of multiplets 1, 3, and (in one case) 5; only multiplet 3 could be measured on "blue" spectra. The Mg II velocities rest solely on the strong and easily measurable Mg II 4481 line. (Two lines of Mg II 10 are visible in the spectra but often give unreliable velocities because of their blending with the He I 4388 line.) The Ca II velocities rest on Ca II K and H lines. Mean Fe II velocities are based mainly on stronger lines of multiplets 27, 28, 37, and 38. The Fe II 4173.45 line gives velocities which are always more positive (for about 10–15 km s^{-1}) than the mean value of the other Fe II lines. It was consequently omitted from the analysis. Velocities from all the above-mentioned groups of selected lines were averaged into the mean value in Table 4. Only the following eight He I lines were used to define the mean stellar He I velocity of star 2: $\lambda\lambda$ 3867, 3871, 3926, 4009, 4120, 4143, 4387, and 4713 Å. All other (often stronger) observed He I lines were affected by the contributions from the circumstellar matter. The last three columns of Table 4 contain the mean velocity of the sharp (interstellar or circumstellar) Ca II K and H lines and the velocities of the strong and sharp metastable He I lines 3888 and 3964 Å.

(x) The old Potsdam spectra were obtained with two different prismatic spectrographs (Nos. 18 and 19 in Tables 1 and 5) attached to 0.80-m (f/15) and 0.325-m (f/10.5) telescopes. The spectra from spectrograph 18 cover the wavelength region from about 3800–5000 Å (dispersions 42 Å mm^{-1} at Hβ, 27 Å mm^{-1} at Hγ and 17 Å mm^{-1} at H8), those from spectrograph 19 cover only the wavelength region from about 4300–4600 Å (dispersion 17 Å mm^{-1} at Hγ). Radial velocities were measured with the Potsdam oscilloscopic comparator and can be found in Table 5. The Si II velocities are again the mean velocities of five lines of multiplets 1 and 3. The mean Fe II velocities rest on three lines ($\lambda\lambda$ 4233, 4549 and 4583 Å), Mg II is the velocity of the 4481 line, and Ca II, the K line only.

To check on the velocity systems of the spectrographs, radial velocities of three standard-velocity stars observed with spectrograph 19 under similar conditions as the β Lyr spectra were also measured. Between 1900 and 1909, no RV standards were observed with spectrograph 18. Instead, several available spectra of α Lyr and α Per obtained with spectrograph 18 were measured. These measurements are collected in Table 6 and demonstrate an excellent

Table 4. Radial velocities of β Lyr measured on the Ondřejov 4 Å mm^{-1} spectrograms obtained in 1991

Plate number	HJD −2400000	RV Si II	RV Mg II	RV Fe II	RV Ca II	RV mean	RV He I	RV Ca II	RV He 3888	RV He 3964
5418	48449.4200	−170.3	−171.6	−170.1	−170.4	−170.2	−168.7	−15.85	−139.9	−134.5
5420	48449.5182	−176.6	−174.5	−175.2	−176.3	−175.8	−178.1	−16.23	−138.9	−128.6
5422	48454.5241	47.5	43.5	49.2	44.6	47.5	45.5	−9.87	−144.2	46.9
5423	48458.4241	125.2	123.8	115.6	131.6	120.0	134.1	−18.02	−142.9	139.5
5430	48461.4282	−126.2	−130.3	−126.6	−127.0	−126.7	−119.2	−16.86	−145.0	−120.3
5435	48466.3566	−47.9	−50.3	−51.2	−44.3	−49.8	−42.0	−20.80	−150.3	−45.9
5438	48468.3739	116.8	113.1	115.8	115.4	115.9	113.3	−16.17	−150.4	119.2
5441	48473.3829	−38.2	−38.8	−38.9	−44.0	−39.2	−53.1	−16.95	−149.9	−87.2
5442	48473.4586	−43.1	−43.3	−44.4	−44.1	−43.9	−49.3	−20.21	−151.3	−81.7
5443	48474.4030	−128.3	−131.0	−126.9	−124.2	−127.6	−124.1	−17.92	−148.3	−117.8
5446	48475.3807	−180.7	−182.4	−178.3	−180.9	−180.2	−172.5	−17.76	−148.4	−129.5
5451	48476.3293	−200.1	−199.1	−199.1	−194.2	−199.2	−202.1	−17.20	−144.5	−146.8
5452	48476.3585	−198.2	−197.8	−201.3	—	−200.5	−201.7	—	—	—
5453	48476.3835	−199.2	−201.0	−199.7	—	−199.2	−203.1	—	—	—
5456	48476.4710	−201.8	−199.9	−205.1	−196.2	−201.6	−201.0	−17.12	−146.9	−126.7
5458	48480.3601	45.1	42.1	42.9	46.9	43.9	44.3	−17.44	−148.0	−112.8
5461	48481.3900	119.9	117.2	121.3	119.7	120.1	117.5	−19.75	−148.3	−102.9
5463	48484.3583	124.7	117.2	116.1	120.5	119.0	128.8	−17.91	−135.6	−109.5
5464	48484.3986	116.6	119.9	113.7	—	114.7	126.4	—	—	—
5467	48486.3325	−40.5	−44.1	−41.3	−43.7	−41.6	−49.2	−17.95	−148.8	−86.4
5468	48486.3735	−40.4	−39.8	−45.1	—	−44.0	−58.5	—	—	—
5469	48490.3456	−177.8	−182.2	−185.9	−181.9	−183.9	−177.5	−16.84	−142.3	−156.0
5470	48490.3883	−175.5	−180.2	−182.8	—	−181.4	−180.1	—	—	—
5476	48491.3560	−123.0	−124.8	−129.6	−122.0	−126.4	−114.2	−17.06	−146.5	−111.1
5477	48491.3764	−117.4	−119.9	−126.0	—	−124.6	−116.0	—	—	—
5482	48491.5323	−110.5	−110.1	−115.0	−105.5	−112.6	−105.9	−17.97	−145.1	−106.5
5483	48491.5834	−104.8	−109.4	−110.1	—	−109.3	−108.3	—	—	—
5489	48496.3311	160.6	157.2	152.2	—	153.5	164.2	—	—	—
5490	48496.3818	157.3	156.5	149.2	160.6	153.2	161.3	−16.72	−141.4	−126.3
5491	48496.4988	157.7	155.6	153.6	—	154.3	156.9	—	—	—
5492	48498.3640	50.1	42.7	38.9	57.9	46.0	46.6	−16.63	−143.4	−80.8
5500	48499.5306	−70.6	−77.8	−72.6	—	−72.8	−101.9	—	—	—
5504	48500.3987	−133.8	−139.9	−136.0	−123.0	−135.5	−131.9	−13.70	—	—
5505	48500.4542	−140.4	−146.8	−141.5	−140.3	−141.4	−132.2	−19.51	−143.5	−132.5
5508	48501.3174	−181.2	−180.2	−176.0	−170.3	−177.6	−174.4	−15.21	−141.0	−132.5
5509	48501.3368	−176.8	−181.1	−180.3	—	−179.8	−173.9	—	—	—
5515	48502.3305	−201.5	−197.3	−201.6	−194.9	−200.4	−197.2	−16.82	−141.7	−132.0
5516	48502.3503	−200.6	−200.2	−203.2	–	−202.4	−198.4	—	—	—
5519	48502.5520	−197.6	−194.2	−210.8	−196.2	−204.1	−193.5	−16.32	−144.0	−150.6
5521	48503.2961	−175.5	−179.2	−185.6	−171.4	−180.6	−174.7	−16.54	−141.5	−164.3
5522	48503.3145	−176.1	−176.5	−183.7	—	−182.1	−170.4	—	—	—
5526	48505.3436	−29.4	−32.2	−35.2	—	−34.0	−26.7	—	—	—
5527	48505.4227	−26.8	−26.1	−30.8	−29.7	−28.8	−26.3	−20.09	−146.5	−45.8
5528	48506.3463	53.8	54.8	56.9	—	56.1	50.5	—	—	—
5529	48509.3184	159.8	156.7	151.1	149.1	153.6	164.2	−18.31	−137.3	−126.6
5530	48509.3406	158.3	156.8	150.0	—	151.8	166.7	—	—	—
5535	48510.3892	118.3	109.2	103.7	110.3	108.7	117.3	−16.94	−137.9	−119.9
5536	48510.4093	114.3	112.3	109.5	—	110.5	118.8	—	—	—
5538	48512.3398	−53.8	−61.0	−54.6	−53.0	−54.7	−72.5	−13.70	−147.2	−135.8
5543	48513.2911	−131.8	−134.2	−132.9	−129.2	−132.5	−124.0	−15.45	−145.5	−132.1
5550	48516.3042	−173.9	−178.7	−181.2	—	−180.4	−175.7	—	—	—
5551	48516.3292	−177.6	−177.0	−184.1	−179.1	−181.0	−172.1	−16.66	−146.7	−156.9
5552	48519.2901	52.5	51.2	50.3	56.2	51.0	51.5	—	−146.3	−144.0
5553	48519.3207	55.3	55.8	55.3	—	55.4	51.9	—	—	—
5554	48520.2783	142.1	138.7	129.2	—	132.0	116.0	—	—	—
5555	48520.4463	142.3	133.7	138.1	—	138.3	134.3	—	—	—
5556	48521.2622	168.0	164.2	166.0	—	166.0	164.8	—	—	—
5557	48521.2761	167.7	163.3	166.7	165.8	166.9	166.0	−15.87	−137.9	−132.7
5560	48521.5108	167.7	165.0	164.1	—	165.1	169.4	—	—	—

Table 5. Radial velocities of β Lyr from the old Potsdam prismatic spectrograms

Plate number	HJD −2400000	RV Mg II	RV Ca II	RV Si II	RV Fe II	RV mean	No. of lines	RV He I 3888	Spectrograph No.
3	15184.4766	—	—	−87.8	−73.5	−83.0	3	—	18
1439	16246.4136	−191.2	—	—	—	−191.2	1	—	19
1461	16270.3728	−145.1	—	—	—	−145.1	1	—	19
1467	16290.3796	139.1	—	—	124.7	131.9	2	—	19
1469	16292.3775	149.5	—	—	137.0	141.2	3	—	19
1473	16294.3824	−14.9	—	—	−2.0	−6.3	3	—	19
1479	16298.4116	−192.4	—	—	−188.3	−189.7	3	—	19
568	16298.4116	−172.9	−190.2	−180.0	—	−180.6	6	−110.3	18
573	16300.4074	−72.4	−81.4	−75.0	−81.5	−76.3	8	−105.2	18
801	17489.3962	−29.0	−46.7	−41.0	−44.0	−40.4	5	−99.4	18
885	18214.3999	76.2	83.4	87.4	83.5	84.7	7	−113.0	18
907	18240.2513	77.6	83.7	88.7	82.0	85.8	8	−110.5	18
909	18243.2352	163.6	165.2	165.5	166.8	165.4	8	−88.4	18
918	18262.2294	−176.3	−176.6	−172.4	−197.1	−178.9	5	—	18
920	18264.2140	−77.6	−80.3	−71.6	−83.7	−74.9	8	−110.1	18
921	18266.2028	95.8	89.2	92.3	80.7	90.7	7	−108.3	18
922	18270.2172	94.6	105.1	107.2	101.1	103.8	6	−103.3	18
925	18273.1865	−160.2	−142.1	−144.5	−148.9	−146.7	8	−107.6	18
927	18275.2059	−183.0	−185.4	−181.6	−188.0	−183.2	7	−99.5	18
933	18292.1915	109.3	95.3	105.1	107.1	104.5	6	−108.6	18
936	18293.1762	166.8	156.4	175.8	189.1	173.3	6	−100.0	18
940	18302.1920	−149.3	−143.3	−135.6	−117.1	−136.2	5	−109.5	18
993	18572.2892	−200.5	−172.4	−190.8	−205.9	−192.0	5	−92.5	18
996	18573.2697	−150.5	−149.5	−134.2	−146.5	−140.5	7	−92.0	18

Table 6. Radial velocities of some constant-velocity stars measured on the spectra obtained with the Potsdam spectrographs at about the same period as the β Lyr spectra listed in Table 5. Usually 7–8 stellar lines were measured on the spectra from spectrograph 18, and 18–19 on those from spectrograph 19

Star name	Plate number	HJD −2400000	Measured RV	Catalogued RV	Spectrograph No.
β Gem	1418	16203.3660	3.3 ± 1.1	3.3	19
α Boo	1472	16294.3424	-5.3 ± 1.3	−5.3	19
α Lyr	617	16382.4181	-14.8 ± 3.2	−14.1	18
α Ari	1514	16452.2590	-14.5 ± 2.1	−14.3	19
α Lyr	653	17022.4278	-14.9 ± 0.7	−14.1	18
α Per	669	17182.2306	-1.1 ± 2.5	−2.1	18
α Per	698	17297.3243	-3.0 ± 0.8	−2.1	18
α Lyr	743	17377.4451	-14.5 ± 2.6	−14.1	18
α Per	854	17656.3910	-2.1 ± 3.7	−2.1	18
α Per	917	18244.4382	-3.3 ± 1.4	−2.1	18

agreement of the velocity systems of both spectrographs with the standard one.

A weight w was assigned to each RV datum according to formula

$$w = (10/D) \cdot (N/N_{\max}), \qquad (1)$$

where D is the mean plate dispersion, N the number of lines used to calculate the mean value of the considered group of lines on the spectrogram in question, and $N_{\max}$ is the maximum number of lines measured for the respective group within the given data set. In cases when $N_{\max} < 5$ for the Si II, Fe II or mean metallic velocities, we set $N_{\max} = 5$. In practice, the individual weights ranged from 0.05 to 4.17.

4. The new ephemeris

Various ephemerides used to calculate orbital phases of β Lyr which can be found in the literature usually have the form of a polynomial in epoch, E, with terms up to the third or fourth power. None of them was particularly successful in predicting correct phases outside the time intervals covered by the data on which it was defined. Klimek & Kreiner (1973) compiled a respectably large body of photometrically observed times of primary and secondary mid-eclipses and derived new ephemerides. They argued rather convincingly that the observed secular increase of the orbital period can be adequately described by an ephemeris containing only linear and quadratic terms in E. They show that the cubic term in E does not improve the fit to the data and varies widely from one determination to another. Their conclusion was later supported by Mandushev (1988).

Both Klimek and Kreiner & Mandushev claim that the quadratic dependence of the times of minima on epoch E implies a strictly linear increase of the orbital period with time. This, however, is not true. Let us assume a quadratic ephemeris in the form

$$T = T_0 + P_0 E + aE^2, \tag{2}$$

where T_0 is the reference epoch (e.g. one particular time of primary mid-eclipse), P_0 is the instantaneous period at time T_0, T is the time corresponding to an arbitrary cycle & phase E, and a is a constant. (Please, note that to be able to calculate phases of individual observations in a consistent way, we treate both time T and cycle & phase E as *continuous* variables. It is possible to verify, however, that the relations which we derive below hold also for T and E treated as *discrete* variables provided all derivatives are properly replaced by corresponding finite differences.)

The instantaneous period P at time T is obviously

$$P = \mathrm{d}T/\mathrm{d}E = P_0 + 2aE \tag{3}$$

and the instantaneous rate of period change, $\dot{P}$, is

$$\dot{P} = 2a(\mathrm{d}E/\mathrm{d}T) = 2a/P. \tag{4}$$

These relations show that the period change is linear in epoch but *not in time*. For positive a, the rate of period increase itself is slowly decreasing with time.

This is in fact exactly the situation one expects for an essentially conservative mass transfer after the initial mass ratio has already been reversed, i.e., when the rate of mass transfer from a less massive to more massive star is decreasing in time. This is the stage one encounters in the vast majority of observed mass-exchanging systems. In this sense, the simple quadratic ephemeris (2) seems to be locally (i.e. over "only" a couple of hundreds of years) a very good representation of the period variations of real interacting systems.

Guided by the above findings, we also analysed the RV data allowing for a free convergence of the rate of period increase $\dot{P}$ which fulfils Eqs. (2)–(4) as one of the orbital elements. Another important feature of our solutions is that the systemic velocity can be calculated separately for each of the spectrographs defined in Table 1 and its attached notes. All the orbital solutions were calculated using the Fortran program FOTEL (Hadrava 1990 and personal com.) kindly put at our disposal by its author.

Before calculating the orbital elements for the various subsets of available data, we had to decide whether to consider an elliptical or a circular orbit. Many original investigators calculated an elliptical orbit and invariably derived a small orbital eccentricity of about 0.02. Batten & Fletcher (1975) argued that the eccentricity is spurious and the orbit is in fact circular. We did some trial calculations and found that one indeed obtains an eccentricity of about 0.02 for older data sets. For recent high-dispersion data, however, we usually found eccentricities smaller than 0.01. Moreover, the values of the longitude of periastron for all trial eccentric orbits clustered near 0°, without changing systematically with time. All this, of course, points towards a spurious eccentricity caused by the effect of mass loss from star 2 (see e.g. Milgrom 1978). Klimek & Kreiner (1973) derived separately the ephemerides for the epochs of primary and secondary minima. These epochs differ by $0\overset{\mathrm{P}}{.}5018 \pm 0\overset{\mathrm{P}}{.}0011$, i.e. nearly exactly by $0\overset{\mathrm{P}}{.}5$ within the limits of errors of their determination. In contrast to this, an elliptical orbit with eccentricity of only 0.008 and $\omega = 9°$ (which results from a solution based on all velocities) predicts a phase difference of $0\overset{\mathrm{P}}{.}505$ between the two minima. This independently indicates an orbit of zero eccentricity. *We therefore adopted a circular orbit throughout the rest of this investigation.*

A problem which we encountered repeatedly in the course of this study is that various authors chose different selections of spectral lines in their RV measurements. To derive a new ephemeris we first decided to include *all* available RV observations. Solution 1 in Table 7 is thus based on all 1532 weighted mean velocities. The corresponding RV curve is shown in Fig. 1a.

Figure 1b is the plot of the O–C velocity deviations from solution 1 versus orbital phase. It is seen that the deviations follow a curve which is quite typical for the RV distortions observed for the Roche-lobe filling stars – see e.g. a fine review by Hill (1993). This corroborates the conclusion that star 2 is filling or nearly filling its critical Roche lobe. A more detailed analysis of this effect will be the subject of a separate study.

To check on the stability of the result, we also calculated several other solutions, which are all presented in Table 7. Thus, solution 2 is based solely on the Si II velocities, solution 3 on Mg II velocities, and solution 4 on Fe II velocities whenever available. An inspection of Table 7 reveals that all these solutions – besides that for Si II – agree within the limits of respective errors of individual elements. A slightly different result for the values of the epoch, period and its rate of change derived from the

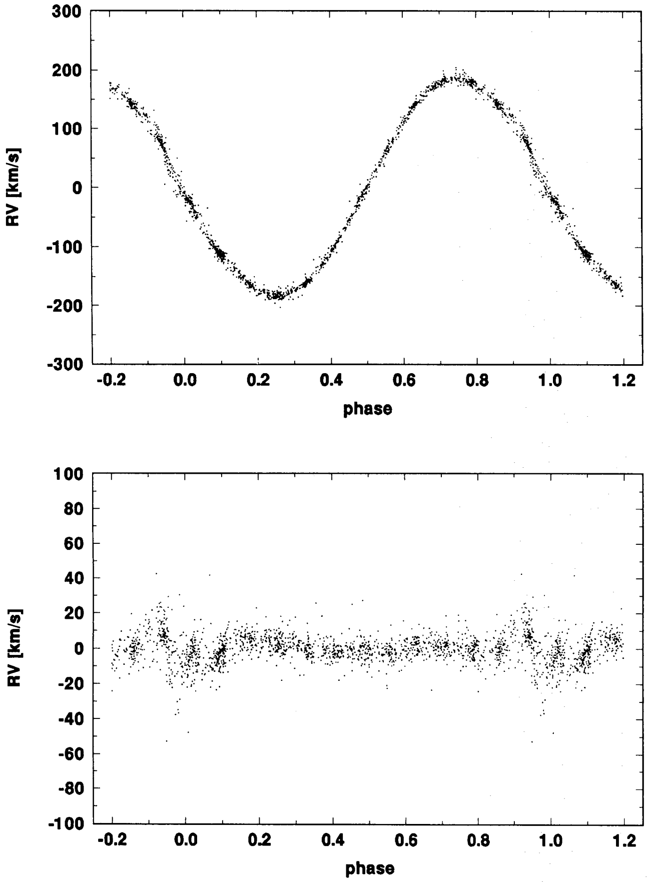

Fig. 1. Radial-velocity curve of the B6-8II component of β Lyr based on all 1532 mean velocities and the O–C velocity deviations from the orbital solution 1 (see Table 7)

Si II RVs is not surprising if one realizes that the only available Si II velocities prior to 1932 are the 18 data from old Potsdam spectrograms.

The final ephemeris which we adopt in all subsequent analyses is therefore based on solution 1 of Table 7 and reads as follows:

$$T_{\mathrm{prim.eclipse}} = \mathrm{HJD}\ 2408247.966 + 12\overset{\mathrm{d}}{.}913780 \times E + 3.87196\,10^{-6}\,E^2. \qquad (5)$$

The corresponding mean rate of period change during the past 100 years amounts to 2.19 10^{-4} d (i.e. 18.9 s) yr^{-1}.

Our ephemeris compares very nicely with the most recent ephemeris by Mandushev (1988). Note, however, that our RV observations span less than 3000 cycles while the photometry used by Mandushev span nearly 6000 orbital cycles. This confirms our assumption [in Sect. 2, (ii)] that for β Lyr the times of minima can be more accurately derived from the RV curve than from the light curve.

Keeping the period and its derivative fixed at values which follow from solution (1) and formulae (3) and (4) for each respective epoch, we also calculated the epochs of primary minima based on local RV data subsets which span no more than 35 days. They are collected in Table 8 and their O–C with respect to ephemeris (5) are plotted in Fig. 2. Figure 2 illustrates a good agreement of all RV data with the new ephemeris.

We also show in Fig. 2 (by different symbols) the O–C deviations calculated with respect to ephemeris (5) for all but 15 out of 1285 photometric primary minima adopted from Klimek & Kreiner (1973) (their Table 1), Klimek & Kreiner (1975) (their Table 4), Bahýl et al. (1979) (their Table 5), Aslan et al. (1987), and Hübscher et al. (1991, 1992). [Those 15 primary minima we decided to omit are obviously very inaccurate, having the O–C deviations larger than 1 day with respect to both, our and Klimek & Kreiner's ephemerides. Note also that the minimum at JD 2415887.62, quoted in Klimek & Kreiner's (1973) Table 1 is in fact a secondary minimum.] Figure 2 shows that the new ephemeris matches even the very first recorded light minima observed by Goodricke in September 1784 equally well as the ephemerides based on the observed times of minima. To quantify this statement, we compare below the r.m.s. errors per single epoch (in days) for all three ephemerides:

Ephemeris	Klimek & Kreiner (1973)	Mandushev (1988)	This paper
Photometry	0.0071	0.0071	0.0071
RVs	0.0096	0.0063	0.0058

Figure 2 and Table 8 also corroborate our conclusion about the generally higher accuracy of epochs derived from RVs over those from even photoelectric photometry. All this justifies us to adopt ephemeris (5) throughout the rest of this study.

5. Variability on timescales different from the orbital period

Previous reports of non-orbital variability of β Lyr included:

(i) long-term variations of the continuous and emission-line spectrum,

(ii) long-term cyclic variability of the systemic velocity of β Lyr; and

(iii) cycle-to-cycle variations of the RV and light curve.

Is there any evidence for some of these suspected variations in our large collection of RV observations of star 2? To address this question, we analyzed the O–C deviations of the epochs of minima derived from RVs, photoelectric observations and all photometric observations as well as the individual O–C deviations of RVs from solution 1. Let us consider the above three types of changes in turn.

5.1. *Long-term spectral and light variability*

There were several suggestions in the past that the light curve of β Lyr and the strength of its emission lines are

Table 7. Some orbital solutions for combined velocities of star 2 of β Lyr

Element	Solution 1 Mean RVs	Solution 2 Si II	Solution 3 Mg II	Solution 4 Fe II
P (days)	12.913780	12.913639	12.913778	12.913794
rms error	0.000020	0.000045	0.000033	0.000051
$\mathrm{d}P/\mathrm{d}t$	$5.9966\,10^{-7}$	$6.0469\,10^{-7}$	$5.9983\,10^{-7}$	$5.9939\,10^{-7}$
rms error	$0.0079\,10^{-7}$	$0.0151\,10^{-7}$	$0.0120\,10^{-7}$	$0.0171\,10^{-7}$
$T_{\mathrm{prim.ecl.}}$	8247.966 ± 0.018	8248.099 ± 0.052	8247.962 ± 0.034	8247.952 ± 0.058
K_2 (km s^{-1})	185.70 ± 0.26	186.15 ± 0.31	185.93 ± 0.43	186.25 ± 0.46
γ_1 (km s^{-1})	-15.7 ± 3.6	—	-7.5 ± 4.5	-18.1 ± 4.3
γ_2 (km s^{-1})	-14.6 ± 2.2	—	-14.6 ± 2.2	—
γ_3 (km s^{-1})	-20.87 ± 1.06	—	-20.2 ± 1.5	—
γ_4 (km s^{-1})	-20.58 ± 0.72	—	—	—
γ_5 (km s^{-1})	-19.28 ± 0.42	—	—	—
γ_6 (km s^{-1})	-5.1 ± 1.7	-4.4 ± 1.7	-7.1 ± 1.8	-5.9 ± 2.7
γ_7 (km s^{-1})	-16.37 ± 0.51	-16.52 ± 0.51	—	—
γ_8 (km s^{-1})	-20.04 ± 0.67	-19.85 ± 0.77	-20.9 ± 1.3	-19.9 ± 1.0
γ_9 (km s^{-1})	-20.12 ± 0.64	-20.09 ± 0.65	-20.08 ± 0.65	-20.15 ± 0.65
γ_{10} (km s^{-1})	-18.59 ± 0.79	—	-17.4 ± 1.0	-18.8 ± 1.0
γ_{11} (km s^{-1})	-17.03 ± 0.34	-17.51 ± 0.34	-18.07 ± 0.45	-18.32 ± 0.45
γ_{12} (km s^{-1})	-22.4 ± 2.5	-22.9 ± 2.7	—	—
γ_{13} (km s^{-1})	-19.3 ± 1.1	-18.9 ± 1.1	—	—
γ_{14} (km s^{-1})	-15.3 ± 1.2	-15.3 ± 1.2	—	—
γ_{15} (km s^{-1})	-18.1 ± 1.7	—	—	—
γ_{16} (km s^{-1})	-21.2 ± 1.7	-21.1 ± 1.7	—	—
γ_{17} (km s^{-1})	-19.69 ± 0.95	-20.07 ± 0.96	—	—
γ_{18} (km s^{-1})	-7.1 ± 1.6	-3.7 ± 2.1	-9.5 ± 2.5	-9.9 ± 2.8
γ_{19} (km s^{-1})	-23.9 ± 5.4	—	-21.2 ± 5.5	-26.1 ± 6.5
γ_{20} (km s^{-1})	-19.48 ± 0.53	-17.02 ± 0.56	-19.04 ± 0.65	-20.68 ± 0.61
No. of RVs	1532	819	743	687
rms (km s^{-1})	6.27	5.70	7.50	7.23

Notes: Row "rms" contains the root-mean-square error of one observation of unit weight. The numbering of individual γ velocities corresponds to data sets defined in column "No. of spg." in Table 1 (see also the remarks on individual data files in Sect. 3).

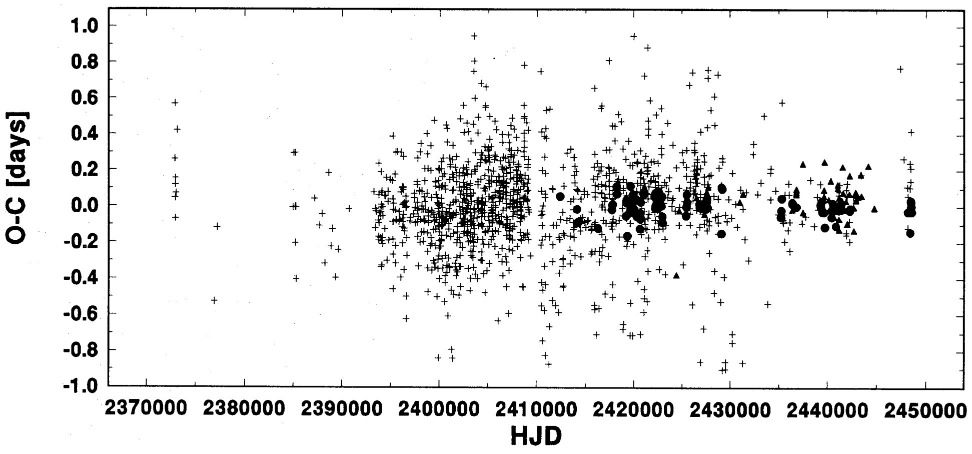

Fig. 2. The O–C deviations of the epochs of primary minima of β Lyr with respect to ephemeris (5). Visual and photographic determinations of epochs of primary minima are shown by crosses, those from photoelectric minima by triangles, and those from local radial-velocity curve fits by black dots

Table 8. Epochs of primary minima and systemic velocities of β Lyr derived from local fits to RV data. Running numbers of individual spectrograms in column "Spg. No." correspond to those of column "No. of spg." from Table 1. Column "N" gives the number of RVs defining the locally derived systemic velocities for each individual spectrograph

Epoch (HJD – 240000)	O–C (days)	γ (km s^{-1})	Spg. No.	N
12380.828 ± 0.044	0.056	–19.9 ± 2.9	1	9
14111.529 ± 0.042	–0.091	–14.8 ± 2.6	2	17
14124.521 ± 0.035	–0.017	–13.3 ± 2.1	2	19
16294.632 ± 0.069	–0.122	–9.6 ± 6.2	18	2
		–23.0 ± 4.2	19	4
17741.668 ± 0.023	–0.018	–18.9 ± 1.5	3	21
17780.452 ± 0.023	0.007	–20.2 ± 1.0	3	23
17806.295 ± 0.013	0.011	–21.0 ± 0.8	3	23
18258.580 ± 0.023	0.109	–4.3 ± 1.5	18	6
18284.380 ± 0.035	0.069	–4.4 ± 2.0	18	6
19201.582 ± 0.028	–0.054	–26.0 ± 2.1	4	13
19227.512 ± 0.029	0.035	–19.7 ± 1.9	4	26
19266.257 ± 0.051	0.019	–20.8 ± 3.4	4	9
19330.676 ± 0.062	–0.164	–28.5 ± 4.9	4	8
19563.372 ± 0.041	–0.037	–18.7 ± 2.6	4	18
19589.262 ± 0.025	0.012	–20.1 ± 1.5	4	21
19628.039 ± 0.042	0.027	–17.6 ± 2.6	4	12
19666.758 ± 0.086	–0.016	–18.3 ± 4.9	4	10
19679.806 ± 0.017	0.112	–15.1 ± 0.8	4	8
19886.449 ± 0.031	0.024	–21.0 ± 2.0	4	14
19925.254 ± 0.016	0.066	–17.4 ± 1.1	4	19
19951.069 ± 0.021	0.040	–17.5 ± 1.2	4	18
19989.780 ± 0.021	–0.012	–19.8 ± 1.5	4	20
20274.090 ± 0.038	0.038	–22.0 ± 2.4	5	9
20286.917 ± 0.030	–0.056	–17.6 ± 2.0	5	10
20351.533 ± 0.030	–0.045	–17.0 ± 1.7	5	6
20622.798 ± 0.016	–0.123	–23.6 ± 1.1	5	6
20648.694 ± 0.024	–0.069	–23.0 ± 1.8	5	8
20687.489 ± 0.023	–0.038	–18.2 ± 1.8	5	11
20713.394 ± 0.016	0.024	–18.0 ± 1.0	5	10
21036.470 ± 0.028	0.066	–18.5 ± 1.9	5	8
22160.655 ± 0.018	0.056	–18.7 ± 0.9	5	18
22199.438 ± 0.013	0.073	–19.4 ± 0.9	5	13
22225.200 ± 0.019	–0.009	–20.9 ± 1.3	5	26
22263.980 ± 0.013	0.004	–19.1 ± 0.7	5	4
22406.152 ± 0.030	0.032	–12.9 ± 1.8	5	5
22444.932 ± 0.008	0.045	–15.8 ± 0.5	5	19
22470.808 ± 0.024	0.076	–16.8 ± 1.5	5	19
22522.410 ± 0.016	–0.011	–21.7 ± 1.1	5	23
22561.182 ± 0.040	–0.006	–19.5 ± 2.6	5	13
22574.193 ± 0.015	0.083	–16.1 ± 1.2	5	9
22845.485 ± 0.031	0.003	–19.6 ± 2.3	5	13
		–22.7 ± 3.0	15	7
22884.302 ± 0.034	0.053	–19.8 ± 2.1	5	7
		–12.4 ± 3.3	15	11
22910.151 ± 0.028	0.056	–17.8 ± 1.8	5	12
22948.806 ± 0.049	–0.056	–21.6 ± 3.9	5	4
		–16.6 ± 4.1	15	8
22974.619 ± 0.059	–0.089	–28.3 ± 5.1	5	2
		–13.4 ± 2.9	15	9
25417.168 ± 0.043	–0.051	–28.4 ± 2.7	15	5
25443.063 ± 0.044	–0.004	–21.8 ± 2.3	15	6
25494.829 ± 0.067	0.066	–19.7 ± 4.1	15	5
25546.482 ± 0.072	0.022	–25.2 ± 4.0	15	6
26800.154 ± 0.061	0.015	–19.9 ± 3.9	9	6
26838.909 ± 0.017	–0.005	–17.4 ± 1.1	9	8
26877.687 ± 0.032	–0.001	–20.9 ± 2.0	9	7
26955.231 ± 0.010	–0.007	–19.8 ± 0.7	9	5
27136.202 ± 0.033	0.013	–20.2 ± 1.9	9	6
27523.933 ± 0.031	–0.012	–21.9 ± 1.7	9	12
27562.746 ± 0.038	0.025	–17.8 ± 2.1	9	14
27588.615 ± 0.031	0.043	–21.8 ± 1.8	9	20
		–24.2 ± 5.7	12	8
27614.418 ± 0.027	–0.005	–20.3 ± 2.3	9	7
		–23.2 ± 2.6	12	11
29100.744 ± 0.032	–0.150	–16.4 ± 2.1	9	11
29126.852 ± 0.030	0.106	–15.8 ± 1.7	9	9
29165.622 ± 0.022	0.097	–14.2 ± 1.4	9	4
35215.833 ± 0.015	–0.057	–17.0 ± 0.8	11	21
35267.589 ± 0.008	–0.021	–15.0 ± 0.6	11	65
35293.514 ± 0.005	0.044	–15.6 ± 0.3	11	102
36379.639 ± 0.016	0.023	–18.7 ± 1.0	10	38
		–18.0 ± 1.9	11	34
36392.562 ± 0.020	0.015	–19.2 ± 1.0	10	12
		–21.5 ± 1.5	11	74
36806.329 ± 0.021	–0.003	–19.3 ± 1.2	13	18
39289.214 ± 0.049	0.008	–25.1 ± 3.1	8	16
39328.019 ± 0.028	0.016	–20.6 ± 2.0	8	27
39353.862 ± 0.027	–0.006	–19.7 ± 1.8	8	18
39405.627 ± 0.032	0.029	–26.8 ± 2.8	8	9
		–19.6 ± 3.1	17	3
39444.366 ± 0.033	–0.029	–18.0 ± 1.6	8	9
39664.218 ± 0.023	–0.030	–23.0 ± 1.6	17	12
39741.731 ± 0.104	–0.113	–24.7 ± 6.2	17	3
40414.294 ± 0.022	–0.059	–19.8 ± 1.5	8	14
40427.270 ± 0.023	–0.016	–23.3 ± 1.7	8	15
40504.859 ± 0.017	–0.025	–19.5 ± 1.1	8	21
40517.837 ± 0.021	0.020	–16.4 ± 1.3	8	15
40556.601 ± 0.025	–0.016	–26.8 ± 1.6	8	10
40866.907 ± 0.028	–0.107	–23.4 ± 1.9	17	4
41151.537 ± 0.010	–0.012	– 4.8 ± 1.7	6	35
		–17.0 ± 0.7	7	9
		–21.1 ± 1.7	16	19
		–19.2 ± 1.3	17	20
41177.397 ± 0.014	–0.019	– 7.0 ± 2.9	6	10
		–17.0 ± 1.0	7	3
		–19.0 ± 6.8	16	4
		–18.9 ± 2.0	17	7
41384.369 ± 0.028	0.016	–17.5 ± 1.9	8	15
41410.207 ± 0.015	–0.014	–12.3 ± 0.8	8	11
42276.772 ± 0.012	–0.021	–17.8 ± 0.7	7	8
42289.715 ± 0.025	–0.012	–15.9 ± 0.8	7	11
48149.676 ± 0.024	–0.028	–18.2 ± 1.1	14	10
48395.380 ± 0.057	–0.141	–11.5 ± 3.2	14	5
48460.242 ± 0.029	0.031	–20.7 ± 3.5	14	5
		–18.5 ± 2.1	20	6
48486.111 ± 0.011	0.025	–14.5 ± 0.8	14	7
		–20.1 ± 0.7	20	23
48499.026 ± 0.014	0.002	–9.3 ± 1.8	14	5
		–19.4 ± 1.0	20	20
48511.964 ± 0.012	0.002	–9.7 ± 2.8	14	3
		–19.7 ± 0.7	20	23
48550.749 ± 0.069	–0.027	–16.9 ± 4.8	14	8

secularly variable. Thus, Blagg (1924) concluded that the light curve varies with a period of 3563 days. Later, Blagg (1928) refined the period to 3634 days and Maury (1935) found a similar period of 3646 days in the variation of the emission-line strength. Large secular variations of the Hα emission were also clearly demonstrated by Batten & Sahade (1973). Analyzing photoelectric photometry from 1958 and 1959 observing campaigns, Larsson-Leander (1970) reported a $0\overset{\mathrm{m}}{.}1$ drop in the brightness of β Lyr between 1958 and 1959. However, a new reduction of 1958 photometry by Wood (1973) led to an agreement in the 1958 and 1959 light level of β Lyr within $0\overset{\mathrm{m}}{.}02$. Wilson (1974) therefore concluded that the light curve of β Lyr had been secularly stable, with the exception of a narrow interval of phases near the primary minimum. Alduseva (1974) detected slow variations in the spectrophotometric gradient of β Lyr with a possible period of 1170 days. So far the most convincing report of long-term variability of β Lyr was presented by Guinan (1989) who observed the light curve of β Lyr systematically over several years and in several bandpasses and found a periodic variation, even at light maxima, with a period 275 ± 25 days.

Let us begin our analysis by inspection of Table 8. First, with the exception of epochs based on only a few velocities and some very old data, the O–C deviations of all epochs from RVs are within 0.05 days. This is, however, a value which is nearly comparable to the true uncertainty of their determination. When we tried to derive the local epochs for the same subset but for velocities based on different ions (Mg II, Si II, Fe II or Ca II) we found scatter amounting up to 0.04–0.10 d for the same epoch. It is obvious that – depending on the actual phase distribution of the observations – the observational errors can affect the epoch, the systemic velocity and the semiamplitude of the RV curve and it is not always easy to separate their effect on each of these elements. We therefore conclude that the scatter observed is close to what one expects and does not indicate any real long-term changes. Yet, we formally ran period analyses of all above-mentioned residuals for periods between 100 and 10000 days. There were some hints of periods close to 145–146 and 550 days (which are – within the limit of errors – a submultiple and first overtone of the period of 275 days, found by Guinan 1989). However, no really consistent periodicity could be found. We therefore conclude that there is no compelling evidence of either periodic or larger aperiodic non-orbital variations in RVs of star 2 or in observed epochs of primary minima. This, in turn, may indicate that the real long-term changes in the brightness of the binary and in the strength of the emission lines are related to the variations of the circumstellar matter and perhaps of star 1, but not star 2.

In passing, we wish to call attention to a good indicator of real long-term changes: RV variations of the strong narrow He I 3888 absorption. This line is clearly arising from some component of the circumstellar matter. It is always blue-shifted and exhibits only small-amplitude double-wave orbital RV variations. However, the mean velocity and the amplitude of the phase-locked variations itself both vary with time. This is illustrated by Fig. 3, which shows that the velocity was definitely less negative in the old spectra. Inspection of data published by various authors reveals secular RV variations over a range from about -60 to $-170\,\mathrm{km\,s^{-1}}$. Sahade et al. (1959) even observed three distinct components at about -170, -130, and $-60\,\mathrm{km\,s^{-1}}$. It seems obvious that the He I 3888 line is very sensitive to secular variations in the circumstellar envelope of β Lyr and should be monitored systematically.

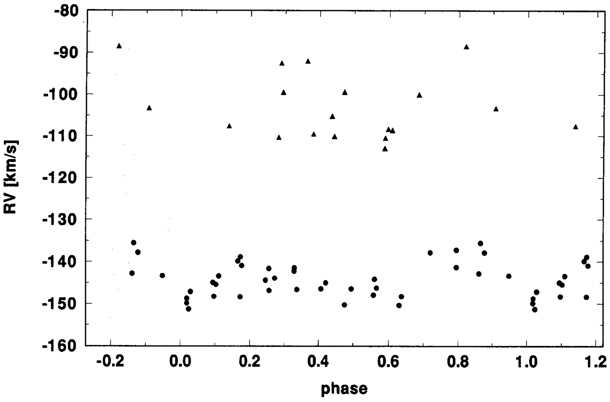

Fig. 3. Radial-velocity curve of the He I 3888 line. Triangles denote the velocities from the Potsdam 1900–1909 spectra, dots the velocities from Ondřejov 1991 spectrograms

5.2. *Variable systemic velocity of β Lyr?*

Rossiter investigated but rejected the possibility of a slow change of the systemic velocity of β Lyr . Also Abt (1962) concluded that no real γ-velocity variations of β Lyr were observed. Later, Skulskij (1969, 1970, 1971) maintained that the γ velocity of β Lyr does vary with a period of 4.2 yr (1530 days). However, Batten & Fletcher (1975) were unable to confirm such a change in their velocity data and Klimek & Kreiner (1975) did not find any 4.2-year variation in the O–C deviations of the observed times of photoelectrically observed minima of β Lyr .

We feel that the problem of possible systemic-velocity changes is a bit more tricky. What Skulskij did was that he collected the γ velocities published by various authors without investigating their homogeneity. This is a risky procedure, for several reasons. Thus, for instance, the wavelengths available to Bélopolsky were only accurate to 0.1 Å, i.e. to some 5–7 $\mathrm{km\,s^{-1}}$. Some of the more recent investigations are based on Si II velocities only but always on Moore's (1945) wavelengths for these lines. (This is what we also accepted to be consistent with older data.) However, the Moore's wavelengths for the Si II lines need a small revision which, for example, for the most often used $\lambda\lambda$ 4128 and 4130 Å lines amounts to -1.02, and

-0.65 km s^{-1}, respectively. Further, the differences in the zero-point of the velocity scale between two different spectrographs of the order of a few km s^{-1} are not uncommon.

We therefore begun our analysis by calculating separate orbital solutions for each spectrograph and each measured ion separately, keeping the period and its rate of change fixed at their corresponding values calulated from (2) to (4) and adopting the ephemeris (5). This exercise showed several important things. First, the ion-to-ion differences in the semiamplitude of the velocity curve are obviously quite accidental, not systematic, and we attribute them to measuring errors, possibly combined with effects of accidental line blending which may vary with the dispersion of the spectrograms in question and with the orbital phase. Second, the ion-to-ion systemic velocity differences are also quite accidental from one spectrograph to another one and can amount up to 7 km s^{-1}. This is comparable to the full amplitude of the 4.2-yr γ-velocity variation advocated by Skulskij. Note also that while we confirm a large difference in the systemic velocity between Skulskij's (1971) and (1973) spectrograph 8 velocities for Si II and Ca II, the corresponding values for Fe II differ for only 1 km s^{-1} . Another hint of a real accuracy of the systemic-velocity determination from older data is provided by the comparison of Curtiss' (1911) RVs of Allegheny plates with their re-measurement by Baxandall & Stratton (1930): using an exactly identical data set, we obtained the systemic velocity of -20.2 ± 0.7 km s^{-1} from original Curtiss' RVs but -15.7 ± 1.5 from Baxandall and Stratton's re-measurement. There is also a very puzzling difference between spectrographs 6 and 16, i.e. the blue and red OHP spectra. The systemic velocities of all ions from the blue spectra are for some 10 km s^{-1} more positive than those from the red ones, the latter roughly corresponding to values obtained from most of other spectrographs. The difference has already been noted by Flora & Hack (1975). As it amounts approximately to the value of the heliocentric correction, we made an inquiry to Dr. Hack about the problem. She ensured us that the correction was properly applied to all of their data. She, too, has no clear explanation for the difference. The only thing we noted is that quite a number of velocities from spectrograph 6 are affected by the Rossiter effect around the primary minimum but this does not seem to be the complete explanation for the peculiar γ velocity observed. In fact, we encountered a similar problem with the Potsdam spectrograph 18 velocities – in spite of the encouraging results obtained on constant-velocity stars [cf. Sect. 3 (viii) and Table 6] and have no clear explanation either. The relatively low prismatic dispersion of spectrograph 18 cannot serve as an explanation since Table 7 shows that our reconstruction of the velocity zero-point of Pulkowa spectrograph 1 (having yet smaller prismatic dispersion) led to the systemic velocity for the Fe II lines (the only lines which were transformed accurately enough to the present-day wavelength scale), which compares nicely with that from the most stable instruments. Whatever is the cause of these two peculiar cases, however, we cannot consider them a proof of real variations in the systemic velocity of the binary since fortunately in both cases there are observations from other instruments leading to obviously normal values.

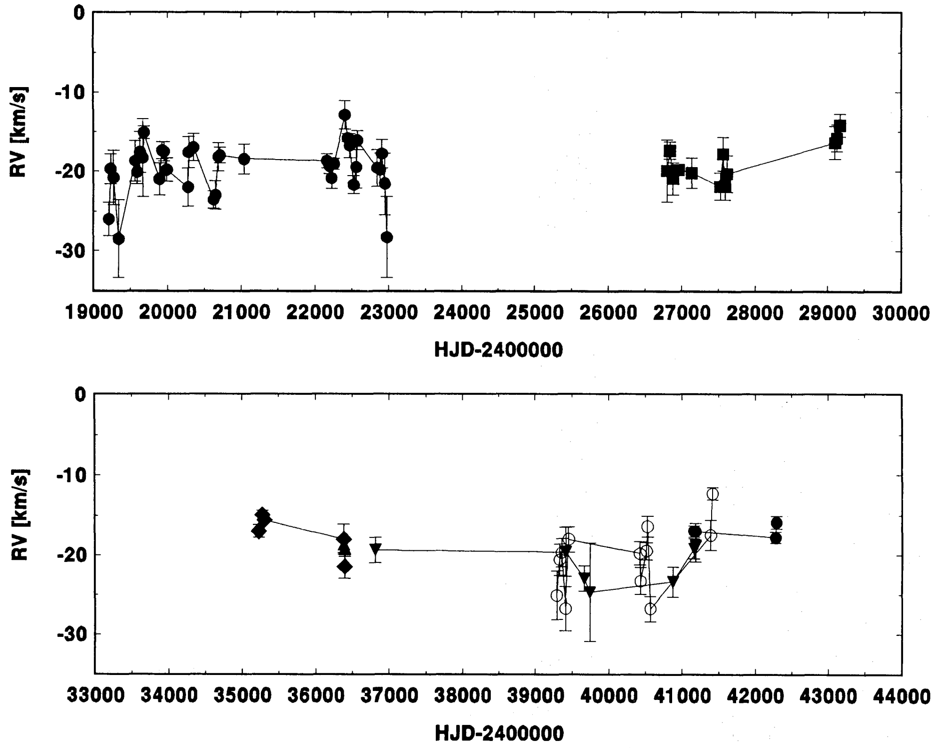

Fig. 4. A plot of locally derived γ-velocities of Table 8 versus time for those spectrograms yielding larger amounts of data. Each spectrograph is denoted by a different symbol and the data from individual spectrographs are connected by lines. The error bars show the rms errors of the γ-velocity determinations

We also conclude on the basis of our results from RV curves of individual ions that forming plate mean metallic velocities from Mg II, Si II, Fe II and Ca II velocities is justified not only for the purpose of deriving a new ephemeris but also to increase the accuracy of plate velocities before an attempt to search for possible real γ-velocity changes.

In our own search for the systemic-velocity variation we therefore used the mean metallic-line velocities (or even mean plate velocities if nothing else was published). In practice, we employed the subsets of velocities observed within no more than 35 days i.e. the data summarized in Table 8.

Figure 4 shows the γ-velocity variations versus time for several different spectrograms (each instrument is denoted by a different symbol and corresponding local determinations are connected by a line). Inspection of Fig. 4 and Table 8 indicates that there are possible real variations of the systemic velocity which exceed the corresponding formal errors of their determination. A period search indicated possible periods of about 1180, 560 and 145 days, but none of these "periods" could be established convincingly. Our conclusion is that possible slight variations of the formally determined systemic velocity of β Lyr may arise from blending effects of the variable circumstellar environment but we have no reason to postulate a real third body in the system of β Lyr A. Nevertheless, we feel that further monitoring of possible slight systemic-velocity variations would be desirable.

5.3. *Cycle-to-cycle variations*

Cycle-to-cycle variations in the amplitude of the Rossiter effect were reported by Struve (1958) and we can only state that our new ephemeris cannot alter his conclusion for the data set he used. We feel, however, that a more detailed study of the effect would require a coordinated multi-longitudinal observing campaign. The trouble is that – given the orbital period of 12.9 days – an observer at given location repeatedly observes only specific intervals of orbital phases during each observing season. This "patchiness" of the RV curve is especially unpleasant around the conjunctions and in extreme cases it can lead to incorrect, or at least incomplete conclusions about the character of the variations observed.

6. Is there a direct evidence of mass exchange between the binary components?

After discussing possible non-orbital effects and verifying that there are no systematic ion-to-ion differences, we can use the whole body of the velocity data from heterogeneous sources to the search for a possible long-term change of the semiamplitude of the velocity curve of star 2 due to continuing mass transfer towards star 1 as discussed in Sect. 2 (iv).

Let us first consider the expected magnitude of the net effect for two extreme cases of conservative and fully non-conservative mass transfer. Neglecting the stellar rotational angular momenta, the orbital angular momentum of the binary is

$$J=2\pi P^{-1}M_1M_2(M_1+M_2)^{-1}A^2, \tag{6}$$

where M_1, M_2 are component masses, A is the separation of their centres (circular orbit is assumed) and P is the orbital period.

Using the third Kepler law in the form

$$A^3=(2\pi)^{-2}P^2G(M_1+M_2), \tag{7}$$

one can re-write (6) as

$$J^3(M_1+M_2)=C_1PM_1^3M_2^3, \tag{8}$$

where $C_1=(2\pi)^{-1}\ G^2$ is a constant.

Calculating the time derivative of (8), one obtains

$$3(M_1+M_2)\,J^2\dot J+J^3(\dot M_1+\dot M_2)$$
$$=C_1M_1^2M_2^2[M_1M_2\dot P+3P(M_1\dot M_2+M_2\dot M_1)] \tag{9}$$

for the general case of variable masses and orbital angular momentum.

For the case of conservative mass transfer (no mass or angular momentum are lost from the system) $\dot J=0$ and $\dot M_1=-\dot M_2$ and formula (9) reduces to

$$\dot PP^{-1}=3(M_1-M_2)(M_1M_2)^{-1}\dot M_1, \tag{10}$$

while for a mass loss from star 2 via spherically symmetric stellar wind with no accretion by star 1 $\dot M_1=0$, $\dot J=2\pi P^{-1}a_2^2\dot M_2=2\pi P^{-1}\dot M_2M_1{}^2A^2(M_1+M_2)^{-2}$ (a_2 is the distance of star 2 from the centre of gravity of the binary system) and

$$\dot PP^{-1}=-2\dot M_2(M_1+M_2)^{-1} \tag{11}$$

(cf. e.g. Pringle 1985).

The variation of the semi-amplitude K_2 follows from the time derivative of the well-known formula

$$M_1^3\sin^3 i=C_2K_2^3P(M_1+M_2)^2, \tag{12}$$

where the constant $C_2=(2\pi G)^{-1}$:

$$3PM_1(M_1+M_2)K_2^{-1}\dot K_2=(M_1+M_2)(3P\dot M_1-M_1\dot P)$$
$$-2PM_1(\dot M_1+\dot M_2). \tag{13}$$

(The term containing $\sin^3 i$ was eliminated with the help of Eq. 12).

For conservative mass transfer one gets

$$\dot K_2=\dot P(2M_2-M_1)\ K_2/[3P(M_1-M_2)], \tag{14}$$

while for the fully non-conservative spherical wind case

$$\dot K_2\equiv 0. \tag{15}$$

The latter is a somewhat unpleasant finding since the absence of observable change in K_2 can only place certain limits on an essentially conservative mass transfer. However, Mazzali et al. (1992) found from their recent analysis of UV line profiles of β Lyr that the rate of *mass loss* via the stellar wind from star 2 is only $7\,10^{-7} m_\odot\,\mathrm{yr}^{-1}$. On the other hand, formula (11) implies $\dot{M}_2 = -1.2\,10^{-4} m_\odot\,\mathrm{yr}^{-1}$ for the component masses adopted by Mazzali et al. ($M_1 = 12 m_\odot$, $M_2 = 2 m_\odot$) and the observed $\dot{P}/P = 1.7\,10^{-5}\,\mathrm{yr}^{-1}$, assuming all material lost by star 2 leaves the system through the stellar wind. This seems to indicate that the mass transfer is essentially conservative, i.e., star 1 should accrete inflowing material at a rate of the order of $10^{-5} m_\odot\,\mathrm{yr}^{-1}$.

For conservative mass transfer, formula (13) predicts $\dot{K}_2 = -0.00083\ \mathrm{km\,s^{-1}\,yr^{-1}}$ for the masses adopted by Mazzali et al., $\dot{K}_2 = -0.00061\ \mathrm{km\,s^{-1}\,yr^{-1}}$ for the masses derived by Harmanec (1990) ($M_1 = 14.1 m_\odot$, $M_2 = 4.25 m_\odot$), and $\dot{K}_2 = -0.00075\ \mathrm{km\,s^{-1}\,yr^{-1}}$ for the masses derived by Skulskij (1992) ($M_1 = 13.2 m_\odot$, $M_2 = 2.9 m_\odot$). In other words, the semiamplitude K_2 should have decreased by 0.06–0.08 $\mathrm{km\,s^{-1}}$ since Bélopolsky's observations in 1892 until the present time, *if* there is a steady and basically conservative mass transfer from star 2 towards star 1 in β Lyr. While this is certainly a very small change to be detected observationally, the good agreement between various determinations of K_2 from individual data sets justifies an attempt to estimate $\dot{K}_2$ from the whole body of RV data.

To this end we calculated another family of orbital solutions based on mean as well as ionic velocities from all spectrographs, for which we kept the period and the rate of its change fixed at the values from solution 1 and allowed for a free convergence of the epoch, semi-amplitude of the velocity curve and its linear change with time, $\dot{K}_2$. The results of these calculations indicated that the semiamplitude and its rate of change are highly correlated (correlation coefficients being higher than 0.95 in all considered cases). Consequently, we also fixed the value of semiamplitude from respective solutions (see Table 7) and calculated a new family of solutions. These are summarized in Table 9. It is seen that we have indeed obtained the values of the rate of the semiamplitude change with time which agree surprisingly well with the predicted ones. Unfortunately, this result is not conclusive since the associated errors are in all cases larger than the value obtained. Nevertheless, it seems encouraging that the result did not change even when we started the convergence with a positive value of $\dot{K}_2$. We therefore conclude that the available RV observations *do not contradict* the hypothesis that β Lyr is indeed in phase of an essentially conservative mass transfer from star 2 towards star 1. Our result also indicates that continuing accurate systematic RV observations of β Lyr are very desirable since they could permit a much more stringent test of the mass transfer theory after no more than some 30 years.

7. Improved physical parameters of β Lyr

Using high-dispersion high-S/N CCD spectra, Skulskij & Topilskaya (1991) and Skulskij (1992) recently confirmed the earlier discovery (Skulskij 1975; cf. also Sahade 1966) of a weaker pair of the Si II 2 lines, obviously originating in material which moves in orbit with star 1. Harmanec (1992) identified these lines with the shell lines from the disk around star 1 and supported the view that they define the orbital motion of the still mysterious star 1 quite well.

Thanks to these findings and to the analyses presented above, we are now in the position to derive the individual masses of β Lyr with an unprecedented accuracy. To this end, we calculated yet another family of orbital solutions, in which we only used star 2 velocities based on spectrograms with dispersion 10 Å mm^{-1} or better and star 1 mean Si II 2 velocities from Skulskij (1975, 1992). Once more, we kept the period and its rate of change fixed at their values obtained from solution (1) and formulae (2) to (4) for the considered epoch. We also treated the photographic and CCD star 1 velocities as data from different spectrographs (Nos. 21 and 22). The results can be found in

Table 9. Orbital solutions for velocities of star 2 of β Lyr with the secular change of the semi-amplitude of the RV curve as a free parameter

Element	Mean RVs	Si II	Mg II	Fe II
P (days)	12.913780 fixed	12.913780 fixed	12.913780 fixed	12.913780 fixed
$\mathrm{d}P/\mathrm{d}t$	$5.9966\,10^{-7}$ fixed	$5.9966\,10^{-7}$ fixed	$5.9966\,10^{-7}$ fixed	$5.9966\,10^{-7}$ fixed
$T_{\mathrm{prim.ecl.}}$	8247.966±0.022	8247.953±0.034	8247.962±0.037	8247.973±0.027
K_2 (km s^{-1})	185.70 fixed	186.13 fixed	185.93 fixed	186.25 fixed
$\mathrm{d}K_2/\mathrm{d}t$ [km s^{-1} (100 yr)$^{-1}$]	−0.05±0.34	−0.12±0.37	−0.04±0.53	−0.20±0.56
No. of RVs	1532	819	743	687
rms (km s^{-1})	6.27	5.74	7.50	7.23

Note: Row “rms” contains the root-mean-square error of one observation of unit weight.

Table 10. Orbital solutions for combined velocities of stars 1 and 2 of β Lyr

Element	Solution 5 Mean RVs	Solution 6 Si II	Solution 7 Mg II	Solution 8 Fe II
P [days]	12.933504 fixed	12.933504 fixed	12.933504 fixed	12.933504 fixed
$\mathrm{d}P/\mathrm{d}t$	$5.9875\,10^{-7}$ fixed	$5.9875\,10^{-7}$ fixed	$5.9875\,10^{-7}$ fixed	$5.9875\,10^{-7}$ fixed
$T_{\mathrm{prim.ecl.}}$	41164.4823 ± 0.0038	41164.4671 ± 0.0038	41164.4808 ± 0.0054	41164.4910 ± 0.0057
K_1 (km s^{-1})	41.4 ± 1.3	41.4 ± 1.3	41.4 ± 1.1	41.4 ± 1.3
K_2 (km s^{-1})	185.90 ± 0.33	186.30 ± 0.35	186.07 ± 0.50	186.40 ± 0.52
γ_6 (km s^{-1})	-5.0 ± 1.7	-4.4 ± 1.7	-7.2 ± 1.8	-5.9 ± 2.7
γ_7 (km s^{-1})	-16.35 ± 0.52	-16.51 ± 0.52	—	—
γ_{10} (km s^{-1})	-18.57 ± 0.79	—	-17.46 ± 0.99	-18.8 ± 1.0
γ_{11} (km s^{-1})	-16.99 ± 0.34	-17.66 ± 0.34	-18.12 ± 0.45	-18.33 ± 0.45
γ_{13} (km s^{-1})	-19.3 ± 1.1	-19.0 ± 1.1	—	—
γ_{14} (km s^{-1})	-15.3 ± 1.2	-15.3 ± 1.1	—	—
γ_{17} (km s^{-1})	-19.72 ± 0.95	-20.02 ± 0.96	—	—
γ_{20} (km s^{-1})	-19.46 ± 0.54	-16.91 ± 0.62	-19.02 ± 0.65	-20.67 ± 0.62
γ_{21} (km s^{-1})	-17.7 ± 2.7	-17.8 ± 2.7	-17.7 ± 2.7	-17.6 ± 2.7
γ_{22} (km s^{-1})	-19.5 ± 1.1	-19.6 ± 1.1	-19.5 ± 1.1	-19.5 ± 1.1
$M_1 \sin^3 i$ ($M_\odot$)	12.87	12.95	12.91	12.97
$M_2 \sin^3 i$ ($M_\odot$)	2.87	2.88	2.87	2.88
$A \sin i$ ($R_\odot$)	58.08	58.19	58.13	58.21
No. of RVs	612	562	474	464
rms (km s^{-1})	5.18	5.31	6.80	7.04

Notes: Row "rms" contains the root-mean-square error of one observation of unit weight. Systemic velocities γ_{21} and γ_{22} refer to the Si II 2 absorption-line velocities of star 1 from the Crimean photographic and CCD spectrograms, respectively, while all other systemic velocities refer to star 2 and their numbers correspond to numbers of data sets defined in column "No. of spg." in Table 1.

Table 10 and the corresponding RV curve for the mean metallic plate velocities of star 2 and Si II 2 velocities of star 1 is shown in Fig. 5.

Our determination of the component masses and the mass ratio reads as follows:

$$M_1 \sin^3 i = 12.94 \pm 0.05 M_\odot, \quad M_2 \sin^3 i = 2.88 \pm 0.10 M_\odot,$$

$$M_2/M_1 = 0.223.$$

Any substantial progress in the accurate knowledge of the masses and all other basic physical elements of β Lyr must now come from a more sophisticated analysis of the light curves over a large range of wavelengths to decrease the remaining uncertainty in the accurate value of the orbital inclination i. Note, however, that for inclinations between 80° and 90° (the true inclination lies in all probability within these limits), the projected rotational velocity of star 2, calculated on the assumption that this star is just filling its Roche lobe, would be invariably 56.9 km s^{-1} which nicely agrees with the observed value of 55 ± 10 km s^{-1}. It thus appears that – after one hundred years of concentrated effort of many investigators – we are quite close to consistent physical elements of β Lyr A.

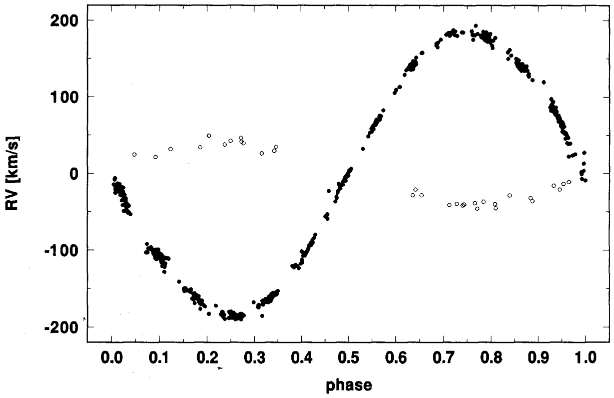

Fig. 5. Radial-velocity curves of both binary components of β Lyrae based on high-dispersion spectrograms only (see solution 5 of Table 10)

The situation is certainly less satisfactory as far as the dynamical evolution of the system is concerned. Note that for the same range of orbital inclinations as considered above, the rate of mass transfer calculated on the assumption of a fully conservative mass transfer from formula (10) would be between $2.09\,10^{-5}$ and $2.19\,10^{-5} m_\odot\,\mathrm{yr}^{-1}$. This is five times smaller than the rate of *mass accretion* by star 1 required in Hubeny & Plavec's (1991) model of the disk around star 1 treated as an accretion disk. Considering the principal success of Hubeny and Plavec's model in describing the observed continuum radiation from the disk, one must ask again how much mass and angular momentum is being lost from the whole system. It is clear that – besides the mass loss via stellar wind – an outflow from the Lagrangian point L_2 (as suggested by Kuiper 1941) or even via jets (as speculated by Harmanec 1992) may be taking place in β Lyr. Clearly, a future refinement of the test attempted in Sect. 6 as well as any other research aimed at a more accurate determination of the rate of mass transfer and mass loss from the system would be very desirable.

Acknowledgements. With this work, we wish to pay our tribute to three people, who started the story of β Lyr and kept it running: John Goodricke, Aristarch Bélopolsky and Otto Struve.

We are deeply obliged to Drs. Margherita Hack, N. Samus, John R. Percy, Gordon A.H. Walker and Derek Richardson who kindly provided us with copies of several articles dealing with β Lyr and published in journals not readily available to us. The use of the bibliography provided by the Centre for Astronomical Data at Strasbourg is also gratefully acknowledged.

This study would not be possible without the computer code FOTEL kindly put at our disposal by Dr. Petr Hadrava. We are very grateful to him and can only hope that Petr finds our continuing stream of annoying suggestions how to improve the program equally rewarding – especially now that the program is ready and running so satisfactorily! Similar thanks also belong to Dr. J. Horn who very kindly put his user-friedly reduction software SPEFO at our disposal.

Our thanks are also due to Drs. P. Hadrava, J. Horn, K. Juza, P. Koubský, P. Škoda and S. Štefl who on our request obtained some of the new Ondřejov spectrograms used in this study.

We are obliged to the referee, Dr. A.H. Batten, for his careful reading of the manuscript and constructive criticism and to Drs. P. Hadrava and I. Hubeny for their helpful comments.

Finally, a patient and careful proofreading of an earlier version of the paper by Dr. J.M. Matthews is gratefully acknowledged.

References

Abt H.A., 1962, ApJ 135, 424
Abt H.A., Levy S.G., 1976, AJ 81, 659
Abt H.A., Jeffers H.M., Gibson J., Sandage A.R., 1962, ApJ 135, 429
Alduseva V.Y., 1969, AZh 46, 363
Alduseva V.Y., 1974, AZh 51, 403
Aslan Z., Derman E., Engin S., Yilmaz N., 1987, A&AS 71, 597
Bahýl' V., Kreiner J.M., 1981, Bull. Astron. Inst. Czechosl. 32, 52
Bahýl' V., Pikler J., Kreiner J.M., 1979, Acta Astron. 29, 393
Batten A.H., Sahade J., 1973, PASP 85, 599
Batten A.H., Fletcher J.M., 1975, PASP 87, 237
Baxandall F.E., Stratton F.J.M., 1930, Ann. Solar Phys. Obs. Cambridge 2 (Part 1), 1
Bélopolsky A., 1893a, Bull. Acad. Impér. Sciences St.-Pétersbourg 7, 423
Bélopolsky A., 1893b, Mem. Soc. Spett. Ital. 22, 101
Bélopolsky A., 1894, Mem. Soc. Spett. Ital. 23, 80
Bélopolsky A., 1897a, ApJ 6, 328
Bélopolsky A., 1897b, Mem. Soc. Spett. Ital. 26, 135
Blagg M.A., 1924, MNRAS 84, 629
Blagg M.A., 1925, MNRAS 85, 484
Blagg M.A., 1928, MNRAS 88, 162
Curtiss R.H., 1911, Publ. Allegheny Obs.2, 73
Flora U., Hack M., 1975, A&AS 19, 57
Guinan E.F., 1989, Space Sci. Rev. 50, 35
Guthnick P., 1945, IAU Circ. 1002
Hadrava P., 1990, Contr. Astron. Obs. Skalnaté Pleso 20, 23 and pers.com.
Harmanec P., 1990, A&A 237, 91 (Paper 1)
Harmanec P., 1992, A&A 266, 307 (Paper 2)
Heard J.F., 1959, J. R. Astron. Soc. Can. 53, 188
Hill G., 1993, PASP (in press)
Hubeny I., Plavec M.J., 1991, AJ 102, 1156
Hübscher J., Agerer F., Wunder E., 1991, BAV Mitt. No. 59
Hübscher J., Agerer F., Wunder E., 1992, BAV Mitt. No. 60
Klimek Z., Kreiner J.M., 1973, Acta Astron. 23, 331
Klimek Z., Kreiner J.M., 1975, Acta Astron. 25, 29
Korytnikov S.N., Lavrov M.I., Martynov D.J., 1963, Bibliography of Spectroscopic binaries (IAU Com. 5 and 42), Vol. 4, Astronomical Council, Soviet Academy of Sciences, Moscow,
Kuiper G.P., 1941, ApJ 93, 133
Larsson-Leander G., 1970, Vistas in Astronomy 12, 183
Mandushev G.I., 1988, Astron. Circ. No. 1533
Maury A., 1935, Harvard Col. Astron. Obs. Bull. 899, 25
Mazzali P.A., Pauldrach A.W.A., Puls J., Plavec M.J., 1992, A&A 254, 241
Merrill P.W., Burwell C.G., 1933, ApJ 78, 87
Merrill P.W., Burwell C.G., 1943, ApJ 98, 153
Merrill P.W., Burwell C.G., 1949, ApJ 110, 387
Milgrom M., 1978, A&A 70, 763
Moore C.E., 1945, Contr. Princeton Univ. Obs. No. 20
Paczyński B., 1971, ARA&A 9, 183
Pringle J.E., 1985, in: Pringle J.E. Wade R.A. (eds.) Interacting Binary Stars. Cambridge Univ. Press, London, p. 1
Rossiter R.A., 1933, Publ. Obs. Univ. Michigan 5, 67
Sahade J., 1966, Trans. IAU, Vol. XIIB, 494
Sahade J., 1980, Space Sci. Rev. 26, 349
Sahade J., Huang S.-S., Struve O., Zebergs V., 1959, Trans. Amer. Philosoph. Soc.(New Ser.) 49, Part 1, 1
Skulskij M.Yu., 1969, Astron. Circ. No. 505, 3
Skulskij M.Yu., 1970, Astron. Circ. No. 587, 1
Skulskij M.Yu., 1971, AZh 48, 766 ≡ 1972 SvA 15, 606
Skulskij M.Yu., 1972, Astron. Circ. No. 707, 3
Skulskij M.Yu., 1973, Perem. Zvezdy 18, 609
Skulskij M.Yu. 1975, AZh 52, 710
Skulskij M.Yu. 1992, Pisma AZh 18, 711
Skulskij M.Yu., Topilskaya G.P., 1991, Pisma AZh 17, 619
Struve O., 1941, ApJ 93, 104
Struve O., 1958, PASP 70, 585
Struve O., Sahade J., 1958, PASP 70, 313
Tikhoff G., 1897, Mem. Soc. Spett. Ital. 26, 107
Wilson R.E., 1974, ApJ 189, 319
Wood D.B., 1973, ApJ 186, 615