The Schwarzschild model and the virial theorem

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Outline

1) The Schwarzschild Model:
   A favorite for modeling galaxies.

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2) Virial equilibrium or not?
   The virial theorem and its consequences.

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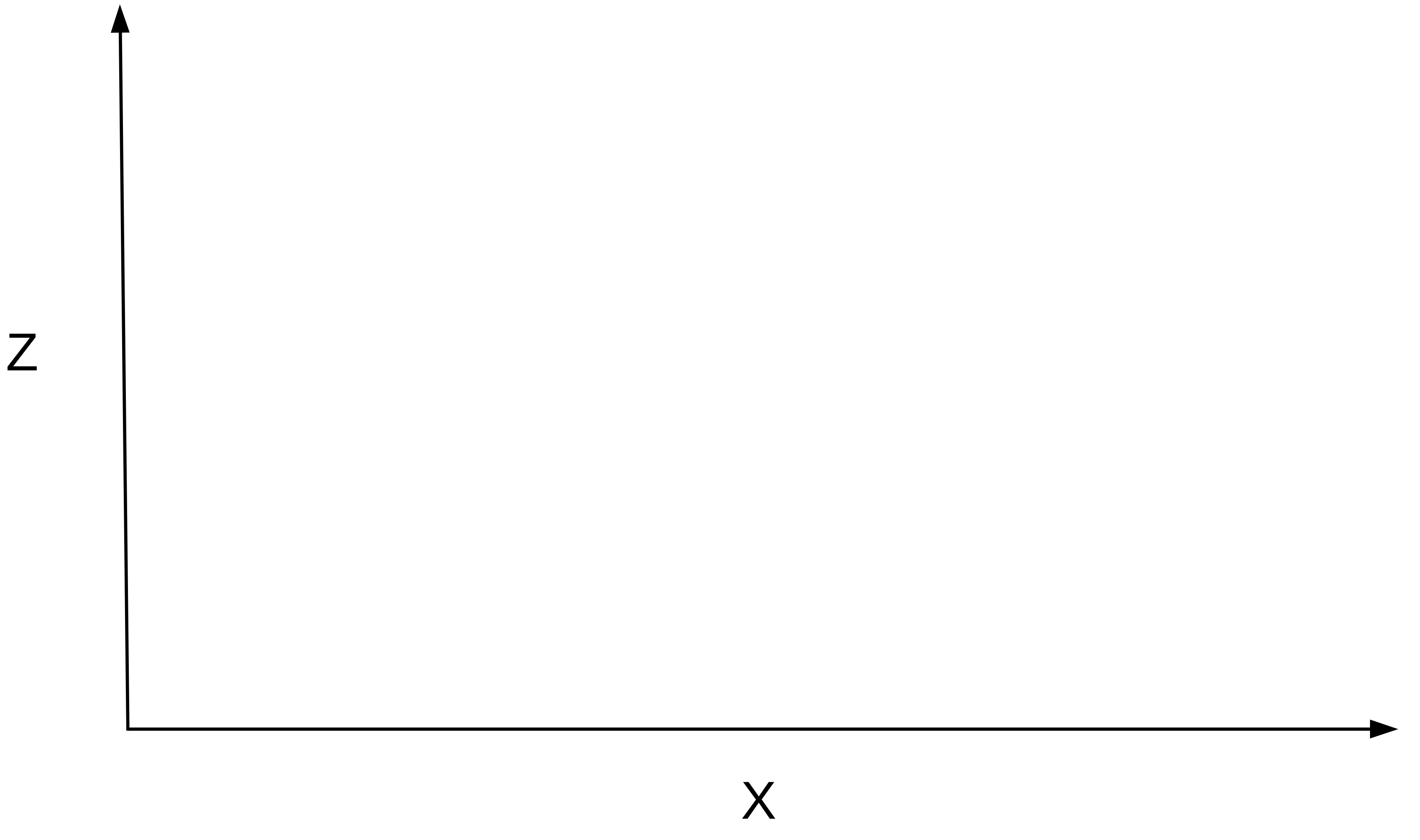
2) Virial equilibrium or not?
   The virial theorem and its consequences.

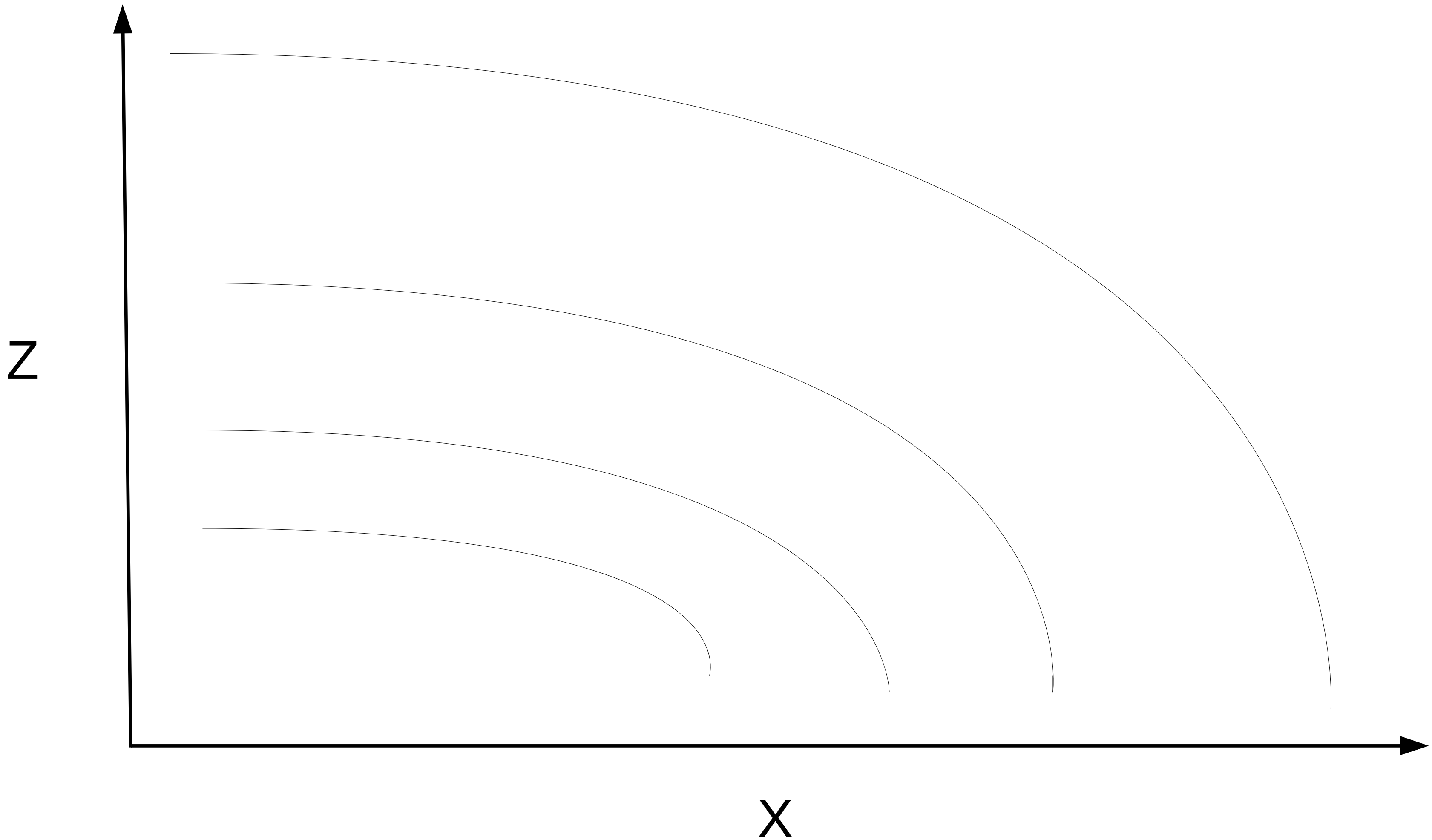
3) Virializing the Schwarzschild Model.
   Application to the Galactic center.

The Schwarzschild Model



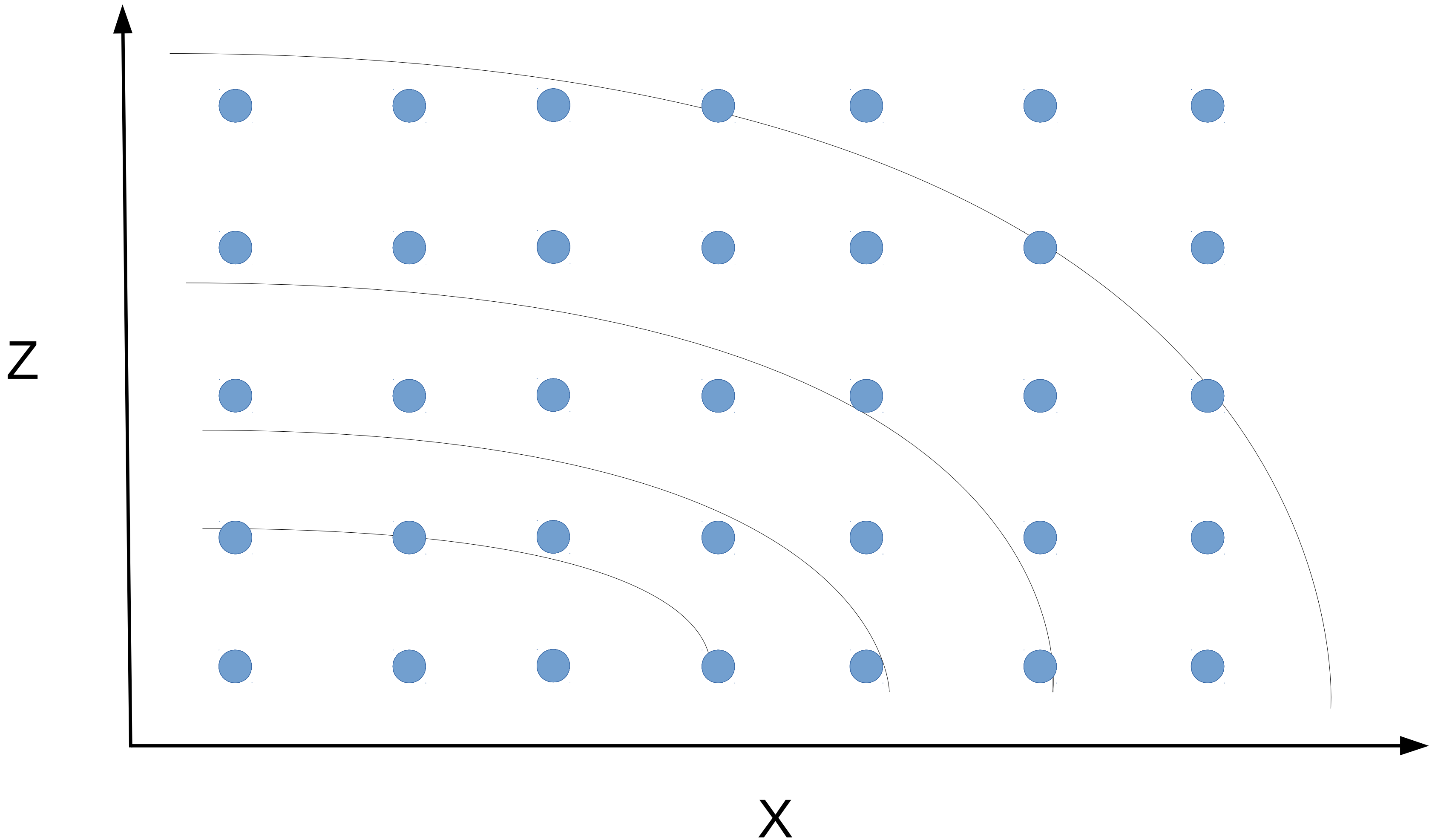
Set coordinate system

The Schwarzschild Model



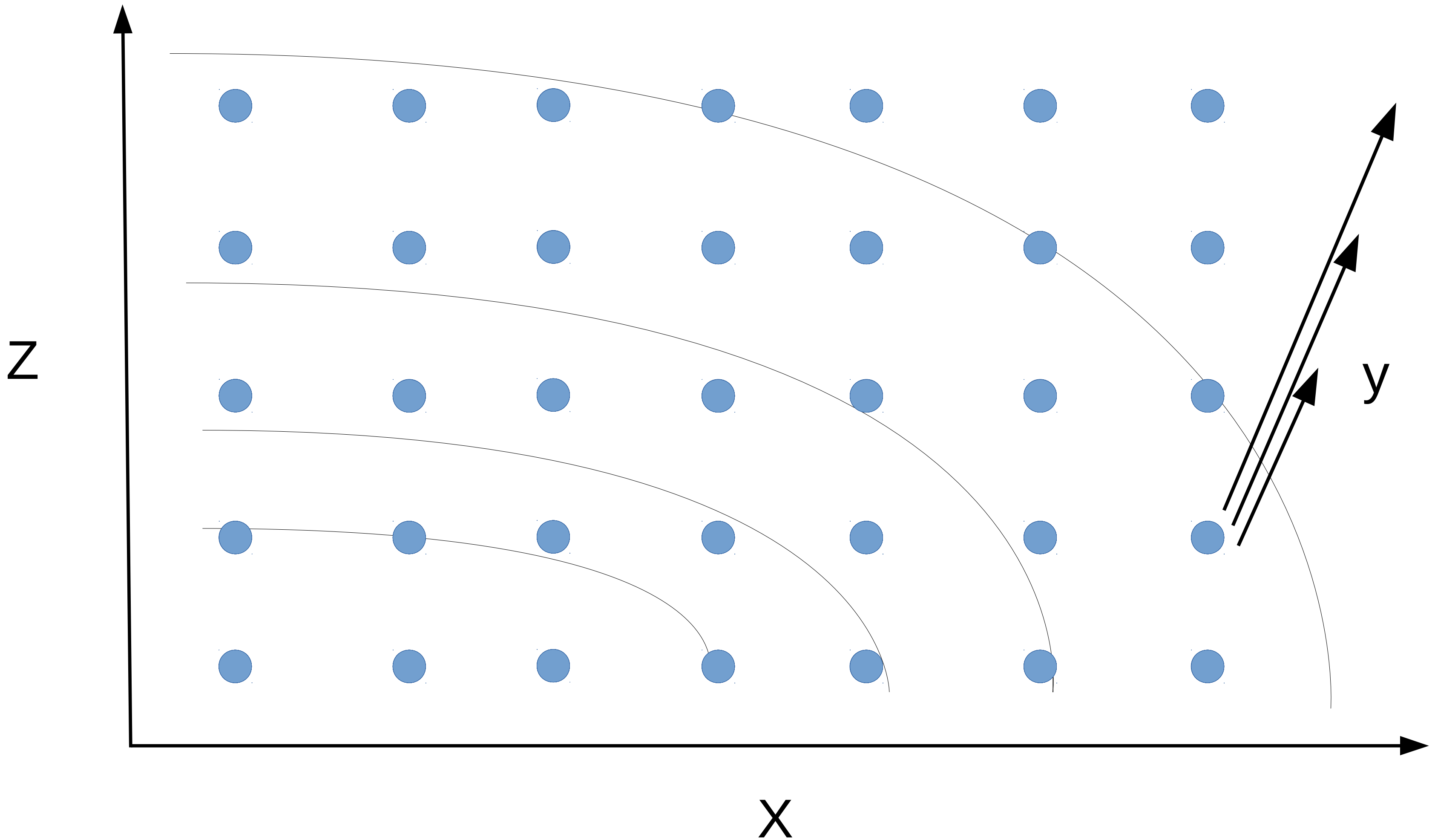
Define potential

The Schwarzschild Model



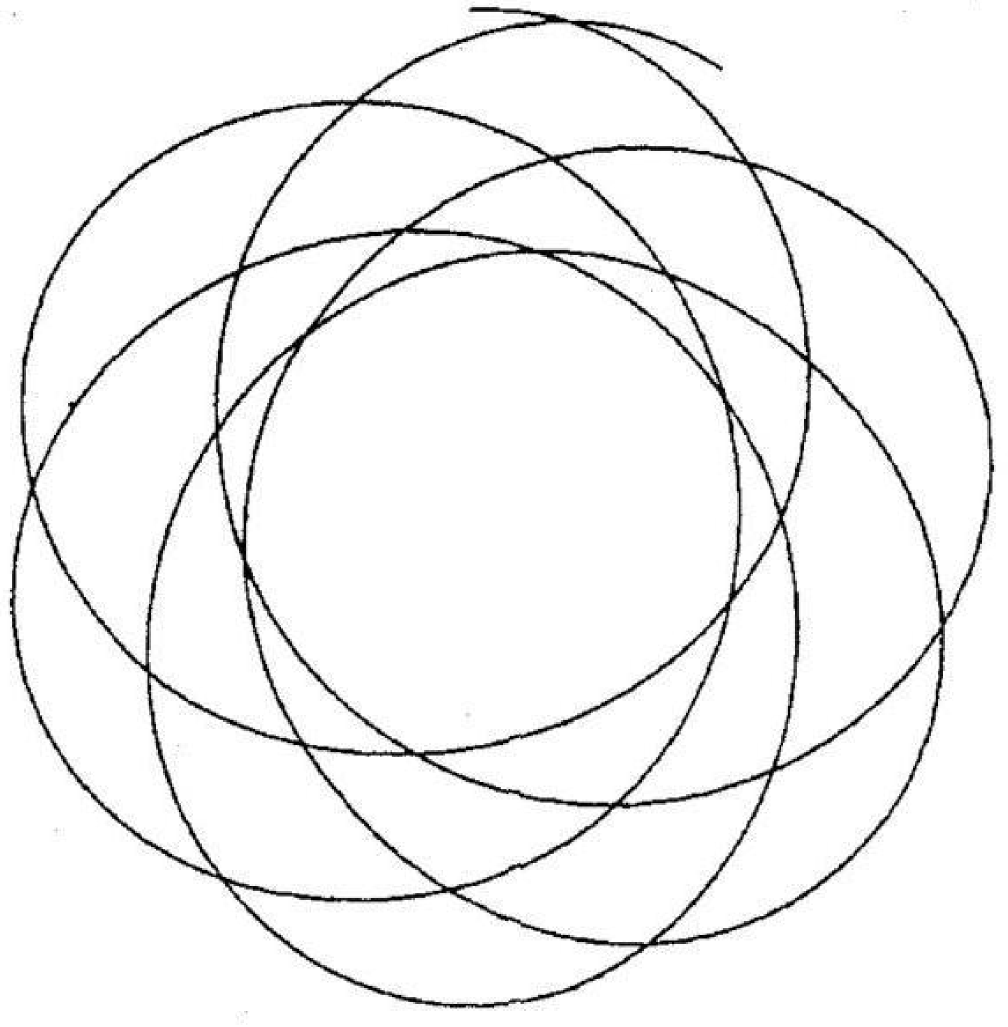
Define starting positions of massless particles

The Schwarzschild Model



Set initial velocities of particles

The Schwarzschild Model



For each set of initial conditions, a characteristic orbit is the Consequence.

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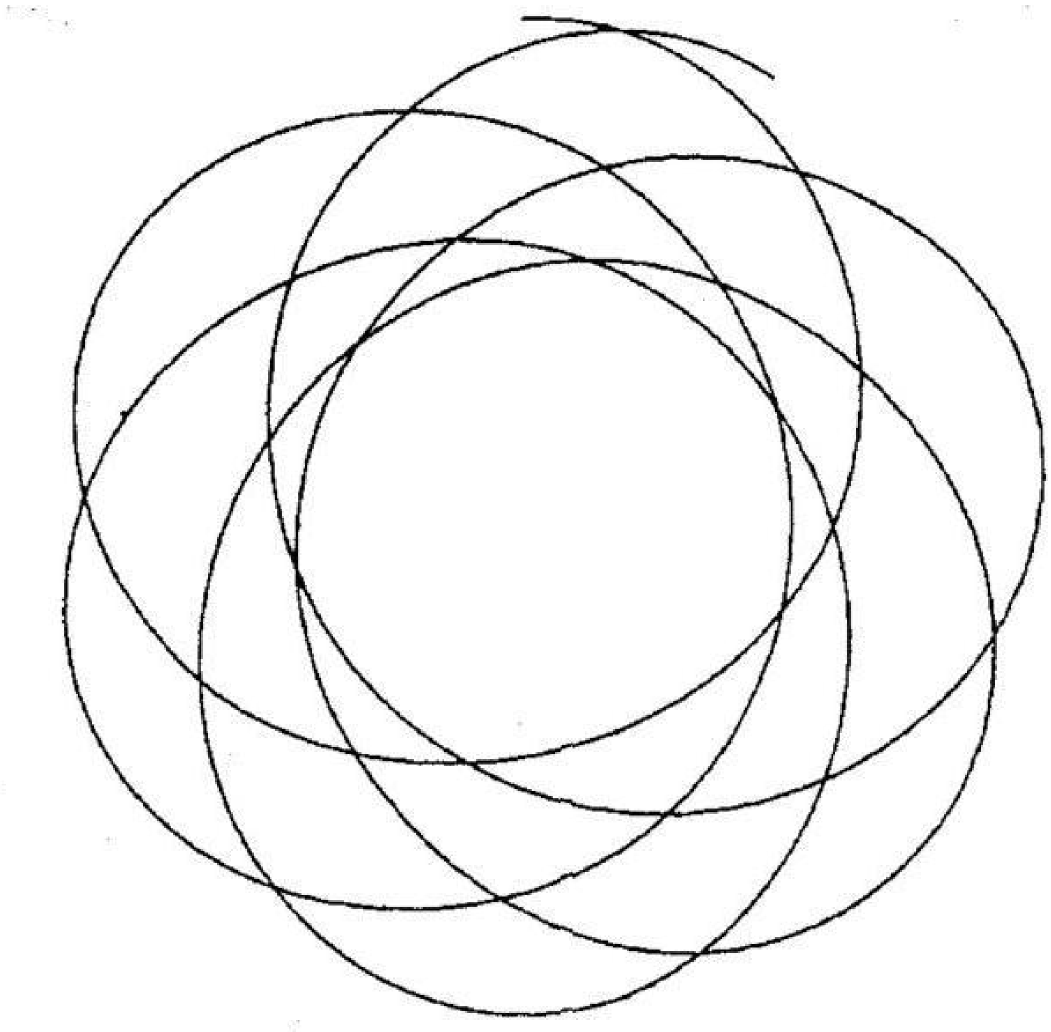

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Integration over many orbital periods leads to density profiles for the orbits.

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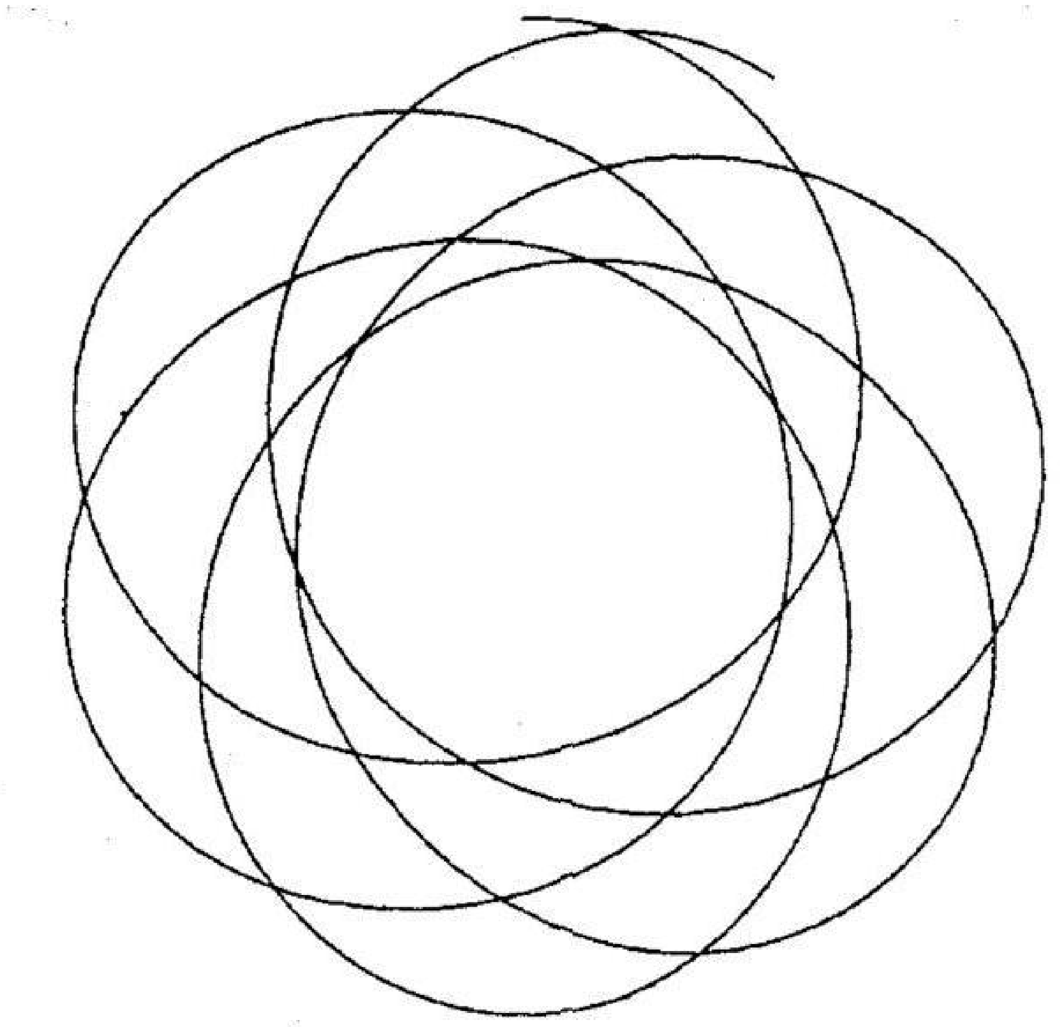

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Each orbit can be treated as a stationary quantity.

The Schwarzschild model

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- How many orbits are considered
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- Which level of agreement between the model and the data is required for the model to be accepted

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The task to find agreement between model and data gets more difficult with the number of order of the velocity profile number of boundary conditions, but the virial equilibrium should not be neglected!

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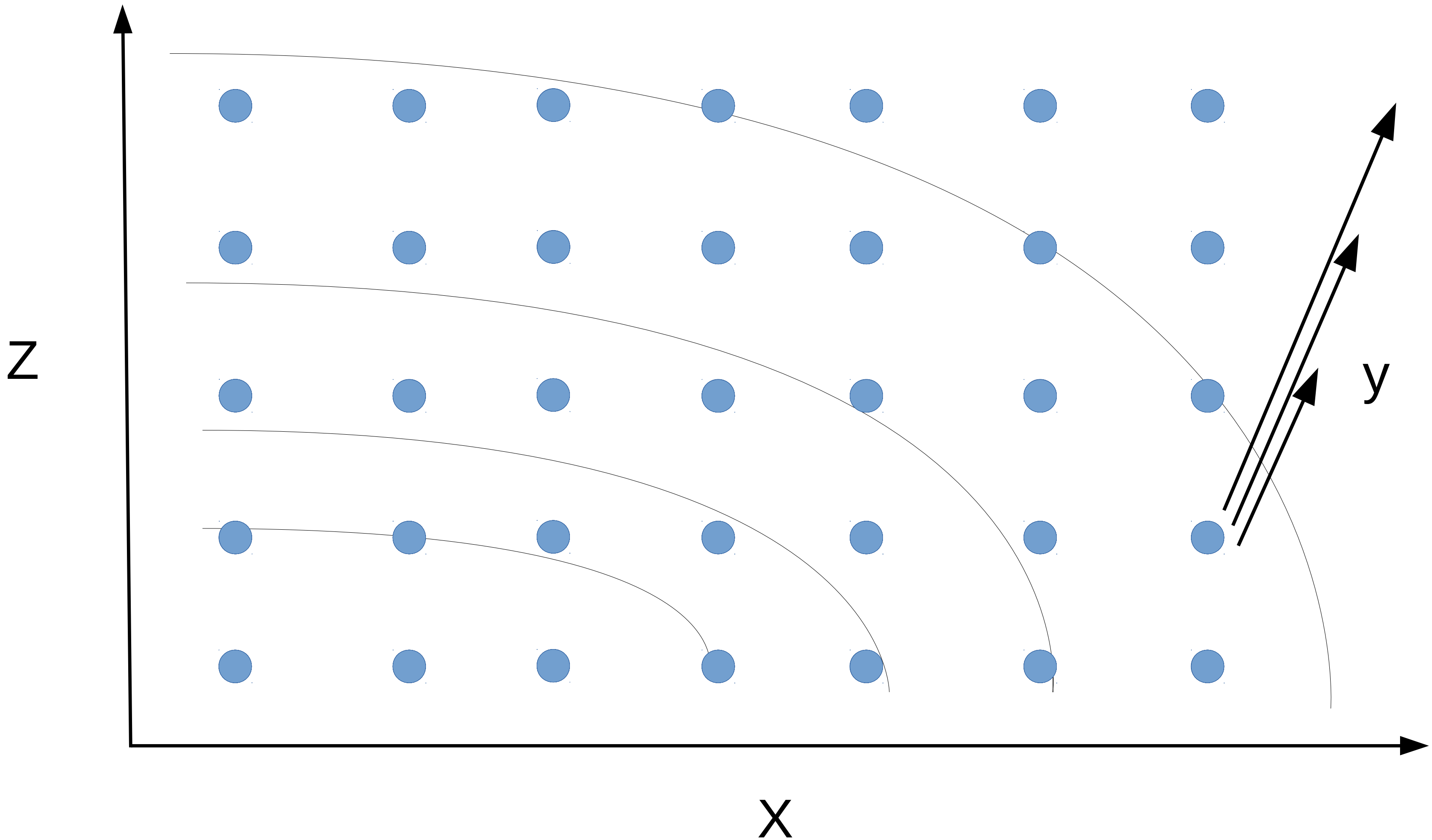
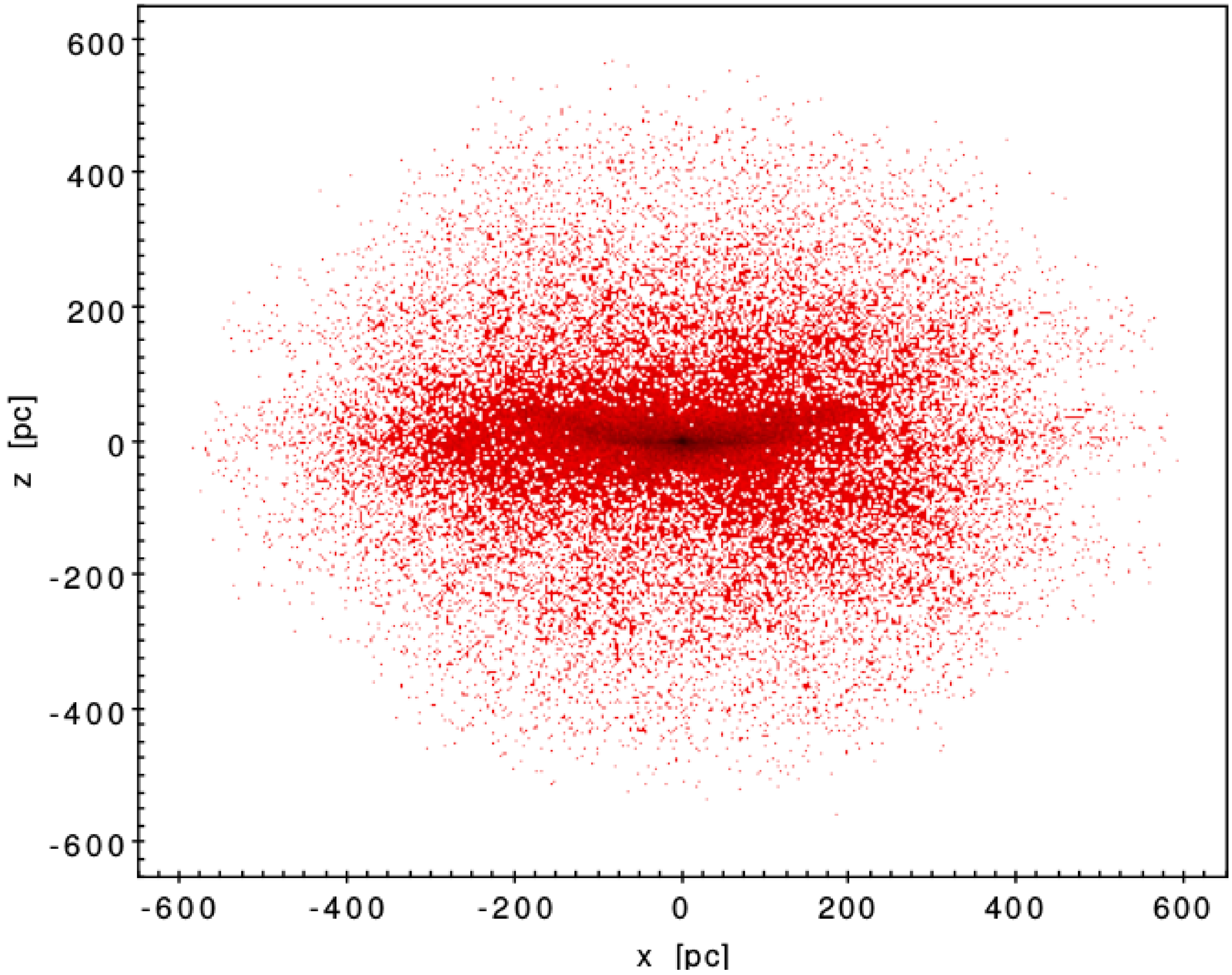


THE KEY STEP: Determine the weights of the different orbits!

The Schwarzschild Model



The Galactic center – Feldmeyer-Krause et al. 2017

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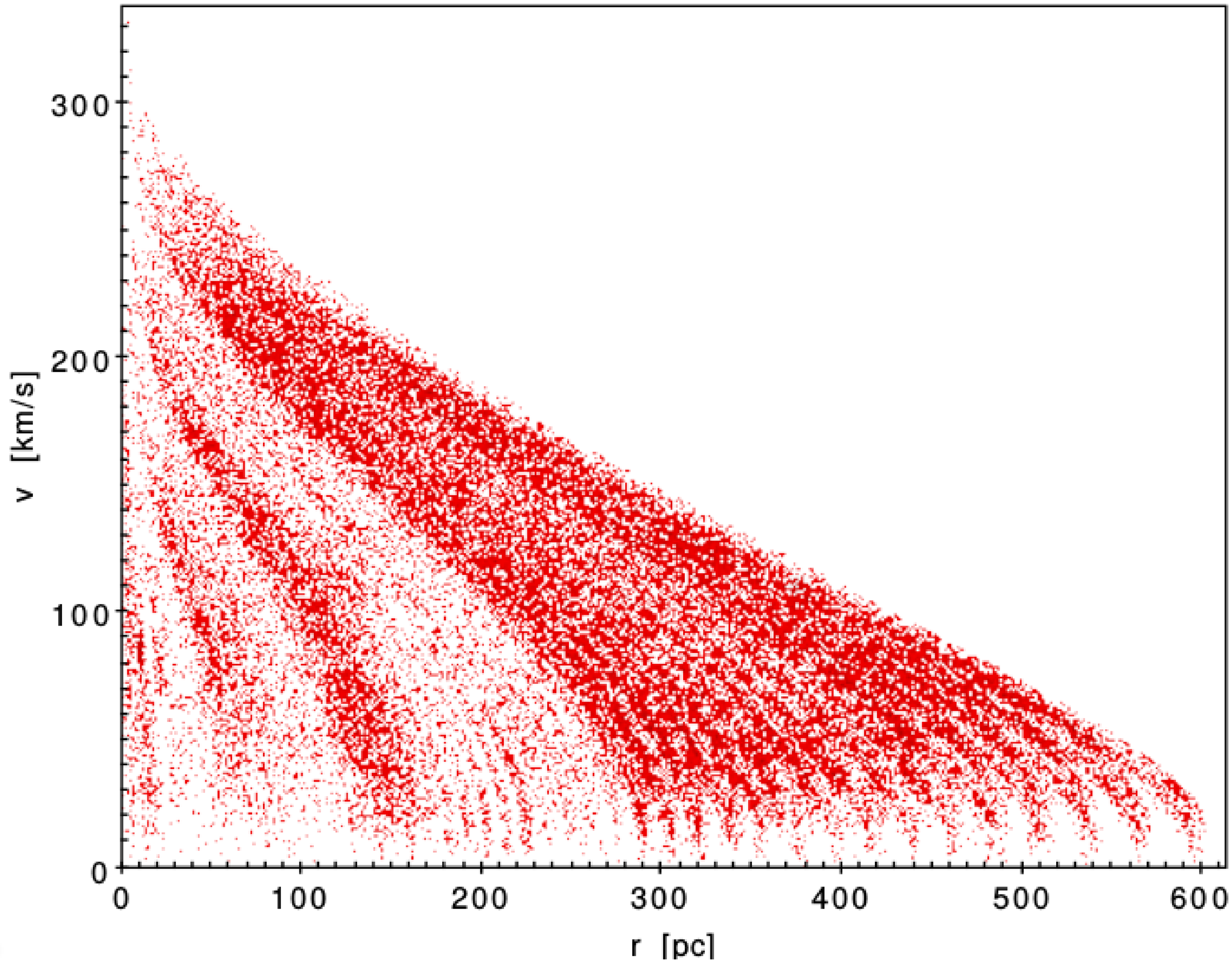

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The Virial Theorem

The virial theorem in its most general form:

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If forces are additionally homogeneous ($\mathbf{r} \cdot \nabla\Phi(\mathbf{r}) = k\Phi(\mathbf{r})$):

$$-k\left\langle \frac{1}{2} \int_V \rho(\mathbf{x})\Phi(\mathbf{x})\, d^3\mathbf{x} \right\rangle + 2\langle T \rangle = 0$$

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$$\frac{1}{2}\int_V \rho(\mathbf{x})\Phi(\mathbf{x})\, d^3\mathbf{x} = U$$ is the potential energy!

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For the virial theorem in this form, the potential must be conservative and homogeneous and stationary!

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Virial equilibrium under gravitation has to be introduced as additional condition in Schwarzschilds Method!

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An external Plummer profile with particles in it which have the same density profile, but are practically mass-less.

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Both systems are stable with an external Plummer potential, but as self-gravitating system **case 1 is stable** and **case 2 is not**

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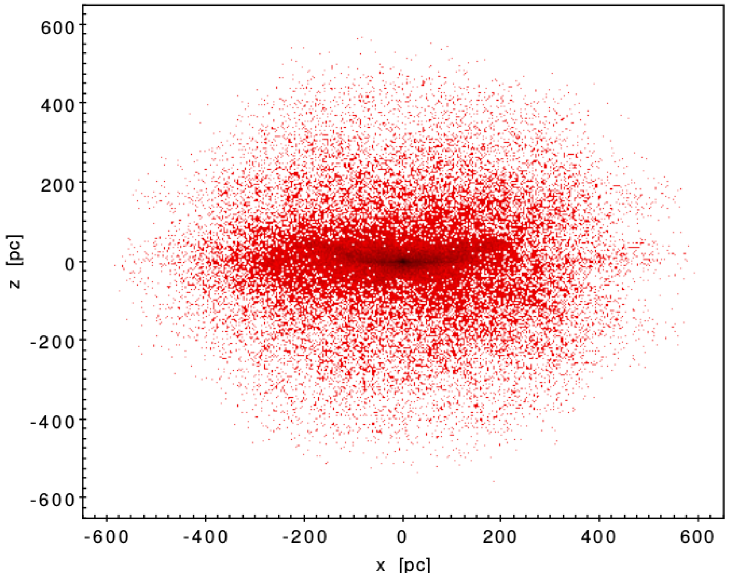
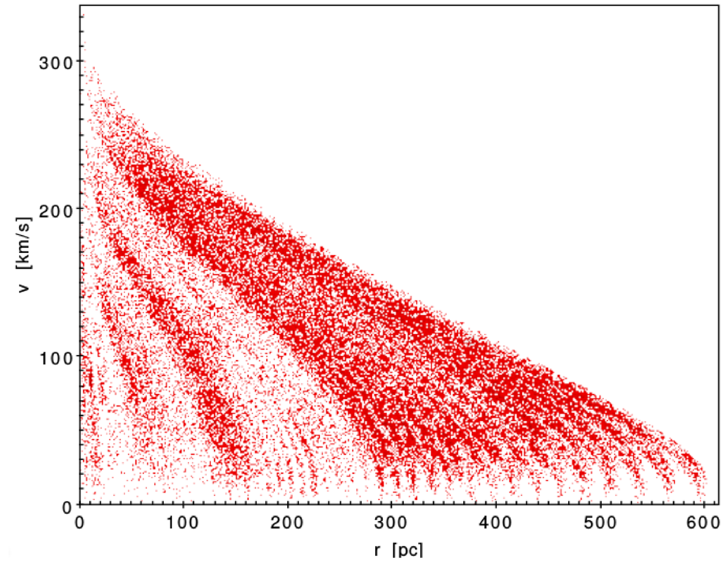
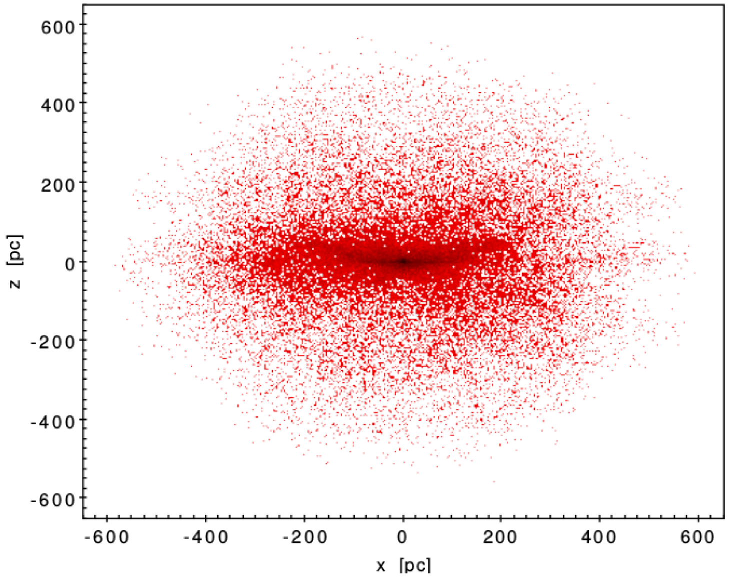
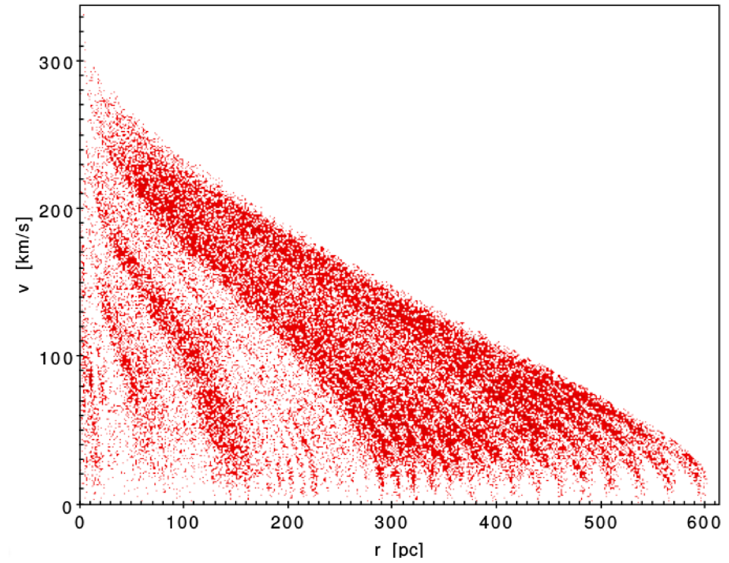




Take a Schwarzschild Model with *M* orbits with weights.

The Virial Theorem and Schwarzschilds Method





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Discretize the orbits into *N* particles and let them fill randomly the space according to their weights

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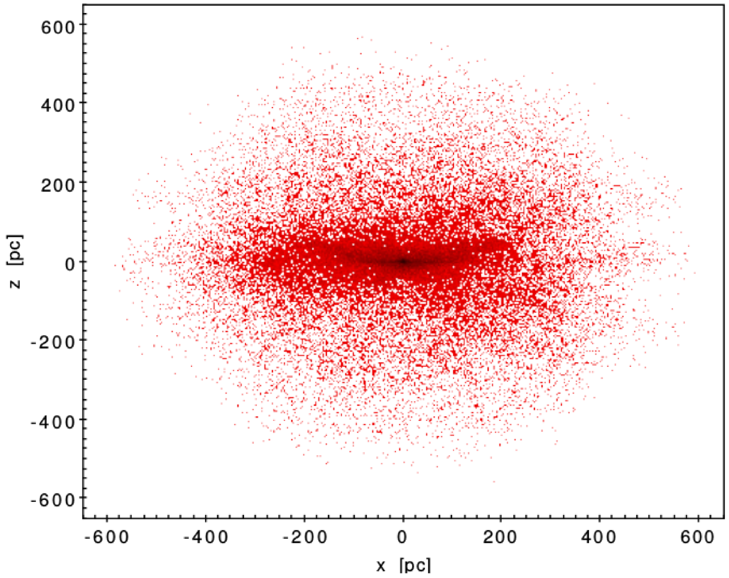

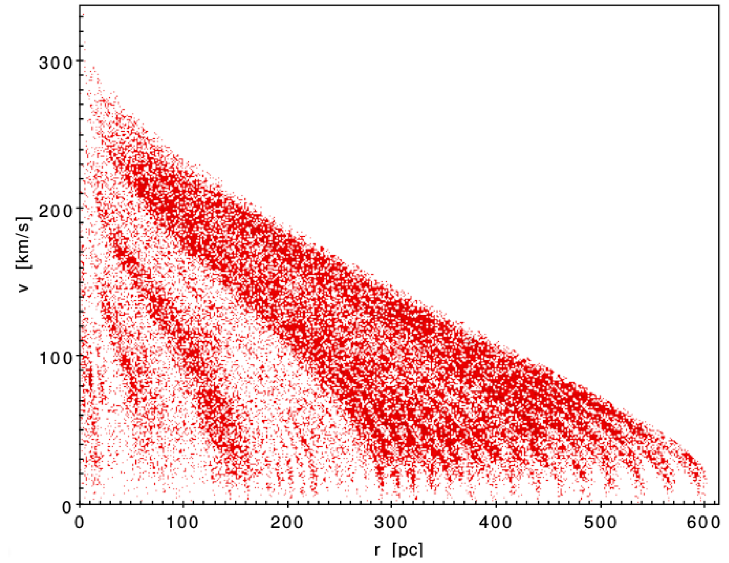

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Attribute the mass of the potential equally to the *N* particles

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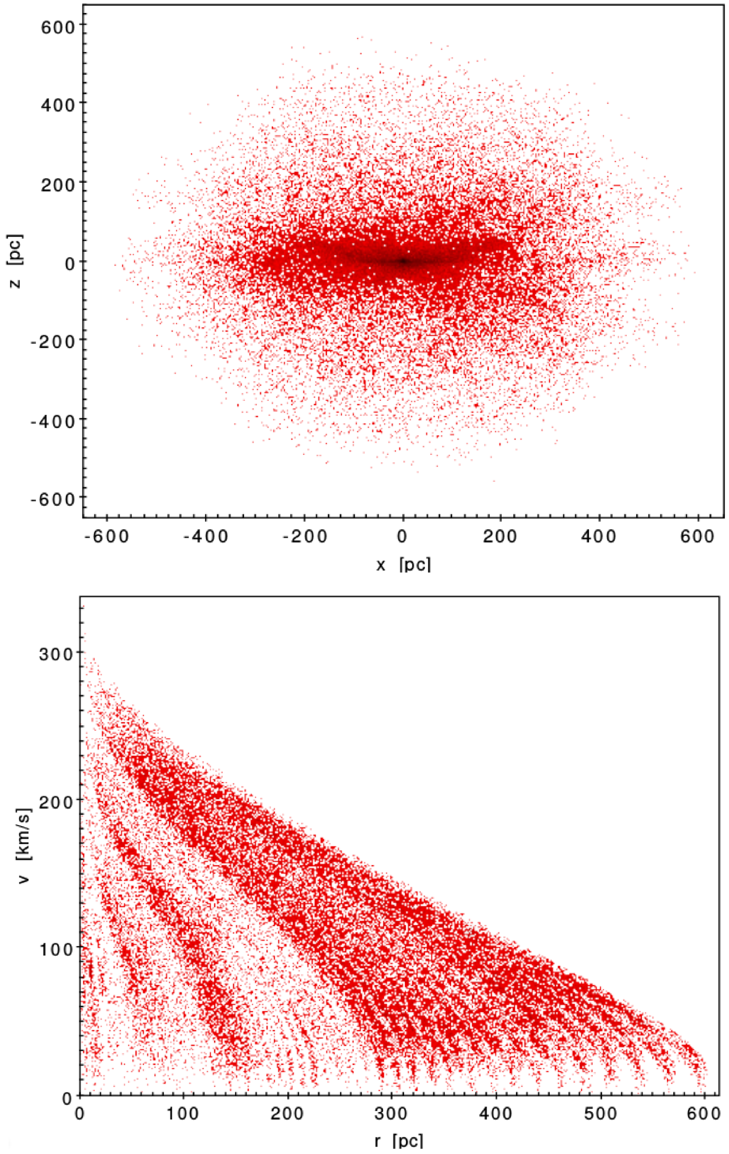

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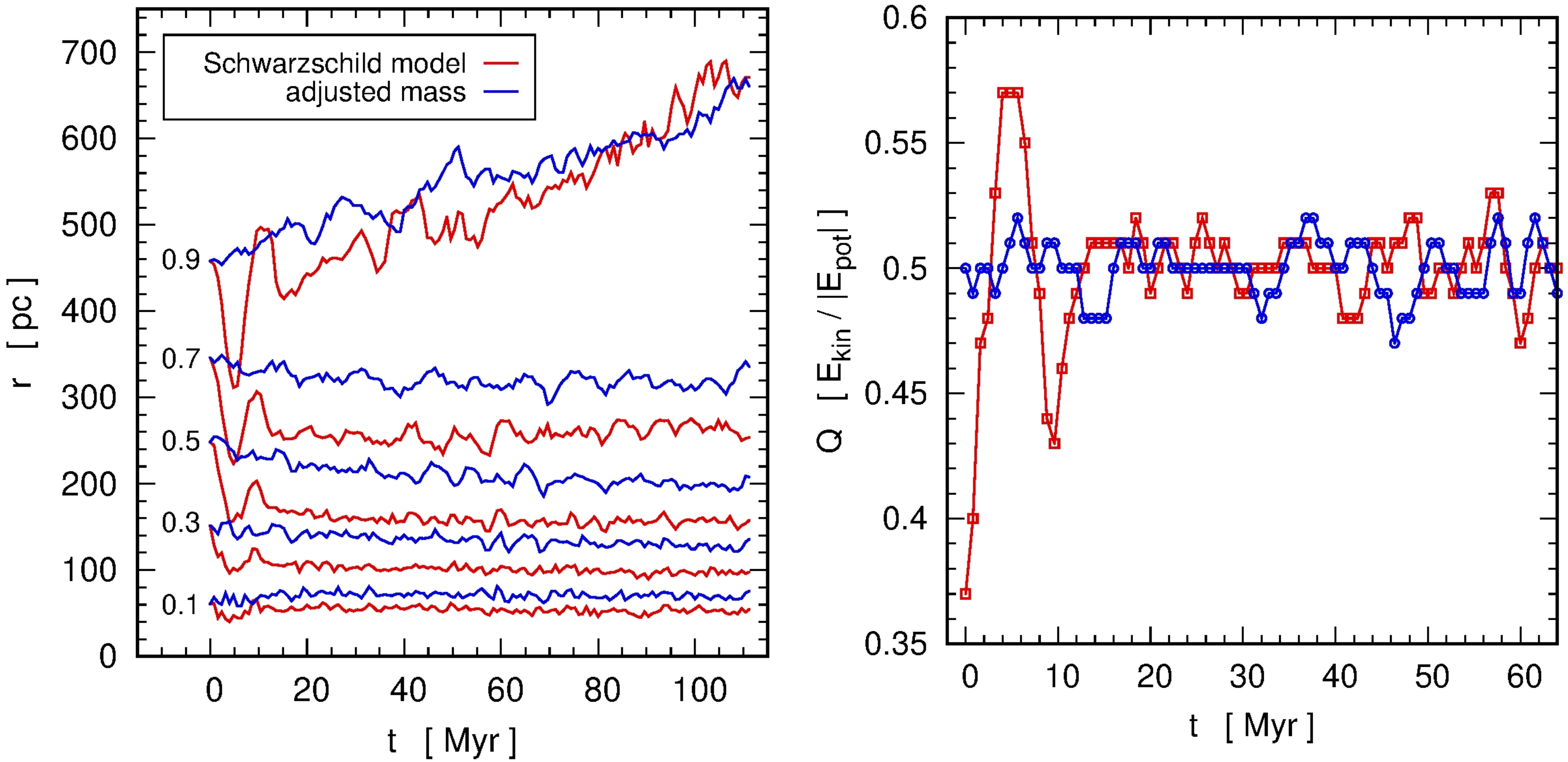
Integrate the model with an *N*-Body integrator (S. Aarseth's NBODY 6) and see what happens

The best-fitting SM for the Galactic Center by Feldmeyer-Krause+(2017)



1000 Particles

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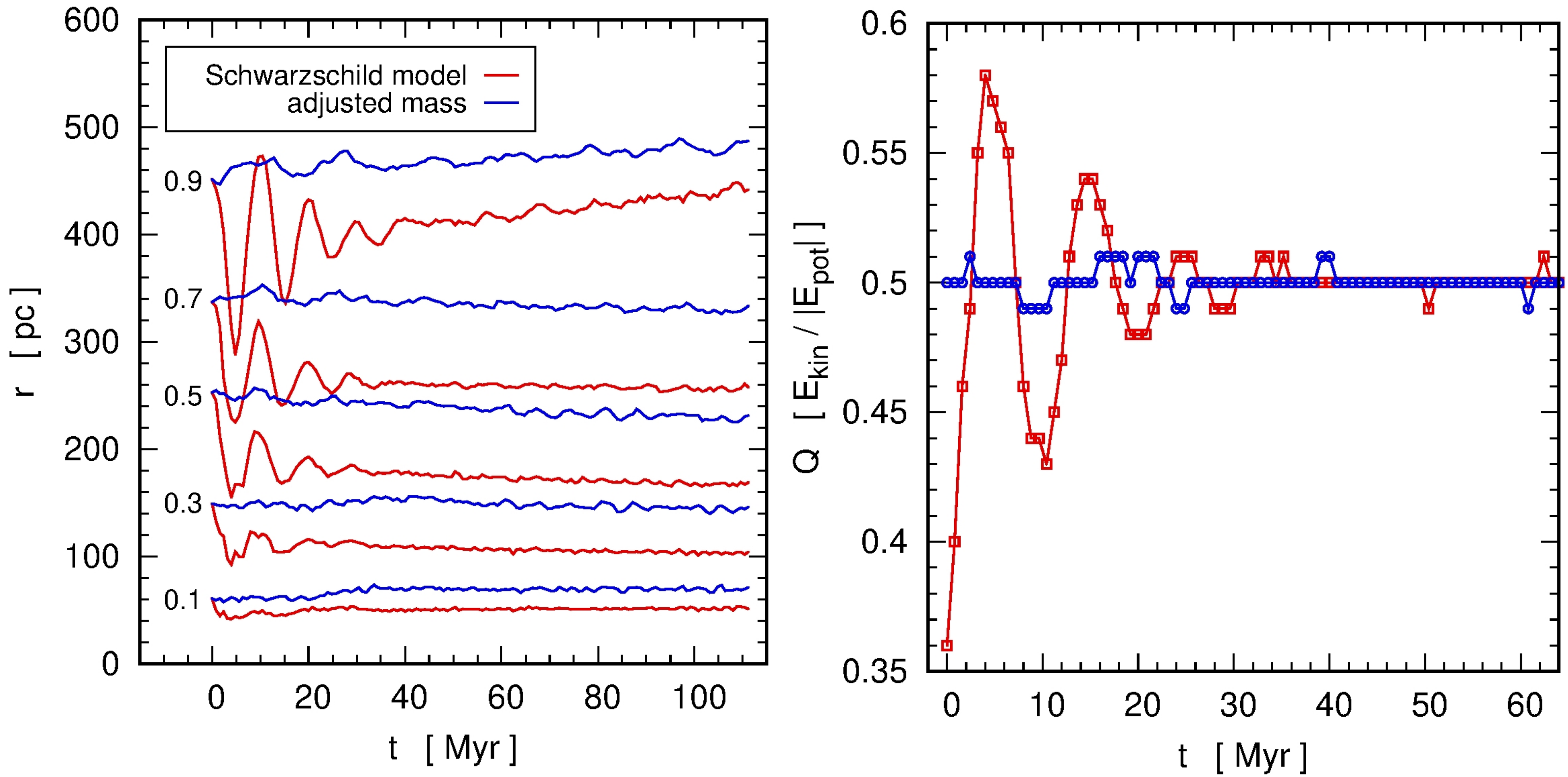

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1.) A posteriori:
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2.) A priori:
Make the Fulfillment of virial equilibrium as an additional Requirement when searching for the Schwarzschild solution

(better)