The Astronomical Journal, 132:1950–1958, 2006 November

NEW CANDIDATES FOR RECENT ASTEROID BREAKUPS

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Received 2006 February 28; accepted 2006 July 26

ABSTRACT

Asteroids in our solar system formed in a dynamically quiescent disk, but their orbits became gravitationally stirred enough by Jupiter to lead to high-speed collisions. As a result, several dozen large asteroids have been disrupted by impacts over the past several gigayears and have produced groups of fragments called asteroid families. Here we report three new candidates for asteroid families that were formed by collisions occurring in the last 1 Myr. According to our modeling of the past orbital histories of known cluster members, we estimate that the Emilkowalski, 1992 YC2, and Lucascavin clusters are 220 ± 30, 50–250, and 300–800 kyr old, respectively. Together with the previously identified Datura cluster, estimated to be 450 ± 50 kyr old, they are the most recent asteroid breakups ever discovered in the main belt. Astronomical observations of identified family members can be used to better understand impact physics, asteroid composition, and surface-aging processes. Discovered breakups may also be important sources of interplanetary dust.

Key words: minor planets, asteroids

1. INTRODUCTION

Up to now, ejecta from a few tens of major collisions have been discovered in the main belt (e.g., Bendjoya & Zappalà 2002; Cellino et al. 2002). These groups of fragments are called asteroid families. To identify an asteroid family, researchers look for clusters of asteroid positions in the space of the *proper* orbital elements: proper semimajor axis (a_P), eccentricity (e_P), and inclination (i_P) (Milani & Kneževic 1994; Kneževic et al. 2002). Proper orbital elements, being more constant over time than instantaneous orbital elements (Milani & Kneževic 1994), provide a dynamical criterion for whether or not a group of bodies has a common ancestor. Unfortunately, most of the observed asteroid families are old enough (hundreds of millions to billions of years; e.g., Vokrouhlický et al. 2006a, 2006b, 2006c) that they have been substantially eroded and dispersed by (1) secondary collisions (e.g., Marzari et al. 1999; Bottke et al. 2005a, 2005b), (2) chaotic orbital evolution (e.g., Nesvorný et al. 2002b), and (3) semimajor axis mobility due to radiation effects (e.g., Bottke et al. 2001; Vokrouhlický et al. 2006a, 2006b, 2006c). These effects make it problematic to determine the conditions that existed immediately after the family break-up event. Fragments produced by recent collisions, on the other hand, would suffer little erosion in the interim period and would therefore provide more direct information about the break-up event itself.

So far we know of three asteroid families with ages between 1 and 10 Myr. These are (1) the Iannini cluster (1–5 Myr old), (2) the Karin cluster (5.75 ± 0.05 Myr old), and (3) the Veritas family (8.3 ± 0.1 Myr old). The ages of these families were determined by numerical integrations of asteroid orbits backward in time (e.g., Nesvorný et al. 2002a, 2003; Nesvorný & Bottke 2004) and by showing that the orbits were nearly identical at the time of family formation. Using the same technique we identified another recently formed family, the Datura cluster, and found that it is 450 ± 50 kyr old (Nesvorný et al. 2006a). To date, the Datura cluster is the only known asteroid family younger than 1 Myr.

The Datura cluster is located in the part of the main belt where high-order mean motion resonances with Mars produce chaos (Fig. 1) with Lyapunov times that range between 25 and 100 kyr. This is unfortunate because the effects of chaos do not allow us to reliably track positions of objects in their orbits (as defined by mean anomaly M) or to show that their positions converged to a single point about 450 kyr ago. The age determination for the Datura cluster thus relies on showing the convergence of nodal (Ω) and perihelion longitudes (ϖ), because these angles are generally less susceptible to the effects of chaos and can be tracked more easily over a megayear timescale.[2]

Here we describe three new candidates for asteroid families formed within the last 1 Myr. We show that these families reside in a dynamically quiet zone of the main belt and have ages that are comparable to or only slightly longer than the Lyapunov times. For two of these families, we accomplish "the ultimate goal" of orbit reconstruction by showing the convergence of Ω, ϖ, and M. Therefore, for the first time a definitive proof is given that a group of asteroid fragments comes from a single parent object. This result required appropriate modeling of the Yarkovsky thermal effects similar to that described in Nesvorný & Bottke (2004).

We first explain the identification method and the general approach that we used to determine the ages of the new families (§§ 2 and 3). In § 4 we describe our results for the three candidate clusters individually. Implications of this work are discussed in § 5.

2. IDENTIFICATION METHOD

To identify very young asteroid families we used a new approach that differs from the traditional family-identification methods (e.g., Bendjoya & Zappalà 2002 and references therein). Instead of using a catalog of proper orbital elements, we used the osculating

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[2] One known object in the Datura cluster, 89309 2001 VN36, resides in the exterior 9:16 mean motion resonance with Mars (Fig. 1). The past history of Ω and ϖ for this object critically depends on whether, and for how long, the orbit interacted with this resonance. This object cannot yet be used for the determination of the age of the Datura cluster. Future observations of 89309 2001 VN36 may improve orbit determination and help to better reconstruct its past orbital history.

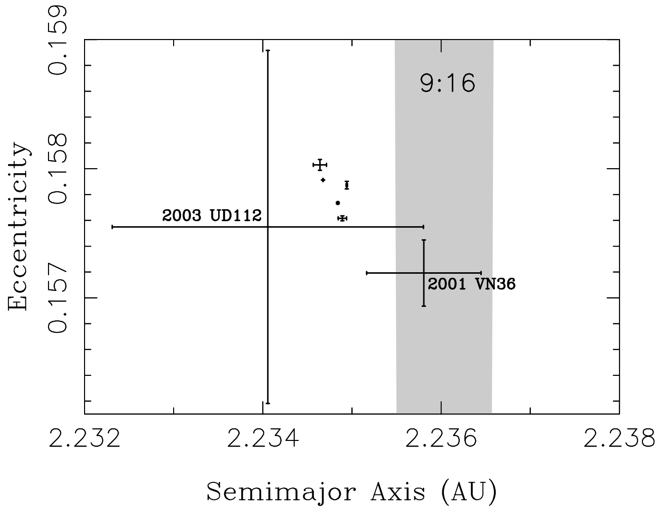

FIG. 1.— Members of the Datura cluster projected onto the plane of the proper semimajor axis a_P and proper eccentricity e_P. The error bars were calculated from 10 determinations of the synthetic proper elements that used different initial orbits within the orbit uncertainty range. The large uncertainty in a_P and e_P for 89309 2001 VN36 is produced by the exterior 9 : 16 mean motion resonance with Mars. The maximum width of this resonance is shown by the shaded zone. Nesvorný et al. (2006a) did not use 89309 2001 VN36 (or the single-opposition object 2003 UD112) for the age determination of the Datura cluster.

orbital elements directly. Because the osculating elements are not constant over megayear-long time intervals we did not expect to find asteroid families older than about 1 Myr. We anticipated, however, that the very young families formed in the last 1 Myr could show up as clusters in a five-dimensional space of osculating orbital elements: semimajor axis a, eccentricity e, inclination i, perihelion longitude ϖ, and nodal longitude Ω. Note that it typically takes >1 Myr before ϖ and Ω of family members become dispersed by the differential precession induced by the planets and about the same timescale to disperse a compact cluster in osculating a, e, or i. We did not expect to find any clustering in mean anomaly M because this angle becomes dispersed by Keplerian shear within $\sim$10–1000 yr. Therefore, we did not use M in our identification method.

To exclude very poorly defined orbits, we selected 264,403 asteroids from the AstOrb catalog (Bowell et al. 1994) that have an observational arc longer than 10 days.[3] To search for clusters in (a, e, i, ϖ, Ω) space, we used the hierarchical clustering method (HCM; Zappalà et al. 1990). The HCM selects orbits in (a, e, i, ϖ, Ω) space with the length of each link $\leq d_{\rm cutoff}$. We used the standard definition of distance in (a, e, i) space (Zappalà et al. 1990) and various definitions of distance in (ϖ, Ω) space. Our general definition of distance, d, is

$$\left(\frac{d}{na}\right)^2 = k_a\left(\frac{\delta a}{a}\right)^2 + k_e(\delta e)^2 + k_i(\delta \sin i)^2 + k_\Omega(\delta\Omega)^2 + k_\varpi(\delta\varpi)^2, \quad (1)$$

where n is the mean motion and (δa, δe, $\delta \sin i$, $\delta\varpi$, $\delta\Omega$) is the separation vector of neighboring bodies. The standard HCM metrics use $k_a = 5/4$ and $k_e = k_i = 2$. Values of k_Ω and k_ϖ were chosen empirically. We required that $k_\Omega \simeq k_\varpi$, because secular precession rates of Ω and ϖ are comparable across the main belt (the rates of precession of Ω and ϖ are equal under the approximations of linear perturbation theory; e.g., Brouwer & Clemence 1961). Given that we expected $\delta\Omega$ and $\delta\varpi$ on the order of degrees and $(\delta a/a) \sim \delta e \sim \delta \sin i \sim 0.001$–$0.01$, we experimented with k_Ω and k_ϖ ranging between 10^{-4} and 10^{-7}. The best results were obtained with $k_\Omega = k_\varpi = 10^{-6}$. We also performed analyses with different values of $d_{\rm cutoff}$ to test the sensitivity of the method to this parameter.

We found three new and three previously known asteroid families. The Iannini and Karin clusters, known to be 1–5 and $\approx$5.75 Myr old, respectively, showed up as several groups with 5–10 members each representing a small part of the two families that has maintained a coherent distribution of osculating orbits until the present epoch. In addition, we identified the Datura cluster as a group of seven asteroids located around 1270 Datura. According to the results of Nesvorný et al. (2006a) the Datura cluster is 450 ± 50 kyr old.

The three new families are groups of three objects each (Tables 1 and 2). Following the usual terminology we name these families after their lowest numbered asteroid members. The Emilkowalski and 1992 YC2 clusters are located in the middle part of the main belt at $\approx$2.6 AU. The Lucascavin cluster is located in the inner main belt at $\approx$2.3 AU.

The asteroids in the new clusters have been observed at several oppositions: out of nine member asteroids identified here, four asteroids are numbered, four have been observed in at least three oppositions, and one (2003 VM9 in the Lucascavin cluster) has been observed during two oppositions. T. Spahr (2006, private communication) has found three prediscovery observations of 2004 XL40 from 2002 April, which allowed us to determine the orbit of 2004 XL40 with a higher accuracy.

We used OrbFit9 public software for orbit determination.[4] Table 1 lists osculating a, e, i, ϖ, and Ω for asteroids in the new clusters and the uncertainties of these elements. The orbit uncertainty was taken into account in every step of our work, including the age determination and computation of proper elements. Each cluster shows a very narrow spread of orbital elements. In fact, each of the four clusters listed in Table 1 is much more compact than any other known asteroid family.

The osculating elements listed in Table 1 include the effects of planetary perturbations that accumulated in the orbits since the formation of these clusters. To eliminate these effects, we determined the synthetic proper elements for all asteroids listed in Table 1. We adopted the technique of Knežević & Milani (2000, 2003). The orbits were numerically integrated forward in time for 2 Myr. The nonsingular eccentricity and inclination vectors were passed through a Fourier filter, which identified the proper and principal forced (planetary) frequencies. The proper eccentricity (e_P) and proper inclination (i_P) were defined as the amplitudes of the proper terms. The proper semimajor axis (a_P) was defined as the mean value of a.

We integrated 10 clones of each object with initial orbits that were chosen from the uncertainty range of the current orbit determination (see Table 1). The values of a_P, e_P, and i_P listed in Table 2 were determined as arithmetic means of proper elements determined for individual clones. We also estimated 1 σ uncertainties of a_P, e_P, and i_P from the differences of proper elements as standard deviations and list these values in Table 2.

[3] In our original analysis we used the AstOrb (ftp://ftp.lowell.edu/pub/elgb/astorb.html) catalog from 2005 December 20. We verified that no new members in the identified clusters appeared in a more updated version of this catalog on 2006 March 1.

[4] Available at http://newton.dm.unipi.it/orbfit/.

TABLE 1

Osculating Orbital Elements

Asteroid	$a/\delta a$ (AU)	$e/\delta e$	$i/\delta i$ (deg)	$\omega/\delta\omega$ (deg)	$\Omega/\delta\Omega$ (deg)
Datura Cluster					
1270 Datura	2.235553491(11)	0.20750424(13)	5.9876230(86)	258.68066(14)	97.90401(14)
60151 1999 UZ6	2.23470878(21)	0.20758512(36)	5.993636(21)	260.77803(27)	96.80238(25)
89309 2001 VN36	2.23468843(36)	0.20731883(50)	6.020404(19)	266.77862(23)	92.99943(19)
90265 2003 CL5	2.23486438(16)	0.20817849(29)	5.995205(30)	262.00113(32)	95.70379(30)
2001 WY35	2.2343433(14)	0.2083830(12)	5.990231(29)	260.39857(88)	96.89528(22)
2003 SQ168	2.235532(74)	0.207482(12)	5.98895(14)	259.229(20)	97.4898(30)
2003 UD112	2.2342(18)	0.20614(55)	5.9999(83)	263.14(31)	95.474(67)
Emilkowalski Cluster					
14627 Emilkowalski	2.598477717(75)	0.15061485(17)	17.733545(26)	44.56383(13)	41.568301(38)
126761 2002 DW10	2.59896767(28)	0.15225080(65)	17.756607(22)	42.47166(40)	41.39285(16)
2005 WU178	2.5988781(29)	0.151551(10)	17.75416(22)	42.8675(19)	42.34248(20)
1992 YC2 Cluster					
16598 1992 YC2	2.618814068(53)	0.22081556(12)	1.628026(16)	105.13893(53)	287.06871(53)
2000 UV80	2.6187945(31)	0.2209835(84)	1.626909(25)	105.6433(27)	286.7448(13)
2005 TT99	2.6197232(16)	0.2200800(30)	1.628248(31)	105.8049(25)	286.6487(21)
Lucascavin Cluster					
21509 Lucascavin	2.280668397(38)	0.11297372(15)	5.986972(15)	4.11772(18)	70.28200(16)
2003 VM9	2.280624(42)	0.113186(12)	5.98728(15)	3.6219(88)	70.52488(27)
2004 XL40	2.2815430(79)	0.111693(21)	5.981458(89)	4.5356(42)	70.12860(42)

Notes.— Keplerian orbital elements are given for epoch MJD 2,453,700.5. The values of mean anomaly, not listed here, show no clustering. The 1 σ uncertainties of the orbital elements in the last two decimal digits are shown in parentheses. They are roughly inversely proportional to the length of the observational arc. Asteroid 1270 Datura, first discovered in 1930, has the most accurately determined orbital elements. Single-opposition asteroids 2003 SQ168 and 2003 UD112 in the Datura cluster have the largest uncertainties. For 2003 UD112, the orbit uncertainties in a, e, and i are comparable to the orbital spread of the Datura cluster.

The new asteroid groups are extremely compact in the space of proper elements (Table 2). Dispersions in a_P, e_P, and i_P are $\approx 2\times10^{-4}$ AU, $\approx 10^{-4}$, and $\approx 2\times10^{-3}$ deg for the Emilkowalski and 1992 YC2 clusters. The Lucascavin cluster shows even smaller dispersions: $\approx 4\times10^{-5}$ AU, $\approx 10^{-5}$, and $\approx 2\times10^{-5}$ deg in a_P, e_P, and i_P, respectively. These dispersions are 1–3 orders of magnitude tighter than those of the Karin cluster! We estimate that the probability that the newly identified clusters are random fluctuations of orbit density is significantly less than one part in a million (see the Appendix for a discussion of the results of our statistical tests). Based on these results, we propose that the Emilkowalski, 1992 YC2, and Lucascavin clusters were produced by recent collisional breakups of three parent asteroids.[5]

3. AGE DETERMINATION

A detailed analysis of the new clusters in terms of impact conditions that produced them must await until more than three members in each cluster are identified (many more members will be discovered after the Pan-STARRS telescopes start operations in 2009). As we describe below, however, it is already possible with the current data to estimate the formation ages of clusters via orbit integrations. Our new method of age determination accounts for the effects of chaos, orbit uncertainties, and Yarkovsky thermal drag. We describe this method below.

The origin of cluster members in a breakup event implies a link between their orbit elements and the ejection speeds by which they were ejected from the breakup location. This link is given by the Gauss equations (e.g., Bertotti et al. 2003):

$$\delta a = \frac{2}{n\eta}\left[V_R\, e \sin f + V_T(1 + e\cos f)\right], \tag{2}$$

$$\delta e = \frac{\eta}{na}\left(V_R\,\sin f + V_T\frac{e + 2\cos f + e\cos^2 f}{1 + e\cos f}\right), \tag{3}$$

$$\delta i = \frac{\eta}{na} V_Z \frac{\cos(\omega + f)}{1 + e\cos f}, \tag{4}$$

$$\delta\Omega = \frac{\eta}{na\sin i} V_Z \frac{\sin(\omega + f)}{1 + e\cos f}, \tag{5}$$

$$\delta\varpi = \frac{\eta}{nae}\left(-V_R\,\cos f + V_T\,\sin f\,\frac{2 + e\cos f}{1 + e\cos f}\right) + 2\sin^2\frac{i}{2}\,\delta\Omega, \tag{6}$$

where $\eta = (1 - e^2)^{-1/2}$. Equations (2)–(6) relate the osculating element differences (δa, δe, δi, $\delta\varpi$, $\delta\Omega$) of members' orbits to their ejection velocity vectors $\boldsymbol{V} = (V_R,\ V_T,\ V_Z)$. Here we use the usual notation in which the velocity vector is projected into radial (V_R), transverse (V_T), and normal (V_Z) directions defined by a reference orbit with elements a, e, i, Ω, and ϖ.

[5] An additional indication of the recent collisional origin of these clusters is that elements δi and $\delta\Omega$ are strongly correlated (Table 1). Such a correlation is expected from Gauss eqs. (4) and (5), which can be used to yield $\sin i\,\delta\Omega = \tan(\omega + f)\,\delta i$. Formally, ω and f in this equation represent the values of these quantities at the time of the breakup event. We verified that this relation persists for a limited time after the breakup.

TABLE 2
SIZES AND PROPER ORBITAL ELEMENTS

Asteroid	H (mag)	D (km)	a_P (AU)	e_P	i_P (deg)
Datura Cluster					
1270 Datura	12.5	10.8	2.2346757(28)	0.15791254(74)	5.335155(30)
60151 1999 UZ6	16.3	1.9	2.234892(47)	0.157616(19)	5.3254(20)
89309 2001 VN36[a]	16.3	1.9	2.23580(64)	0.15719(26)	5.3047(74)
90265 2003 CL5	15.4	2.9	2.2349417(81)	0.157872(29)	5.3350(16)
2001 WY35	17.0	1.4	2.2348413(59)	0.1577349(60)	5.32732(65)
2003 SQ168	16.9	1.4	2.234640(74)	0.158029(42)	5.3392(27)
2003 UD112	17.9	0.9	2.2340(17)	0.1575(14)	5.334(67)
Emilkowalski Cluster					
14627 Emilkowalski	13.1	8.2	2.59929998(78)	0.18005281(80)	17.223933(25)
2002 DW10	15.0	3.4	2.599537(53)	0.179960(74)	17.22545(18)
2005 WU178	16.5	1.7	2.599307(12)	0.179928(19)	17.22370(30)
1992 YC2 Cluster					
16598 1992 YC2	14.7	3.9	2.619929(17)	0.179722(39)	2.86413(40)
2000 UV80	16.5	1.7	2.620139(63)	0.179645(26)	2.862519(92)
2005 TT99	17.1	1.3	2.620211(77)	0.179611(78)	2.862588(70)
Lucascavin Cluster					
21509 Lucascavin	15.0	3.4	2.2811632(14)	0.13171791(22)	5.314984(13)
2003 VM9	16.8	1.5	2.281104(40)	0.131716(16)	5.31498(13)
2004 XL40	17.0	1.4	2.281122(67)	0.131709(43)	5.31500(77)

NOTES.—All currently known asteroid members of the four clusters are listed here. Absolute magnitude values H were taken from the Lowell asteroid database AstOrb. Effective diameters D were calculated assuming albedo $p_V = 0.15$. The last three columns give the synthetic proper orbital elements: semimajor axis a_P, eccentricity e_P, and inclination i_P. We determined these elements from a 2 Myr long numerical integration of orbits (see § 2). The uncertainties of a_P, e_P, and i_P (shown in parentheses) were determined by integrating different orbit clones of the same object.

[a] Proper elements of 89309 2001 VN36 are less accurate because the orbit of this object is affected by the 9 : 16 exterior mean motion resonance with Mars (Fig. 1).

Strictly speaking, it is only meaningful to use the Gauss equations (2)–(6) for the time of the breakup event. In a generalized sense, however, these relations can also help to determine how close the current orbital configuration of a cluster is to its initial state. For example, the current dispersion in osculating Ω and ϖ on the order of a few degrees (Table 1) corresponds to speeds $V \gtrsim 10$ m s^{-1}. These values are an order of magnitude larger than those suggested by the clustering of proper elements a_P, e_P, and i_P. This comparison shows that the orbits of fragments have been substantially modified by secular planetary perturbations. In addition, the values of M for individual cluster members are currently distributed evenly between 0° and 360°, showing that the Keplerian shear had enough time to operate. Given these results, we estimate that the ages of new clusters, $t_{\rm age}$, cannot be younger than ~10,000 yr or older than ~1 Myr.

We developed the following strategy to determine $t_{\rm age}$ more precisely. As in Nesvorný et al. (2002a, 2003) the essence of our approach was to numerically integrate the orbits of cluster members backward in time and to identify the time of their convergence. Unlike in Nesvorný et al., however, we worked directly with the osculating orbital elements that include short- or long-periodic oscillations.

The initial orbits and conducted integrations were designed to account for two important factors: (1) the current orbits of cluster members are determined with finite accuracy, which means that all initial orbits within the (sometimes broad) orbit uncertainty distribution are statistically equivalent, and (2) the asteroids are small (see Table 2), so that the thermal Yarkovsky force can change their semimajor axis and therefore also affect the past evolution of the orbital angles.

Effect 2 is especially important here because the slow drift of bodies in a due to the Yarkovsky effect (e.g., Bottke et al. 2002) produces an amplified effect on angles. For example, we estimate that the mean anomaly is modified by $\sim(1/2)(\partial n/\partial a)(da/dt)t^2 \sim (3/4)[(da/dt)/a]nt^2$. Therefore, for the maximum estimated drift rates of kilometer-sized objects, $da/dt \sim 2\times10^{-4}$ AU Myr^{-1} (e.g., Morbidelli & Vokrouhlický 2003, their Fig. 5), the mean anomaly can rotate with respect to an orbit with fixed a by 360° in ~200 kyr. The same effect on Ω and ϖ is about 1000 times smaller.

To deal with (1) we cloned orbits of each cluster member assuming the normal distribution of orbit elements and 1 σ uncertainties listed in Table 1. In total, 20 orbit clones were numerically integrated for each asteroid. We observed that the past trajectories of these orbit clones slowly diverge due to regular and chaotic effects. All these evolutions were used for the age determination. In addition, to cope with (2), we used 41 clones for each of the 20 initial orbits that were assigned different values of da/dt. The range of these values was determined from the linearized theory of the diurnal Yarkovsky effect (e.g., Vokrouhlický 1999).

Specifically, we used maximum drift speed $da/dt = \pm 1.4\times10^{-4}$ AU Myr^{-1} for a diameter $D = 2$ km object in the Emilkowalski and 1992 YC2 clusters ($a \approx 2.6$ AU), and $da/dt = \pm 1.7\times10^{-4}$ AU Myr^{-1} for a $D = 2$ km object in the Lucascavin cluster ($a \approx 2.28$ AU). These values were calculated

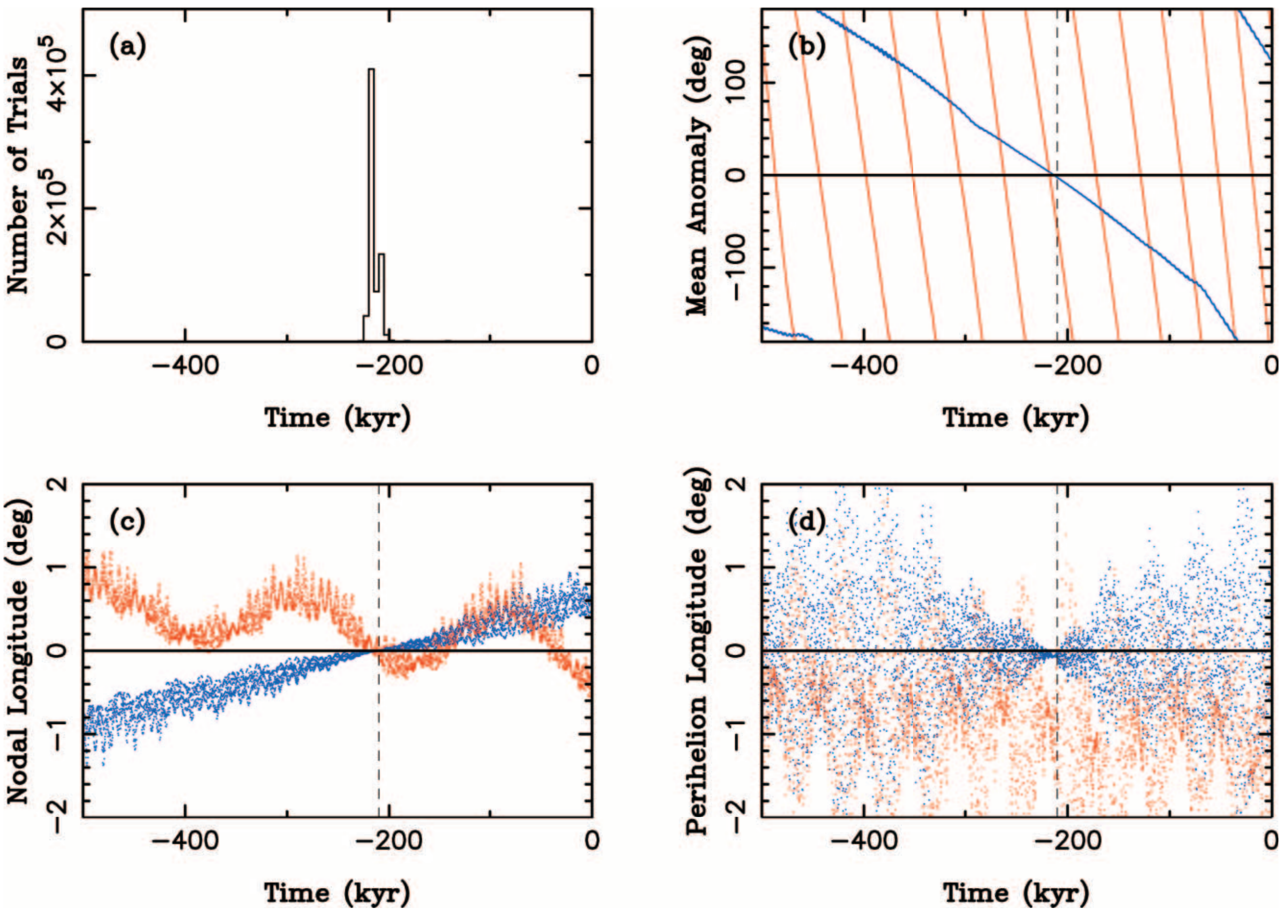

Fig. 2.—Emilkowalski cluster. Panel *a* shows the distribution of $t_{\rm age}$ that meets our criteria of convergence. According to this result, the Emilkowalski cluster is 220 ± 30 kyr old. The other three panels show the past evolution of orbital elements for the best-converging solution: 126761 2002 DW10 (*red*) and 2005 WU178 (*blue*). Values relative to asteroid 14627 Emilkowalski are shown. Panels *c* and *d* demonstrate the convergence of Ω and ϖ, respectively. The dispersion of these angles at $t \approx -210$ kyr, marked by the dashed vertical line, corresponds to speed $\Delta V \approx 0.5$ m s^{-1}, which is a value comparable to ΔV inferred from distributions of a_P. The long-term variations of the nodal longitude for 126761 2002 DW10 suggests an effect of yet-to-be-identified resonance. Panel *b* shows the convergence of mean anomalies that occurs at $t \approx -215$ kyr, i.e., about 5 kyr earlier than the convergence of Ω and ϖ. This discrepancy is small compared to the range of $t_{\rm age}$ allowed by different trials.

assuming a combination of thermal parameters that maximize the Yarkovsky force. The range of actual da/dt is likely to be smaller than the one we conservatively adopted here. We discuss the effect of these different assumptions on $t_{\rm age}$ in § 4.

Assuming random orientations of spin axes of member asteroids, drifts da/dt of individual clones were drawn from a uniform distribution between the maximum negative and maximum positive drift speeds listed above. In total, we produced 840 alternative orbit histories for each known asteroid that differ in the starting orbit and magnitude of Yarkovsky thermal drag.

For orbit integrations, we used the numerical code known as SWIFT-MVS (Levison & Duncan 1994), which we modified to apply the required da/dt for each body. The SWIFT-MVS code is a symplectic, state-of-the-art N-body code based on the Wisdom-Holman map (Wisdom & Holman 1991). We tracked all orbits of cluster members from MJD 2,453,700.5 for 2 Myr backward in time.

To determine $t_{\rm age}$, we searched for the minimum of function $\Delta V(t)$, defined as

$$\Delta V(t) = na\sqrt{k_1(\sin i\,\Delta\Omega)^2 + k_2(e\Delta\varpi)^2}, \qquad (7)$$

where $na \approx 20$ km s^{-1} and $\Delta\Omega$ and $\Delta\varpi$ are dispersions of angles at time t as determined from the numerical integration. We defined these latter quantities in the following way. For example, $(\Delta\Omega)^2 = \sum_{ij}(\delta\Omega_{ij})^2/N_{\rm pairs}$, where $\delta\Omega_{ij}$ are the differences between values of Ω for the ith and jth orbits and $N_{\rm pairs}$ is the total number of pairs.

The values of coefficients k_1 and k_2 can be set arbitrarily. The first term in equation (7) represents the contribution of the off-orbit velocity component, and the second term in equation (7) represents the contribution of the in-orbit velocity components. Therefore, with $k_1 = k_2 = 1$ the second term would have twice the weight of as the first one if fragments were ejected from the site of breakup with near-isotropic velocities. To compensate for this factor, we take $k_1 = 1$ and $k_2 = 1/2$. We verified that the results do not sensitively depend on the choice of k_1 and k_2 as long as these values are comparable.

To apply equation (7), we randomly selected three recorded orbital histories produced in our orbital integrations, each corresponding to one used member asteroid, and determined $t_{\rm age}$ for this trial from two criteria: we required that (1) the dispersion in ϖ and Ω at $t_{\rm age}$ correspond to $\Delta V < V_{\rm max}$, and (2) that the dispersion of M at $t_{\rm age}$, ΔM, be $<M_{\rm max}$.

The range of plausible $t_{\rm age}$ values was determined from 10^7 trials, which is about 2% of all possible clone combinations (840^3). Our tests showed that 10^7 random trials capture satisfactorily the distribution of $t_{\rm age}$. This result stems from the fact that the effects of orbit uncertainties for all asteroids in the three clusters are

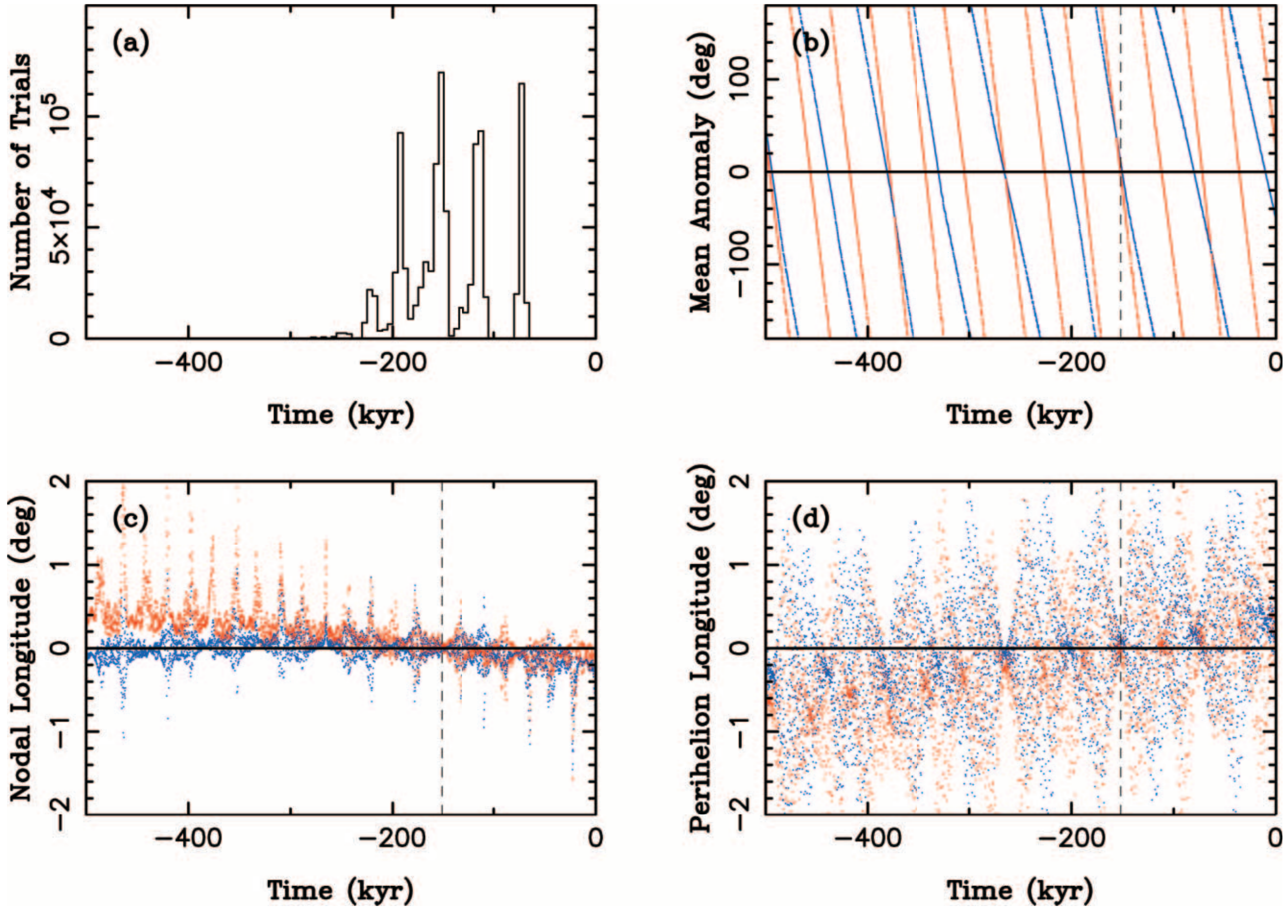

Fig. 3.—Same as Fig. 2, but for the 1992 YC2 cluster. The orbit angles of 2000 UV80 (*red*) and 2005 TT99 (*blue*) in panels *b*, *c*, and *d* are referenced to those of 16598 1992 YC2. The best-fit solution shown here has a nearly perfect convergence of all orbit angles at $t \approx -152$ kyr (*dashed vertical line*). The distribution of $t_{\rm age}$ in panel *a* shows that the 1992 YC2 cluster is 50–250 kyr old. The peaks in the distribution with about 40 kyr spacing in time correspond to the $t_{\rm age}$ at which the mean anomalies can frequently converge.

relatively small (except for 2003 VM9; Table 1). Therefore, trials with different orbit clones (and the same "yarko" clones) show only a small variation of $t_{\rm age}$. A larger spread of $t_{\rm age}$ is produced by different "yarko" clones.

We experimented with the value of $V_{\rm max}$. The distribution of a_P, e_P, and i_P of cluster members suggests ejection speeds of $\sim$1 m s^{-1}. Therefore, we typically used $V_{\rm max} \sim 1$ m s^{-1}. For each of the analyzed clusters, we found many solutions for $t_{\rm age}$ with $V_{\rm max} = 1$ m s^{-1} (or even smaller values of $V_{\rm max}$). For the final fits, however, we decided to use $V_{\rm max} = 2$ m s^{-1}. The distributions of $t_{\rm age}$ with this larger $V_{\rm max}$ are slightly broader and represent the true uncertainty in the age determination more conservatively. In this aspect, the results described below do not overestimate the precision of $t_{\rm age}$. We use $M_{\rm max}$ between 1° and 10°. A more ideal convergence of M is difficult to achieve in some cases due to the limited number of orbital paths and trials used here. We discuss this issue in more detail in § 4.

4. RESULTS

Our results show that all new clusters formed within the past 1 Myr. Older ages can be excluded because secular angles Ω and ϖ of cluster members start to gradually diverge and typically become more dispersed for $t > 1$ Myr ago than they are today. Similarly, very young ages are also unlikely, because the values of M of asteroids in the clusters are widely distributed today and need at least $\sim$50 kyr to converge. We describe the results for the three clusters below.

4.1. *Emilkowalski Cluster*

The Emilkowalski cluster is a group of three asteroids: 14627 Emilkowalski, 126761 2002 DW10, and 2005 WU178 (Tables 1 and 2). All these asteroids were observed at multiple oppositions. The orbit uncertainties in a are 7.5×10^{-8}, 2.8×10^{-7}, and 2.9×10^{-6} AU, respectively, more than 2 orders of magnitude smaller than the semimajor axis spread of the cluster.

The tight clustering of a_P within $\approx 2\times10^{-4}$ AU indicates velocity perturbations $\delta V \approx 1$ m s^{-1} relative to 14627 Emilkowalski. These values are slightly smaller than the escape speed from a 5 km diameter asteroid ($\approx$2.5 m s^{-1}). Spreads in Ω and ϖ at the present epoch correspond to speeds of $\approx$60 m s^{-1}, so one needs 60-fold tighter distributions of Ω and ϖ at $t_{\rm age}$. The Lyapunov time of 14627 Emilkowalski is 750 kyr, which allows us to propagate orbits backward in time for $\sim$1 Myr without much uncertainty due to chaos.

We used $V_{\rm max} = 2$ m s^{-1}. This cutoff is about 2 times larger than δV estimated from the dispersion in a_P. This choice of $V_{\rm max}$ may be a good compromise. A much larger cutoff would be unrealistic. A smaller cutoff would restrict $t_{\rm age}$ to a small range of values that might be too restrictive due to the limited number of orbit histories and trials used here.

We found that the Emilkowalski cluster is 220 ± 30 kyr old (Fig. 2). The convergence of Ω and ϖ near $t_{\rm age} \approx 220$ kyr is almost ideal. More clones and more trials would be necessary to achieve a better convergence of M at $t_{\rm age}$.

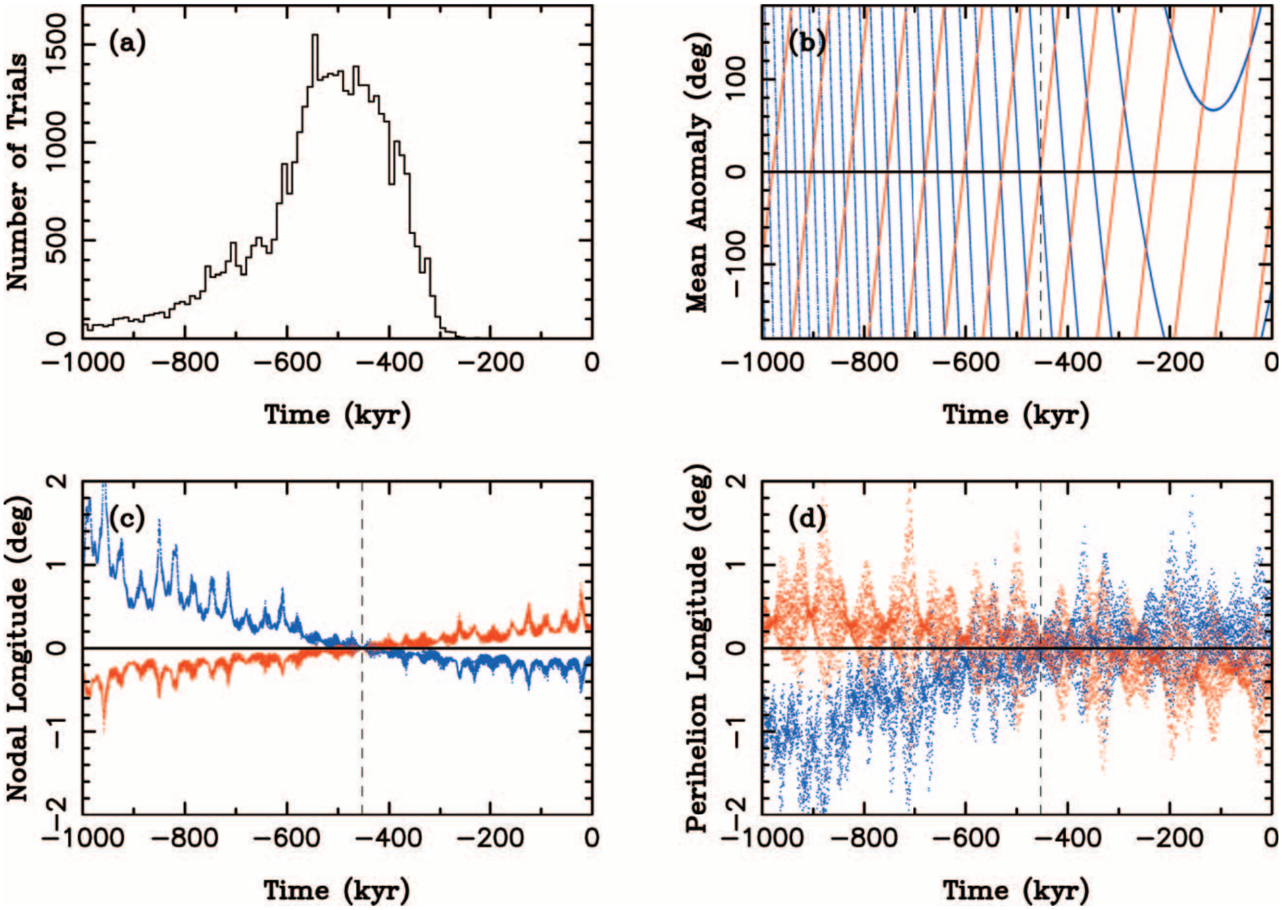

Fig. 4.—Same as Fig. 2, but for the Lucascavin cluster. The orbits of 2003 VM9 (*red*) and 2004 XL40 (*blue*) are referred to that of 21509 Lucascavin. The irregular differential rotation of M is produced by changes in a due to the Yarkovsky effect. The same effect also produces variable differential rotation of secular angles. The results in panel a suggest that the Lucascavin cluster is most likely 300–800 kyr old.

4.2. *1992 YC2 Cluster*

The 1992 YC2 cluster is a group of three asteroids: 16598 1992 YC2, 2000 UV80, and 2005 TT99. Both 2000 UV80 and 2005 TT99 were observed at three oppositions. Their orbit uncertainties in a are $\sim 10^{-6}$ AU (Table 1).

All known member asteroids of the 1992 YC2 cluster are small, having $D \approx 1$–4 km. The effect of Yarkovsky drag on these small bodies is relatively large. In addition, 16598 1992 YC2 has Lyapunov time $\approx$120 kyr, so the effects of chaos may also be important on >100 kyr timescales. These effects make it difficult to determine the age of the 1992 YC2 cluster unambiguously.

The spread of the cluster in a_P is $\approx 3 \times 10^{-4}$ AU, indicating ejection speeds of $\approx$1 m s^{-1} (Table 2). For a comparison, the spreads of Ω and ϖ at the present epoch correspond to speeds of $\approx$7 m s^{-1}. Therefore, we need a sevenfold improvement at $t_{\rm age}$. This indicates that the 1992 YC2 cluster may be younger than the Emilkowalski cluster (for which we required $\approx$60-fold improvement).

We found that the 1992 YC2 cluster is most probably 50–250 kyr old (Fig. 3). The conditions $\delta V < 2$ m s^{-1} and $M < 10°$ set several most probable $t_{\rm age}$ values within this interval that are separated by about 40 kyr (Fig. 3). These values are $t_{\rm age} \approx 72$, 115, 152, 195, and 220 kyr.

When we set a more restrictive condition on M, the range of $t_{\rm age}$ becomes more limited. For example, with $\Delta M < 1°$ we find that 160 kyr $< t_{\rm age} <$ 220 kyr. We are not sure whether this implies that $t_{\rm age} \sim 200$ kyr or whether we sampled an insufficient number of orbit histories. When the range of drift rates in a is limited to half of the nominal one, three values of $t_{\rm age}$ become most likely: 72, 152, and 195 kyr. The solution with $t_{\rm age} = 72$ kyr persists even if da/dt is set to 0.

Our additional tests have shown that it is difficult to constrain the age of the 1992 YC2 cluster better. We anticipate that additional members of the 1992 YC2 cluster that will be identified in the future will help to determine $t_{\rm age}$ with better precision. It would also be important to obtain some information about the spin-axis orientations for the member asteroids (e.g., from light-curve observations) because that would help us to narrow the range and direction of da/dt of individual members due to the Yarkovsky effect.

4.3. *Lucascavin Cluster*

The Lucascavin cluster is a group of three asteroids: 21509 Lucascavin, 2003 VM9, and 2004 XL40. Asteroid 2003 VM9 was observed only at two oppositions and has a relatively large orbit uncertainty (Table 1). Asteroid 21509 Lucascavin has Lyapunov time $\approx$250 kyr. Therefore, the effects of chaos are slightly weaker for the Lucascavin cluster than for the 1992 YC2 cluster. Similarly to the 1992 YC2 cluster, however, all members of the Lucascavin cluster are small, 1–3 km in diameter, and might have significantly drifted in a due to the Yarkovsky effect. The drift magnitude in a over 1 Myr of a kilometer-sized object with $a \approx 2.3$ AU is comparable to the current uncertainty in a of 2003 VM9.

The spread of the Lucascavin cluster in a_P is 5×10^{-5} AU, indicating extremely small ejection speeds, $\approx$0.3 m s^{-1}. Unfortunately, these values are comparable to the current uncertainty in a of 2003 VM9 (Table 1) and the inherited uncertainty in a_P (Table 2). Observations of 2003 VM9 would be very helpful in diminishing the orbit uncertainty and getting a better constraint on the magnitude of the ejection velocity.

To determine the age of the Lucascavin cluster, we used $V_{\rm max} = 2$ m s^{-1} and $\Delta M < 10°$. We found that the Lucascavin cluster is 300–800 kyr old (Fig. 4). Earlier formation dates than 800 kyr ago cannot be strictly excluded, because the distribution of $t_{\rm age}$ in Figure 4 has a tail that extends beyond 800 kyr ago.

This large uncertainty in $t_{\rm age}$ stems from the fact that these asteroids are small and minimally separated in a_P. Therefore, the convergence of ϖ, Ω, and M can be obtained with many combinations of da/dt values and for a large range of $t_{\rm age}$. Detection of new asteroid members of this cluster and improved orbital determinations will help to define $t_{\rm age}$ with better precision. Most solutions require $da/dt > 0$ for 2003 VM9 and $da/dt < 0$ for 2004 XL40, suggesting that these asteroids may have prograde and retrograde spins, respectively.

5. DISCUSSION

We found that the Emilkowalski, 1992 YC2, and Lucascavin clusters are 220 $\pm$ 30, 50–250, and 300–800 kyr old, respectively. The large uncertainty of $t_{\rm age}$ for the 1992 YC2 and Lucascavin clusters is mainly produced by the uncertainty of da/dt due to the Yarkovsky effect. The age determination for these clusters will be significantly improved when (1) additional member asteroids are identified, (2) orbit uncertainties are reduced, and/or (3) we have some information about the spin states of the currently known member asteroids so that we can restrict the range of da/dt. Out of these possibilities, factor 1 would be especially helpful. We believe that a careful analysis of the orbit history of a reliable fourth member of any of these clusters would help to significantly reduce the uncertainty of $t_{\rm age}$.

A dedicated observational search for the fourth member in these clusters is difficult. Perhaps the best strategy would be to look toward the pericenter of the orbits listed in Table 1 with a wide-field camera that is able to detect kilometer-sized asteroids that come to within a distance of 1–1.3 AU from Earth. The general-purpose automated programs (e.g., Stokes et al. 2002) probably have the best chances to succeed by scanning the sky near the ecliptic. Future programs, such as Pan-STARRS, will significantly help these efforts. In retrospect, the work presented in this paper may illustrate the kind of scientific research that will be possible in a decade or so with the Pan-STARRS data.

The Emilkowalski, 1992 YC2, and Lucascavin clusters are conveniently placed in the inner part of the main belt, making it an attractive target for observations. Some of these asteroids might have taxonomic type within the asteroidal S complex (Bus et al. 2002) and might be compositionally related to the ordinary chondrite meteorites (e.g., Gaffey et al. 1993; Binzel et al. 1993). It is generally difficult, however, to determine the precise mineralogical composition of asteroids because optical properties of asteroid surfaces are altered on long time spans by solar wind sputtering and micrometeorite impacts. These processes, known as space weathering (e.g., Clark et al. 2002; Chapman 2004), can mask signatures of different minerals on an old asteroid surface. The asteroids identified here probably suffered minimal effects of space weathering, due to their very young ages. Spectroscopic observations of these objects may help us to determine the rate of spectral alterations by space weathering (extending the analysis of Jedicke et al. [2004] and Nesvorný et al. [2005] to ages $t_{\rm age} < 1$ Myr), the mineralogical composition of asteroids, and their relation to ordinary chondrite meteorites.

The distribution of orbits and sizes of several known fragments in each cluster can be used to deduce the impact parameters. We estimate that the disrupted parent bodies were $\sim$7–15 km in size, where smaller values in this range apply to the 1992 YC2 and Lucascavin clusters and where larger values apply to the Datura cluster. The deduced values of the dispersion velocity, $\approx$1–3 m s^{-1}, roughly correspond to the escape speed from parent bodies of this size. This relation has been noticed before (e.g., Nesvorný et al. 2002a). Here, we verify that it also applies to disruptions of $\sim$10 km diameter asteroids.

Conceivably, a large part of the parent body could have been ejected to space as fragments ranging in size down to micron-sized dust particles. It is therefore possible that the clusters discussed here are sources of some of the material in the circumsolar (zodiacal) dust cloud. Depending on the exact value of $t_{\rm age}$ and location in the main belt, they may be related to some dust trails (see, e.g., Sykes 1986; Sykes & Walker 1992; Nesvorný et al. 2006c) or to dust bands (see, e.g., Dermott et al. 1984; Sykes & Greenberg 1986; Nesvorný et al. 2006b). For example, Sykes (1986, 1988) has identified a number of faint dust bands in observations by the *Infrared Astronomical Satellite* that have never been successfully linked to their sources in the main belt (except those discussed in Nesvorný et al. 2003). Based on the Emilkowalski cluster's inclination ($\approx$17.°7), we speculate that this cluster may be the source for the 17° dust band (Sykes 1986, 1988). Similarly, the Datura cluster may be the source for the E/F dust band pair with 5° inclination. A detailed modeling will be required to probe these possible links.

We estimate that micron-sized particles produced in the clusters reported here migrate by radiation effects from their source locations to 1 AU in only $\approx$3–5 kyr. Therefore, a wave of micron-sized particles might have reached Earth only a few thousand years after the breakup events. Signatures of these events may be found by analyzing tracers of extraterrestrial dust in deep ocean sediments and Antarctic ice cores (e.g., Petit et al. 1999; Brook et al. 2000; Augustin 2004; Farley et al. 2006).

Note that all of our new asteroid clusters have formation ages $\leq$1 Myr for which the ice-drilling programs provide very detailed information about the overall content of dust and the abundance of tracers of extraterrestrial dust, such as ^{3}He in different layers. This detailed analysis of Antarctic ice cores opens potentially unforeseen possibilities to study the dust deposition from specific extraterrestrial sources and to determine whether (or not) it may be correlated with climatic indicators. A fine resolution may be needed (e.g., Brook et al. 2000) to accomplish this, because breakups of $\sim$10 km asteroids should produce only very brief episodes of increased accretion rate of the extraterrestrial material on Earth (lasting only a few kiloyears to a few tens of kiloyears).

The number of recent breakups is also an important constraint on the current collisional activity in the main belt. Collisional models like those developed by Bottke et al. (2005a, 2005b) predict that 10 km diameter asteroids disrupt somewhere in the main belt every $\sim$100 kyr. This rate of catastrophic disruptions is comparable to the one determined here. Specifically, we identified four such breakups occurring in the past $\sim$600 kyr.

This research was supported by NASA's Planetary Geology and Geophysics Program (grant NAG-513038). The work of D. V. was also partly supported by a grant of the Czech Grant

Agency (205/05/2737). We thank Tim Spahr for bringing to our attention the prediscovery observations of 2004 XL40 that allowed us to improve the orbit determination for this object.

APPENDIX

STATISTICAL SIGNIFICANCE OF THE NEW FAMILIES

We performed the following tests to demonstrate the high level of statistical significance of the new families. First, we generated 3.2×10^5 different synthetic asteroid belts. Each synthetic belt was represented by $N = 316{,}599$ orbits that were drawn from uniformly random distributions of elements. The same number of orbits, N, was included in the AstOrb catalog from 2006 March 1 that we used in § 2. We used artificial ranges of the orbit elements that roughly correspond to the real asteroid belt: 2 AU $< a <$ 3.3 AU, $e < 0.2$, $i < 15°$, and values of ω and Ω between 0° and 360°.

In the next step, we applied the five-dimensional HCM algorithm to this synthetic data. Using equation (1), the Emilkowalski, 1992 YC2, and Lucascavin clusters were identified as five-dimensional clusters of three members each with $d = 20-40$ m s^{-1}. No additional members were identified in any of these clusters with 40 m s^{-1} $< d <$ 200 m s^{-1}. Therefore, we conservatively set $d = 50$ m s^{-1} and used the HCM in an attempt to identify three-orbit clusters in any of the 3.2×10^5 synthetic asteroid belts. These attempts failed. The negative result shows that the probability that any of the three real clusters identified in this work is a random fluctuation is less than $1/3.2\times10^5 \sim 3\times10^{-6}$.

The Lucascavin cluster is located in the Flora family, where the density of orbits in the five-dimensional space of proper elements is larger than in other parts of the asteroid belt. To test the possibility that this larger density may facilitate occurrences of random three-body clusters, we repeated the above-described test for the Flora family. In total, we generated 5×10^5 different synthetic Flora families with $N = 21{,}720$ orbits, 2 AU $< a <$ 2.33 AU, $0.07 < e < 0.21$, $3° < i < 7°$, and any values of ω and Ω, and applied HCM to these distributions. Once again, $d = 50$ m s^{-1} yielded no clusters with three or more members. We therefore conclude that the Lucascavin cluster cannot be a by-product of random fluctuations of asteroid orbit density in the Flora family region.

The negative results of our tests can be easily understood from the following probability estimate. Let us divide the asteroid belt into M equal-sized cells whose total volume in five-dimensional space of orbital elements is comparable to that of the main asteroid belt. Let the volume of each cell be comparable to the volume occupied in five-dimensional space by members of our three-body cluster. By comparing the total five-dimensional volume of the asteroid belt to the size of clusters (Table 1), we conservatively estimate that $M \sim 10^{12}$. The probability p_3 that three out of $N \approx 3\times10^5$ orbits (i.e., the total number of orbits in the asteroid belt) fall into the same cell (i.e., produce a tight three-body cluster) is

$$p_3 = \binom{N}{3}\frac{1}{M^2}. \tag{A1}$$

(This equation gives p_3 in the relevant case where $M \gg N$.) Therefore, $p_3 \sim 10^{-8}$. This estimate explains the results of our tests described above, because more than 10^8 trials would be needed to get one positive identification of a random three-orbit cluster. We conclude that the probability that the newly identified clusters are random fluctuations of orbit density is significantly less than one part in a million.

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