Icarus **150**, 78–93 (2001)
doi:10.1006/icar.2000.6552, available online at http://www.idealibrary.com on IDEAL®

Yarkovsky-Driven Leakage of Koronis Family Members

I. The Case of 2953 Vysheslavia

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Received May 24, 2000; revised September 28, 2000

The orbit of the asteroid 2953 Vysheslavia is currently locked in a tiny chaotic zone very close to the 5 : 2 mean motion jovian resonance. Its dynamical lifetime is estimated to be of the order of only about 10 Myr. Since Vysheslavia is a member of the Koronis family, such a short dynamical lifetime opens a variety of interesting questions concerning its origin and evolution. A. Milani and P. Farinella (1995, *Icarus* 115, 209–212) considered a number of plausible scenarios and suggested that most probably Vysheslavia is an outcome of a recent secondary fragmentation event in the family. Here we propose that Vysheslavia might have been placed on its peculiar orbit by a slow inward drift of the semimajor axis due to the Yarkovsky effect. Numerical simulations confirm that such evolutionary processes can take 100–500 Myr, a period comparable to but still shorter than the probable age of the family (on the order of a Gyr), depending on the thermal properties of Vysheslavia's surface, the orientation of its spin axis, and its size. We have also integrated orbits of the asteroids 7340 (1991 UA$_2$) and 1993 FR$_{58}$, located very close to but outside the chaotic zone that triggers Vysheslavia's orbit instability, and we show that the orbits of these asteroids may also slowly evolve toward the chaotic zone. Such an erosion of the asteroid families, caused by a slow leakage to the nearby powerful resonances, could be fairly common in the main asteroid belt. © 2001 Academic Press

Key Words: **asteroids; Yarkovsky effect; chaotic motion; dynamical lifetime.**

1. INTRODUCTION

In a broad sense, the aim of this paper is to contribute to the understanding of the evolution and fate of the asteroid families. Starting from birth, caused by a large collisional event in the main asteroid belt, a typical family suffers various kinds of "erosion" processes. The most obvious causes of such effacement effects are collisional grinding (e.g., Marzari *et al.* 1995, 1999) and chaotic diffusion (see Milani and Farinella 1994, who studied the case of the Veritas family; also Milani *et al.* 1997, Nesvorný and Morbidelli 1998, Knežević 1999), but there are others. An interesting mechanism, recently proposed by Farinella and Vokrouhlický (1999), is the semimajor axis diffusion of the Yarkovsky effect, which spreads out otherwise closely packed family members. This nongravitational phenomenon, due to the recoil force of the thermal radiation of an asteroid (e.g., Rubincam 1995, 1998; Farinella *et al.* 1998), alters secularly the semimajor axes of small asteroids up to a few kilometers. Farinella and Vokrouhlický (1999) observed that the small members of the Astrid family exhibit a larger scatter of the semimajor axes than the big members. Since the Yarkovsky effect is size dependent (larger mobility for small bodies), they concluded that the Astrid data support the idea of the Yarkovsky spreading of the families. In this paper, we consider another case where the Yarkovsky mobility of small family members might result in observable consequences.

[1] In the course of preparing this paper, Paolo Farinella passed away on March 25, 2000. He was at the peak of his scientific productivity and was a driving force in shaping the scenario presented in this paper. This concept was only one of the many innovative ideas of Paolo, whose scientific style combined creativity with deep physical insight and mathematical rigor. In publishing this paper, we honor Paolo's memory.

0019-1035/01 $35.00

The key argument for the present analysis derives from a particularly "lucky" position of the asteroid 2953 Vysheslavia, a small member of the Koronis family (we estimate its size to be about 15 km from the absolute magnitude and a typical geometric albedo of 0.2 observed in the family). Milani and Farinella (1995) noticed that Vysheslavia is located in a tiny chaotic zone (about 10^{-3} AU wide) very close to the border of the strong 5 : 2 mean motion resonance with Jupiter, and that this peculiar location makes the orbit of Vysheslavia rather unstable with respect to falling into the 5 : 2 resonance. Given the very short lifetimes of bodies residing inside this resonance (Gladman *et al.* 1997), and the probability of a Vysheslavia-like orbit transfer to this resonance (Milani and Farinella 1995), the expected dynamical lifetime of this body is estimated to be on the order of about 10 Myr. This is an extremely short timescale, since Vysheslavia is a member of the Koronis family, whose age is, according to the available evidence, presumably much longer, of the order of 1–2 Gyr (Chapman *et al.* 1996, Greenberg *et al.* 1996, Giblin *et al.* 1998). Milani and Farinella (1995), and later Knežević *et al.* (1997), pointed out this apparent contradiction and suggested several possible solutions, but were unable to decisively discriminate between them. In fact, all of the plausible possibilities seem to be related to the "family-aging processes" mentioned above, while it appears that we can safely rule out the possibility that Vysheslavia is an interloper genetically unrelated to the Koronis family. Both the spectroscopic analysis indicating that Vysheslavia is an ordinary S-type asteroid, like other members of the family (Bus 1999), and the interloper analysis of the Koronis family (Migliorini *et al.* 1995) predicting very few interlopers of the size of Vysheslavia make this possibility very unlikely.

According to Milani and Farinella (1995) the present location and lifetime of Vysheslavia can be best explained by assuming that it is an outcome of a secondary fragmentation of a large (possibly originally the largest) member of the Koronis family that occurred several tens of Myr ago. Such a catastrophic event might have placed Vysheslavia into its present orbit, but given the estimated sizes of Vysheslavia and its hypothetical parent body (30–70 km), there is only about $\leq$5% probability that such a collision occurred in the past 100 Myr. Moreover, if this was the case, Vysheslavia's spectrum should be less altered by the space weathering processes and should differ slightly from the typical spectrum of the older Koronis members; also related to the assumed recent collisional origin of Vysheslavia is its possible state of nonprincipal axis rotation that might produce a complex photometric curve (Harris 1994). Vysheslavia was observed for two nights only very recently by L. Šarounová (personal communication, 2000). Apart from the fact that the amplitude of the lightcurve is small (only 0.2 magnitude), which may mean that Vysheslavia is a moderately elongated object, no conclusions can be drawn yet. On the other hand, Vysheslavia might have been placed into its present orbit by a nondisruptive collision against the asteroid itself or by a close encounter with Ceres, but Milani and Farinella (1995) again found these scenarios to be highly unlikely (encounters with Ceres, for example, typically result in a total shift of about 10^{-3} AU in the semimajor axis over the age of the Solar System, so that there is less than 1% probability that a close enough encounter occurred in the last 100 Myr).

In this paper we investigate the possible role of the third family-aging process mentioned above—the semimajor axes spreading by the Yarkovsky effect—in relation to the Vysheslavia case (some preliminary results of this analysis have already been presented in Vokrouhlický *et al.* 1999). In particular, we would like to understand whether Vysheslavia, being originally located further away from the 5 : 2 resonance, could have been placed into its present orbit by a Yarkovsky drift in the semimajor axis. A typical timescale for such a process would be of main concern here. Moreover, to better describe the mechanism of delivery and capture by the tiny chaotic zone in which Vysheslavia is locked, we present also the integrations of the orbits of two other Koronis family asteroids, 7340 (1991 UA_2) and 1993 FR_{58}, that are located in the vicinity of this region (see Knežević *et al.* 1997). We show that for these two bodies, once they enter into the chaotic zone, the subsequent orbital evolution cannot be predicted in a deterministic way; their orbits may remain locked in the region for a long time (of the order of tens of millions of years), or they may quickly cross it, drifting towards the 5 : 2 resonance with Jupiter. We estimate the probability of both outcomes from a number of numerical simulations (each of them starting from slightly different initial conditions), and we propose that the evolution we observed for the two small asteroids might have happened in the past to Vysheslavia itself.

2. NUMERICAL SIMULATIONS

We have implemented both variants of the Yarkovsky effect (diurnal and seasonal) in a number of different numerical integrators. For a precise formulation of the corresponding accelerations, and the way in which we have incorporated them into the codes, see Brož (1999) or Brož, Vokrouhlický, and Farinella in preparation. Long-term evolution studies usually profit from (or even require) a fast integrator based on a mapping method. For that reason, we have implemented the Yarkovsky effect in the `swiftrmvs3` integrator (e.g., Levison and Duncan 1994; in what follows, we use the designation `swiftrmvs3` or `swift` even for our modified version of this integrator). It should be noted that due to weak dissipation through the Yarkovsky effect the symplecticity of the integrator is violated (as it is, in fact, anyway because of handling the close encounters with planets). We may mention papers by Malhotra (1994), Codeiro *et al.* (1997), and Mikkola (1998), in which the authors consider a similar possibility of including nonconservative phenomena in the originally symplectic integrators (our method corresponds to that of Codeiro *et al.* (1997) and Mikkola (1998)). We have carefully tested to ensure that the integrator reproduces analytical results (when available) of the semimajor axis secular evolution. As a check of the results with the modified `swiftrmvs3` integrator, we also performed some simulations using other integrators, in particular (i) the Bulirsh–Stoer integrator (Press *et al.* 1994) and (ii) the ORBIT9 integrator kindly provided by A. Milani. The latter was used by Milani and Farinella (1995) and Knežević *et al.* (1997) in the previous integrations of Vysheslavia.

Since Vysheslavia is located at the outer edge of the 5 : 2 jovian mean motion resonance, we typically include in our integrations the outer planets only (except for Pluto). Their initial state vectors and masses were taken from the JPL DE403 ephemerides, while for the integrated asteroids (2953 Vysheslavia, 7340 (1991 UA_2), and 1993 FR_{58}) the initial osculating elements were taken from Bowell's catalogue (Bowell *et al.* 1994). All the initial data correspond to the epoch of JD2451400.5. Whenever only four outer planets were used in our simulations, the initial data have been corrected for the indirect effect of the inner planets (the so-called "barycentric correction"; see Milani and Knežević 1992, 1994). We typically used a timestep of 36 days, but in several cases where precision of the integration was of particular concern we also used a shorter step.

To check the robustness of conclusions based on the simulations that include the giant planets only, we have performed two separate series of tests. First, by propagating the orbits back in time, we verified that the state vectors of the integrated asteroids match the initial conditions used by Milani and Farinella (1995) and Knežević *et al.* (1997) if referred to the epochs given by these authors. Next, we have also included perturbations of the inner Solar System planets in our integrations. As expected, their influence is small at Vysheslavia's distance, and the main conclusions derived from the integrations with giant planets only do not change. The single major difference in the integrations with and without the inner planets is the shorter lifetime of bodies that eventually terminated in the 5 : 2 resonance in the former case. This observation concurs with the conclusion of Gladman *et al.* (1997) that the 5 : 2 residing bodies are removed principally by close encounters with the inner planets.

Realizing that the "anomalous" fluctuation of Vysheslavia's semimajor axis due to a close encounter with a sufficiently massive perturber could have placed it onto its peculiar orbit (see Section 1), we have performed another series of tests that included gravitational effects of the asteroids Ceres and Pallas. The results of these tests seem to confirm the negative conclusion by Milani and Farinella (1995), notably that the scenario assuming Vysheslavia's orbit to be strongly affected by Ceres is unlikely.

In order to check the proposed scenario, we performed simulations with and without the Yarkovsky effect; the latter essentially coincide with the previous work of Milani and Farinella (1995) and Knežević *et al.* (1997). Let us emphasize that by doing so we actually do not compare two equally plausible approaches. Rather we compare a more complete, and thus more realistic, model (with the Yarkovsky effect included) with a less complete and less realistic model (the purely gravitational model, without the Yarkovsky effect).

There are two major unknown parameters related to Yarkovsky effect modeling: (i) the thermal parameters (the thermal conductivity) of the Vysheslavia surface and (ii) its rotation state (orientation of the spin axis and rotation period). Being aware of this caveat we tried to span the realistic values of these parameters. In particular, we performed simulations with different spin axis orientations, using two different values of the surface conductivity (one at the low end of the plausible range, and the other at the high end). We draw our conclusions only after considering all the results obtained with the full range of values for these quantities. In any case, we believe that taking into account the Yarkovsky effect, even with the unconstrained parameters (and spanning their physically reasonable values), is more realistic than not taking the Yarkovsky effect into account at all.

In order to properly model the Yarkovsky effect, we use the formulation developed by Vokrouhlický *et al.* (2000) and Brož, Vokrouhlický, and Farinella (in preparation). Since both variants—the diurnal and the seasonal (see, e.g., Farinella *et al.* 1998; Vokrouhlický 1999)—of the Yarkovsky effect have been included in our code, we can span the entire range of possible values of the surface thermal conductivity. Remember that the Yarkovsky acceleration in the low conductivity regime is dominated by the diurnal variant, while the high conductivity regime is dominated by the seasonal variant (see also Table IV). The ISO observations reported by Müller and Lagerros (1999) suggest that a comparably higher weight should be given to the case of low surface conductivity (since nearly all asteroids observed by ISO do indicate a very low value for this parameter). Such a conclusion is supported by further observations (such as that of Phobos) indicating that surfaces of small bodies in the solar system might have a regolith (insulating) cover and/or exhibit a high degree of porosity (induced probably by numerous microimpacts). We can thus assume $K = 0.0015$ W/m/K to be the most probable value of the Vysheslavia surface conductivity (this is the lunar regolith value, Rubincam 1995, that also roughly corresponds to the measurements reported by Müller and Lagerros 1999). Nevertheless, we performed a series of test integrations with considerably higher surface conductivity $K = 1$ W/m/K, which corresponds to a solid rock surface (Yomogida and Matsui 1983). In passing we mention that the linearized formulation of the Yarkovsky effect may not be appropriate for very high eccentricity orbits (see, e.g., Vokrouhlický and Farinella 1998a, 1999, Spitale and Greenberg 2000). This drawback is, however, not very important for our conclusions, which are based on transport of asteroids to the 5 : 2 resonance on low-eccentricity orbits (typically $e \leq 0.2$ in our simulations).

The strength of the Yarkovsky effect depends sensitively on the orientation of the spin axis and the rotation period of the body (Rubincam 1995, 1998, Farinella *et al.* 1998). Unfortunately, we currently have no explicit information about these parameters for either Vysheslavia or 7340 (1991 UA_2) and 1993 FR_{58}. We have thus performed a series of simulations with varying assumptions about the orientation of their spin axes. As for the rotation period, we assumed it to be 5 h as a typical value observed in the family

(Binzel *et al.* 1989).[2] Surface geometric albedo was taken to be equal to 0.2 and the thermal emissivity to 0.9. From these values we have derived Vysheslavia's radius of approximately 7.5 km. Similarly, we got 2.8 km for the 7340 (1991 UA_2) radius and 3 km for the radius of 1993 FR_{58}. Given these values, we may estimate a collisional lifetime of these asteroids to be at least 1 Gyr (even 1.4 Gyr for Vysheslavia). Our longest integrations with the Yarkovsky effect accounted for a span of 0.5 Gyr, that is, significantly less than the collisional lifetime. Yet, we shall demonstrate that the Yarkovsky effect has the capability to shift quite significantly the semimajor axes of these objects.

As mentioned above, Vysheslavia's orbit is currently locked in a chaotic zone. In principle, any individual orbit in such a zone quickly loses its deterministic meaning, with a typical timescale for this to occur characterized by the Lyapounov time. Previous numerical experiments (e.g., Milani *et al.* 1997) demonstrated that a typical Lyapounov time of the strongly chaotic orbits in the main asteroid belt is on the order of 10^3–10^4 years. The Lyapounov time for the Vysheslavia orbit is ≈27 kyr (see, e.g., the `AstDyS` Web page maintained in Pisa by A. Milani), indicating that the degree of chaoticity in Vysheslavia's vicinity is somewhat smaller than that in the most chaotic zones of the main belt (as in the 5 : 2 resonance). However, the indicated Lyapounov time of Vysheslavia is still very short with respect to our integrations, which typically cover a time span that is 10^4 to 2.5×10^4 longer. Most of the simulations presented in this paper thus have a statistical meaning only. To explore the stochastic characteristics of the chaotic zone in which Vysheslavia is located, we used the well-known technique of the "fictitious neighbors" (e.g., Milani and Farinella 1995). However, we introduced these fictitious neighbors not only for Vysheslavia itself, but also for the objects on stable orbits, such as the two above-mentioned asteroids 7340 (1991 UA_2) and 1993 FR_{58}; by integrating their orbits with the Yarkovsky effect accounted for, we expected to be able to observe and follow the evolutionary paths of currently stable bodies drifting toward the chaotic zone.

Two levels of "zooming" were used in these simulations. First, to trace statistically the fate of a given orbit, we introduced very close neighbors, which we refer to as the "close clones" (CC). These are assumed to be so tightly clustered around a given orbit that an averaging over their evolutionary states would express the corresponding statistical uncertainty of the orbit itself. Notice that the close clones cover about a 3σ uncertainty area (in both semimajor axis and eccentricity) resulting from the current orbit determination (see, e.g., the AstDyS page at http://newton.dm.unipi.it/ asteroid/). For the sake of minimizing computational expenses, however, we defined the close clones by slightly modifying the initial semimajor axis and eccentricity only. Table I gives the variations of the close clones' initial semimajor axes and eccentricities with respect to the nominal values for the real asteroid itself. Second, we also introduced somewhat less close neighbors—hereafter referred to as the fictitious neighbors (FN)—whose orbits are supposed to map the Vysheslavia zone on a broader scale. In particular, their semimajor axes spanned the entire chaotic zone (about 0.001 AU wide). Increments of their initial semimajor axes and eccentricities with respect to those of the real asteroids are given in Table II. In Vysheslavia's case, the most distant FN^0_{+3} body is located just outside the chaotic zone, thus representing both (i) a suitable delimiter of the chaotic zone and (ii) a suitable probe to clarify the role of the Yarkovsky perturbations in contrast to the gravitational effects only.

[2] In the final stages of preparation of this paper we learned of photometric observations made by L. Šarounová (2000, personal communication). She observed Vysheslavia on Jan 15/16/17, 2000, at the St. Veran Observatory in France. A preliminary reduction of the lightcurve indicates a rotation period of 6.29 ± 0.05 h, roughly consistent with our assumption. The amplitude of the lightcurve is only about 0.2 magnitude which may indicate only a moderately elongated shape of Vysheslavia. Unfortunately, this single observation cannot reveal any information about the Vysheslavia spin axis orientation.

TABLE I
Orbital Parameters of the Close Clones (CC)[a]

Code	Δa ($\times 10^{-7}$ AU)	Δe ($\times 10^{-7}$)
CC^0_0	0	0
$CC^0_{\pm1}$	±1	0
$CC^0_{\pm2}$	±2	0
$CC^0_{\pm3}$	±3	0
$CC^{\pm1}_0$	0	±5
$CC^{\pm2}_0$	0	±10
$CC^{\pm1}_1$	1	±5
$CC^{\pm1}_{-1}$	−1	±5

[a] The increments refer to the nominal orbital elements of the asteroid.

2.1. *Integrations without the Yarkovsky Effect*

Before considering the influence of the Yarkovsky effects on the orbits of the Koronis family asteroids in question, we performed integrations where these effects have not been included. The purpose of the integrations was twofold: (i) to check our software and integration procedure by comparison with previous results (notably Milani and Farinella 1995, Knežević *et al.*

TABLE II
Orbital Parameters of the Fictitious Neighbors (FN)[a]

Code	Δa ($\times 10^{-4}$ AU)	Δe ($\times 10^{-4}$)
FN^0_0	0	0
$FN^0_{\pm1}$	±5	0
$FN^0_{\pm2}$	±10	0
$FN^0_{\pm3}$	±15	0
$FN^{\pm1}_0$	0	±5
$FN^{\pm2}_0$	0	±10
$FN^{\pm1}_1$	5	±5
$FN^{\pm1}_{-1}$	−5	±5

[a] The increments refer to the nominal orbital elements of the asteroid.

1997) and (ii) to contrast these results with those including the Yarkovsky effect. As far as the first item is concerned, we primarily wanted to verify reliability of the results obtained with the `swift` integrator. This is a very important issue, since we need to perform very long integrations (several hundred Myr) with the Yarkovsky effects included. As mentioned above, the `swift` integrator profits in terms of speed from its mapping structure, but `swift` is also known to be less precise in the short term than the slower standard integrators (especially because if does not correctly reproduce the short-periodic part of the perturbation; e.g., see Michel and Valsecchi 1996). On the other hand, since Vysheslavia is located in a chaotic zone, we cannot infer results of a firmly deterministic value anyway, but we still need to be sure that the results obtained with the `swift` integrator are at least statistically correct (i.e., about the same fraction of orbits reaching the same terminal state within a comparable timespan). The comparison of results obtained with the standard, highly precise integrators over shorter timespans (typically tens of Myr) and those obtained with the `swift` integrator thus serves to assess credibility of the long-term results obtained by means of the `swift` integrator itself.

Figure 1 shows the mean semimajor axes of Vysheslavia and its 14 fictitious neighbors (see Table II) as a function of time obtained by means of the `swiftrmvs3` code. We notice the chaotic zone between 2.8275 AU and 2.829 AU in the semimajor axis that has been found by Milani and Farinella (1995). Vysheslavia and its FNs originally residing in this zone (that is, all except FN^{0}_{+3}) fall into the 5 : 2 resonance after a median time of 12.1 Myr, a value which matches well the results of Milani and Farinella (1995). Table III lists the median time of fall in to the 5 : 2 resonance in all Vysheslavia integrations. Despite the short median dynamical lifetime in the chaotic zone, some of the neighbors may remain captured there for a remarkably long time ($\geq$200 Myr for the FN^{+1}_{+1}; see Fig. 1). The FN^{0}_{+3} body, originally put just above the chaotic zone, seems to have a very stable orbit without any noticeable variations of the mean semimajor axis.

TABLE III

Vysheslavia's Median Lifetime (in Myr) against the Fall into the 5 : 2 Resonance Computed from Integration of Fictitious Neighbors and Close Clones in Various Simulations[a]

	Int	NY	Y($\gamma = 135°$)	Y(γ iso.)
CCs	`swift`	14.1	10.6	13.9
	BS	12.9	10.7	10.3
FNs	`swift`	12.1	2.7	9.6
	BS	6.6	7.7	14.9

[a] Three cases are considered: (i) the gravitational interaction with planets only without the Yarkovsky perturbation (NY), (ii) the Yarkovsky perturbation with the obliquity of all particles set to 135° [Y($\gamma = 135°$)], and (iii) the Yarkovsky perturbation with the obliquity of the particles isotropic [Y(γ iso.)]. Two types of integrators (Int column) are also compared: (i) `swiftrmvs3` (and its corresponding extension to include the Yarkovsky perturbation), and (ii) the `Bulirsh–Stoer` (BS) integrator. The case of CCs is more relevant for the particular orbit of Vysheslavia. A good correspondence to the FN results indicates a large degree of chaotic mixing at Vysheslavia's location inside the chaotic zone.

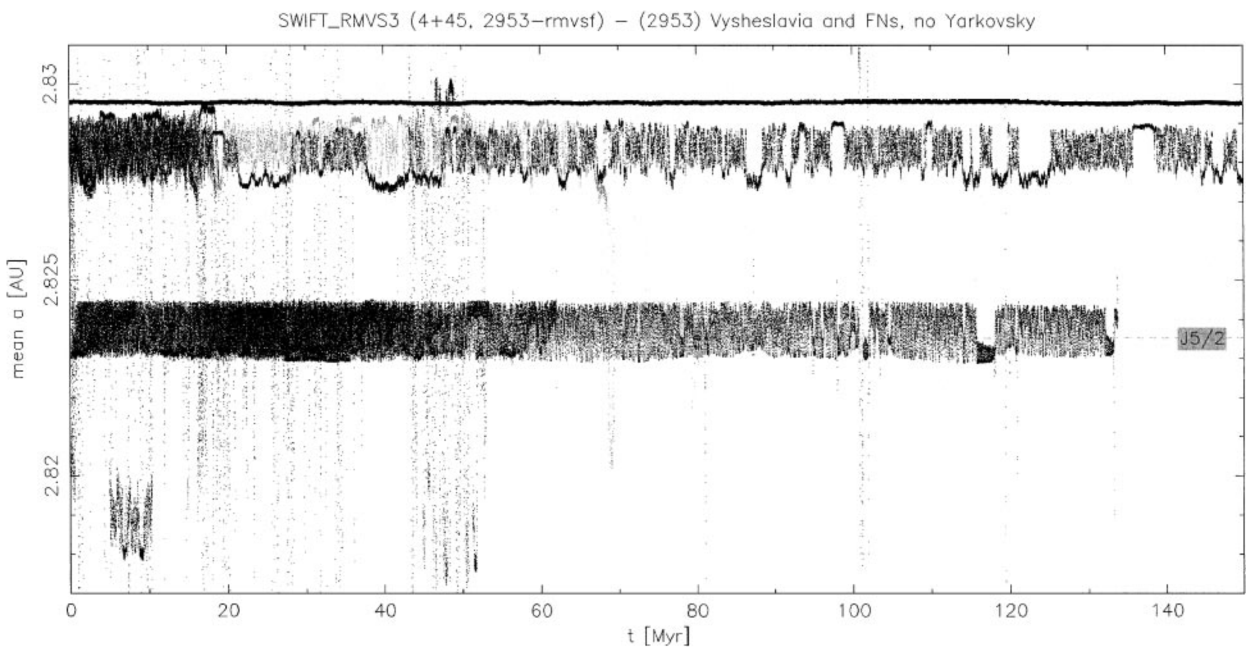

FIG. 1. Mean semimajor axis (in AU) vs time (in Myr) for Vysheslavia (grey curve) and its 14 fictitious neighbors (FN). Perturbations due to outer planets are included, while the Yarkovsky effect is not considered in this simulation. Note the chaotic zone characterized by about 0.001 AU random fluctuations of the semimajor axis. Vysheslavia and FNs originally placed in this zone fall in the 5 : 2 resonance (centered at about 2.823 AU—see the dashed line) typically within 10 Myr.

Notice also a "symmetric" chaotic zone on the opposite side of the 5 : 2 resonance, bracketed by 2.818 AU and 2.820 AU in the semimajor axis, where a chaotic behavior occurs very similar to that in the zone where the present Vysheslavia's orbit resides. One of our objects (FN_0^{+1}) was temporarily captured in that zone before finally falling into the 5 : 2 resonance.

We have repeated the above simulation using the Bulirsh–Stoer integrator with the same initial conditions and dynamical model and covering a time span of 50 Myr. We found qualitatively similar results with a median time to fall into the resonance of 6.6 Myr. The difference of the median times to fall into the 5 : 2 resonance found in the two integrations is well within the uncertainty due to the small statistical sample of objects. Out of the integrated 15 FNs only about 4–5 define effectively the median time, and this is too few. Some of the FNs already lie too close to the bottom part of the chaotic zone, so that they fall into the 5 : 2 resonance in a very short time (in both the simulation with `swiftrmvs3` and with the `Bulirsh–Stoer` integrator). Others lie near the upper border of the chaotic zone and they persist in our simulation for a long time. What thus matters is that the two results have the same order of magnitude. Due to their chaotic nature, the individual orbits of Vysheslavia and its FNs were different from those in the previous simulation (except, again, for the FN_{+3}^0 body that has a stable orbit). Also, some of the FNs were temporarily captured in the symmetric chaotic zone centered at about 2.819 AU.

Much better agreement of the two integrators (in terms of the median time to fall into the 5 : 2 resonance) is found for the close clones (see Table III). Here all 15 particles contribute to the definition of the median and this is already a good statistical sample. In particular, the median time to fall into the 5 : 2 resonance was found to be 14.1 Myr with the swift integrator and 12.9 Myr with the `Bulirsh–Stoer` integrator. Comparison with the corresponding medians for FNs indicates that the chaotic mixing of orbits in the whole zone is very efficient, so that the statistical properties of the orbits very close to that of Vysheslavia match the corresponding statistics taken over orbits departing from a much broader region.

Finally, we noted that the mean lifetime of particles that dropped *in* the 5 : 2 resonance was on the order of 10 Myr in all the integrations. We shall comment more on this issue in Section 2.3.1 where the inclusion of the perturbations caused by inner planets is discussed.

We have also integrated, using both `swift` and `Bulirsh–Stoer` integrators, the orbits of 7340 (1991 UA_2) and 1993 FR_{58} and their FNs. The results of these integrations agreed fairly well. The asteroid 7340 (1991 UA_2) lies close to another tiny chaotic zone between 2.8329 AU and 2.8335 AU. Some of its FNs that were placed in this chaotic zone remained captured in it for the entire timespan of the integration (200 Myr). The asteroid 1993 FR_{58} itself is placed just at the upper boundary of this chaotic zone and might get temporarily captured in it without being ejected to the 5 : 2 resonance. Other FNs that lie outside this zone have remarkably stable orbits. Note that Knežević *et al.* (1997), considering a fictitious neighbor still closer to the 5 : 2 resonance detected the chaotic behavior (placing a body in the same location, we found the same behavior), but by using a bigger displacement of the semimajor axis for the 1993 FR_{58} FNs, they overlooked (skipped) the tiny chaotic zone indicated above.

Generally, however, we can conclude that the results we obtained using both the classical `Bulirsh–Stoer` integrator and the `swift` agree very well with the previous findings of Milani and Farinella (1995) and Knežević *et al.* (1997).

2.2. *Integrations with the Yarkovsky Effect*

As mentioned above, we do not have precise knowledge of a few necessary parameters that tune the Yarkovsky effect. For surface thermal conductivity, we therefore always performed two otherwise identical integrations with a low and a high value for this parameter. The chosen values approximately bracket the interval of physically admissible values, with the lower conductivity appearing more likely to represent reality, according to the ISO measurements. However, we did not notice any really important difference between the simulations with the low and high values of surface thermal conductivity of the bodies, apart from a longer timescale in the latter case. For the sake of illustration we list in Table IV the estimated maximum secular drift of the semimajor axis (we assume zero obliquity for the diurnal variant and ninety degrees obliquity for the seasonal variant of the Yarkovsky effect). We recall that the diurnal variant scales roughly as $\propto \cos\gamma$ and the seasonal variant roughly as $\propto \sin^2\gamma$ (where γ is the obliquity).

We are also lacking information about the orientation of the spin axes of Vysheslavia, 7340 (1991 UA_2), and 1993 FR_{58}. Hence, we again performed two series of simulations: (i) assuming the initial obliquity of the spin axes of all bodies to be 135° and (ii) assuming an isotropical distribution of the spin axes in space. In the former case, we integrated the orbits of the asteroids and of all their fictitious neighbors (from Table II), while in the latter case we integrated the orbits of the asteroids

TABLE IV

Estimated Maximum Values of the Semimajor Axis Secular Drift (da/dt) Due to the Diurnal and Seasonal Variants of the Yarkovsky Effect for the Three Asteroids Considered in This Paper[a]

	$K = 0.0015$ W/m/K		$K = 0.1$ W/m/K		$K = 1$ W/m/K	
Asteroid	Diurnal	Seasonal	Diurnal	Seasonal	Diurnal	Seasonal
Vysheslavia	17.1	0.2	7.2	2.1	2.3	5.8
1991 UA_2	45.8	0.6	19.2	5.5	6.0	15.5
1993 FR_{58}	42.8	0.6	17.9	5.2	5.6	14.4

[a] Units are 10^{-6} AU/Myr; the seasonal drift is always negative, while the diurnal may be either positive or negative. Three values of the surface conductivity K are considered; of these the first—$K = 0.0015$ W/m/K—is the most likely for km-sized objects. Other parameters are given in the text.

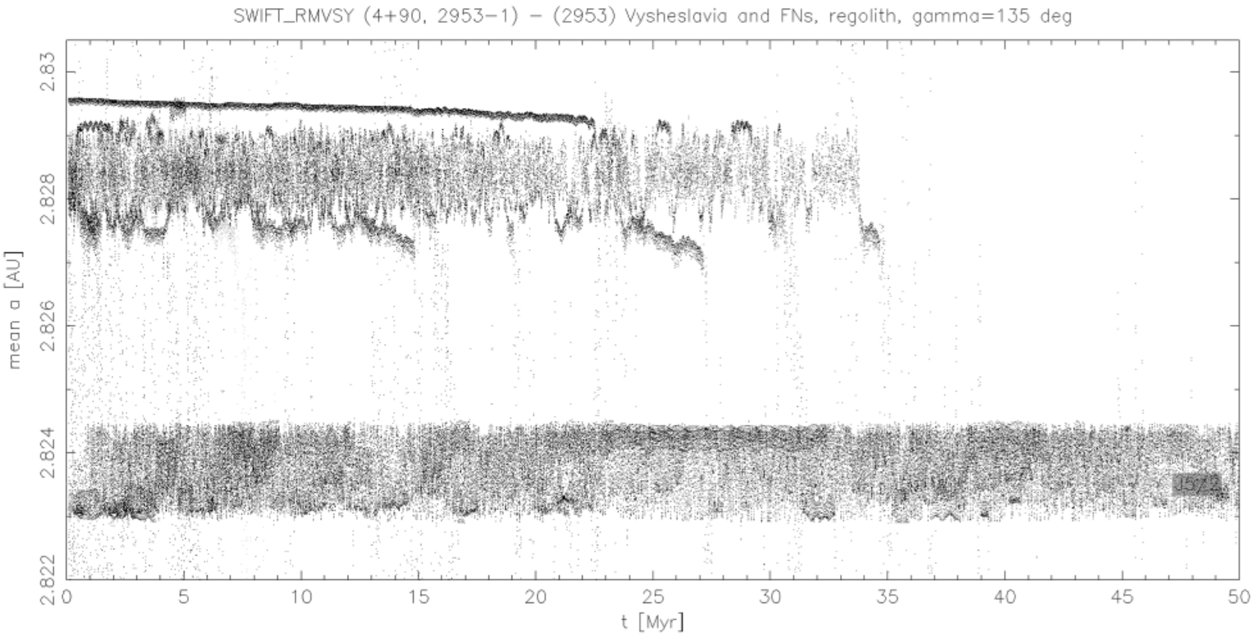

FIG. 2. Mean semimajor axis (in AU) vs time (in Myr) for Vysheslavia (grey curve) and its 14 fictitious neighbors. Planetary perturbations (four outer planets only) and the Yarkovsky effect are included. A low-conductive surface ($K = 0.0015$ W/m/K) is assumed and the initial obliquity is set to 135°.

only, but with 15 different (spatially isotropic) orientations of the spin axis for each of them.

2.2.1. Fixed orientation of the spin axes. Let us first note that the 135° initial obliquity (assumed throughout this section) leads to an average decrease of the semimajor axis of the integrated orbits even in cases dominated by the diurnal Yarkovsky effect (when the surface conductivity is low). We have chosen this value of the obliquity to force drifting of nearby objects toward the 5 : 2 resonance, in order to perform a statistical study of the leakage of the Koronis family members and the family-aging process. However, the 135° value of the initial obliquity does not maximize either the diurnal or the seasonal Yarkovsky mobility.

Figure 2 shows the evolution of the mean semimajor axis as a function of time for Vysheslavia and its 14 FNs. Both planetary and Yarkovsky effects are included and a low value of surface conductivity is assumed ($K = 0.0015$ W/m/K). The general characteristics of the results have not changed significantly (though a somewhat shorter the median time to fall into the 5 : 2 resonance was observed: in the substantial case of CCs 10.6 Myr compared to 14.1 Myr without the Yarkovsky effect, see Table III), except for the behavior of the FN^0_{+3} particle. In contrast to the stable behavior shown in Fig. 1 when only the planetary effects were included, the body now reaches the chaotic zone within 25 Myr due to the Yarkovsky mobility. We also notice that, in contrast with Fig. 1, none of the integrated objects stayed in the chaotic zone longer than 35 Myr.

Figures 3 and 4 show the semimajor axis mobility of the asteroids 7340 (1991 UA_2) and 1993 FR_{58} and their 28 FNs. The same thermal parameters and initial orientation of the spin axis as in the Vysheslavia case are assumed. Note that these bodies are further away from the 5 : 2 resonance and that none of them have fallen there in the course of the 200 Myr integration without the Yarkovsky effect; some fictitious neighbors of 1991 UA_2, e.g., FN^0_{+2}, and the asteroid 1993 FR_{58} itself exhibited temporary chaotic behavior, but the range of their semimajor axis fluctuations was smaller than in the Vysheslavia case. These results changed completely when the Yarkovsky effect was added into our simulation: all the bodies reached the 5 : 2 resonance within 55 to 180 Myr, for the most part by a smooth drift in the semimajor axis due to the Yarkovsky effect. Some of the FNs initially placed above Vysheslavia's chaotic zone were trapped in it for a while before the final fall, while others just quickly passed through on their way to the 5 : 2 resonance. The typical semimajor axis interval crossed by these bodies (of ≃7.5 km) was 2.9×10^{-3} AU per 100 Myr, which corresponds well to the previous analytical estimates (Farinella *et al.* 1998, Farinella and Vokrouhlický 1999, Bottke *et al.* 2000) and also to the data given in Table IV. In a Gyr, comparable to the lifetime of the Koronis family and still shorter than the estimated collisional lifetime, these objects may thus shift as much as 0.029 AU, which turns out to be larger by at least one order of magnitude than the random fluctuations due to the encounters with Ceres and Pallas (see below).

Qualitatively the same results were obtained when the previous simulations were repeated with the high value for surface conductivity ($K = 1$ W/m/K). A slower drift by a factor of about 2.2 was the only quantitative difference between these integrations, which again is in good agreement with the previous analytical estimates.

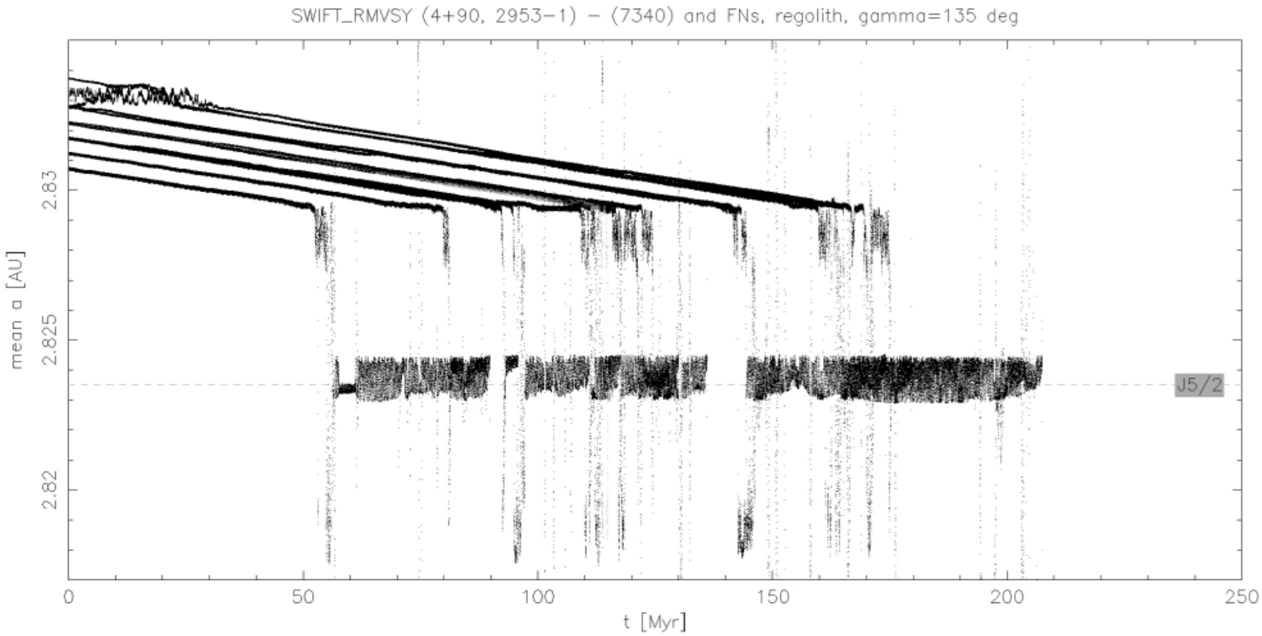

FIG. 3. The same as Fig. 2 but for 7340 (1991 UA_2; grey curve) and its 14 fictitious neighbors.

Note that all the results of this and of all other simulations (many more than it was possible to report herein) can be found at `http://sirrah.troja.mff.cuni.cz/~mira/mp/`.

2.2.2. Isotropic distribution of the spin axes. The Yarkovsky mobility is dominated by the diurnal variant of the effect when low surface conductivity is assumed. As is well known from analytical theory (Vokrouhlický 1998, 1999), in such a case the semimajor axis secular decrease or increase depends on the orientation of the body's spin axis with respect to the orbital plane. If the obliquity value is between $\simeq 90^\circ$ and 180° the orbit decays, while in the complementary half-space orientation of the spin axis the semimajor axis increases. In order to assess the transport of Koronis family members toward the 5 : 2 resonance, in

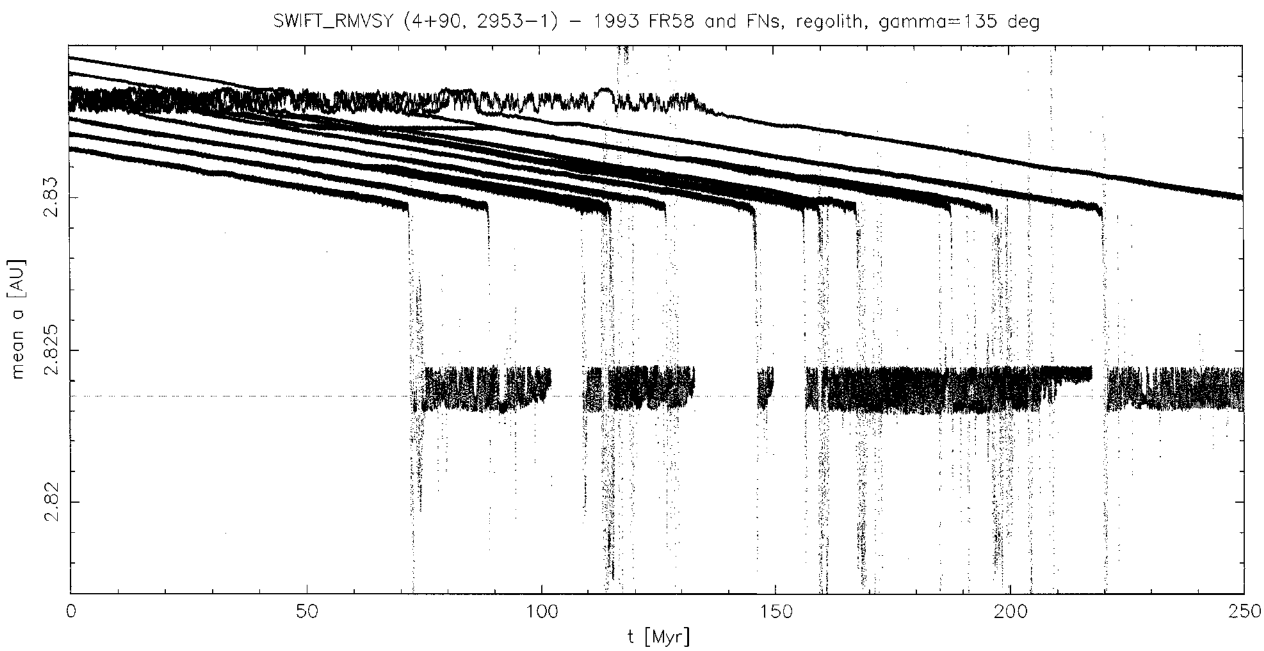

FIG. 4. The same as Fig. 2 but for 1993 FR_{58} (grey curve) and its 14 fictitious neighbors. Despite a possibly long-lasting capture in the chaotic zone near 2.833 AU, all particles eventually fall in the 5 : 2 resonance.

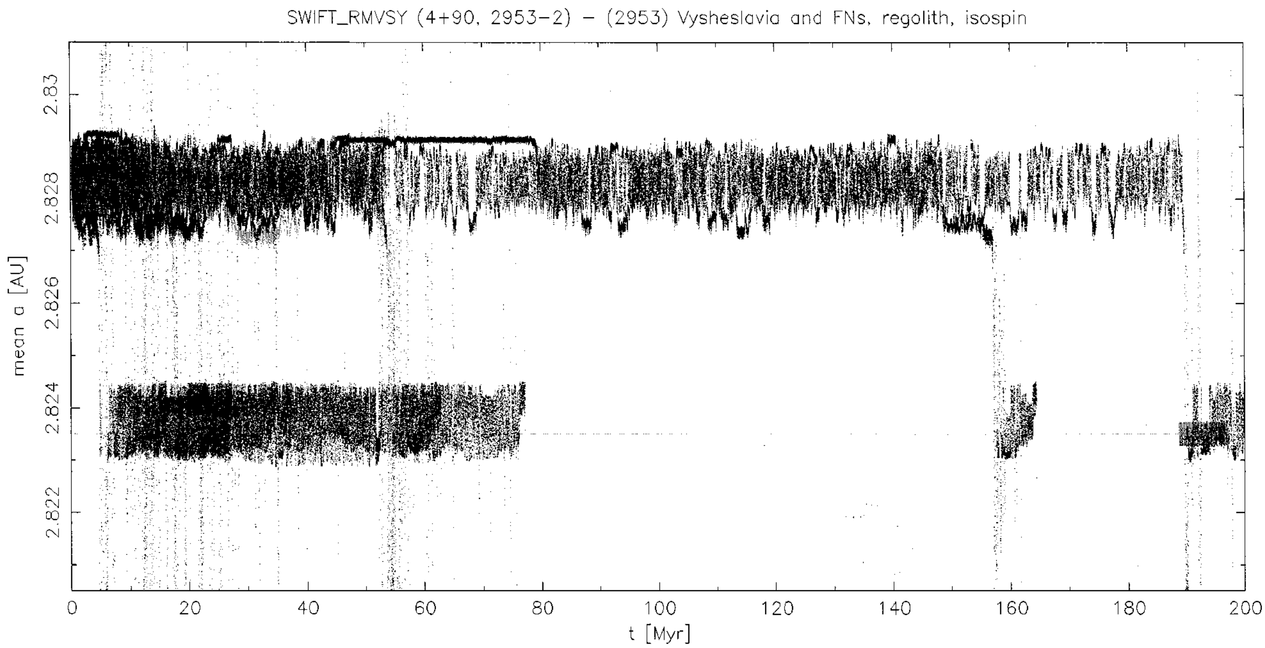

FIG. 5. Mean semimajor axis (in AU) vs. time (in Myr) for 15 fictitious "Vysheslavia-like" asteroids. The initial conditions of each of the integrated bodies correspond to Vysheslavia. The same physical characteristics are assumed (notably the low value $K = 0.0015$ W/m/K of the surface conductivity), except for orientations of the spin axis. The latter are distributed isotropically in space by taking equidistant steps in cosine of the initial obliquity.

Section 2.2.1 above we have assumed 135° for obliquity (note that the very fact that Vysheslavia is currently located in the chaotic zone actually indicates that the obliquity of its spin axis is probably >90). However, it might also be interesting to investigate the role of the Yarkovsky effect in the overall "mixing" of the small Koronis family members at the edge of this resonance. To that purpose we performed another set of simulations, where we assumed isotropic distribution of the spin axes of the integrated asteroids (no fictitious neighbors assumed in this case). Since we do not have any information about the real orientation of the spin axes of Vysheslavia, 7340 (1991 UA_2), or 1993 FR_{58}, each of the integrated "particles" might be a viable representation of the real body.

Figure 5 shows the mean semimajor axis versus time for the 15 Vysheslavia-like objects with various orientations of the spin axis. A low value of the surface conductivity ($K = 0.0015$ W/m/K) has been assumed and perturbations by the outer planets were included. No significant difference between these results and those in Figs. 1 and 2 was observed. None of the "particles" escaped from the chaotic zone toward the stable region of the Koronis family at higher values of the semimajor axis, in spite of the fact that half of the objects have $(da/dt)_{\rm Yark} > 0$. This is due to the fact that the Yarkovsky drift for bodies of this size is rather small, and that therefore it has been overwhelmed and masked by the corresponding chaotic wandering. As in the previous simulations, a majority of the bodies ultimately suffered an abrupt fall into the 5 : 2 resonance, with a median time of the fall of 13.9 Myr (for CCs, see Table III). This value is closer to the corresponding median time in the simulation where the Yarkovsky effect was neglected. This can be easily understood, since half of the objects are drifting outward from the 5 : 2 resonance, thus preventing a rapid fall.

Figures 6 and 7 show the same results as Fig. 5 but for 7340 (1991 UA_2) and 1993 FR_{58}. Contrary to the case of Vysheslavia, half of the orbits, as expected, now shift toward the larger values of the semimajor axes. Temporary interaction with weak resonances might be noticed, but it is the overall Yarkovsky mobility of the two asteroids in the course of time that turns out to be the most interesting result of this test. Starting from the same location in the Koronis family, the two fastest "particles" shifting in opposite directions build up in a Gyr of time their mutual distance to as much as 0.07 AU, which is a fair fraction of the total semimajor axis range occupied by this otherwise rather elongated family. Let us emphasize one more time that we are dealing with objects about 7.5 km in size.

As in Section 2.2.1 above, we have repeated all the above simulations for the case of the high surface conductivity ($K = 1$ W/m/K). The seasonal variant of the Yarkovsky effect then dominates the semimajor axis mobility and most of the orbits decay toward smaller values of a. The results are thus very similar to those we found with the low value of surface thermal conductivity and the initial obliquity in the 90° to 180° range, only the timescale is again "stretched" by a factor of about 2.2 due to a slower seasonal mobility. An example is shown in Fig. 8, where 15 objects with initially the same orbital parameters as 7340 (1991 UA_2), but with isotropic spin axes, are shown. Only

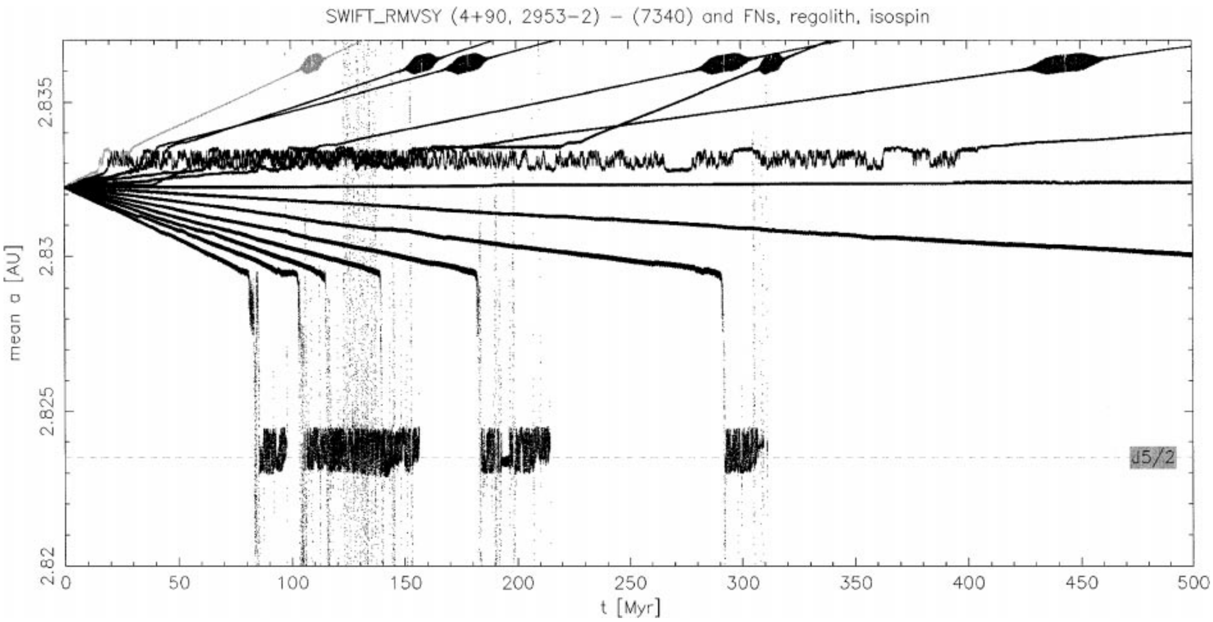

FIG. 6. The same as Fig. 5 but for the asteroid 7340 (1991 UA_2). Contrary to the Vysheslavia case, some particles may now escape from the chaotic region after a very long-lasting period of capture.

when the obliquity is set to zero, and hence the seasonal variant of the Yarkovsky effect vanishes, is there a very slow upward drift due to the diurnal variant (grey particle).

2.3. *Additional Tests*

Hereafter we shall verify that our conclusions from the previous sections warrant several modifications of the (necessarily simplified) dynamical model we used above.

2.3.1. Terrestrial planets included. We first add to the simulation the perturbations due to the inner planets (Mercury through Mars). Figure 9 shows the mean semimajor axis of Vysheslavia and its 14 FNs as a function of time. One obvious difference between these results and the results presented in Fig. 1 is the different residence time in the 5 : 2 resonance. While in the case where only four outer planets were considered (Fig. 1) the bodies could stay in the 5 : 2 resonance typically for tens of Myr, in the integration where all the planets were considered (Fig. 9)

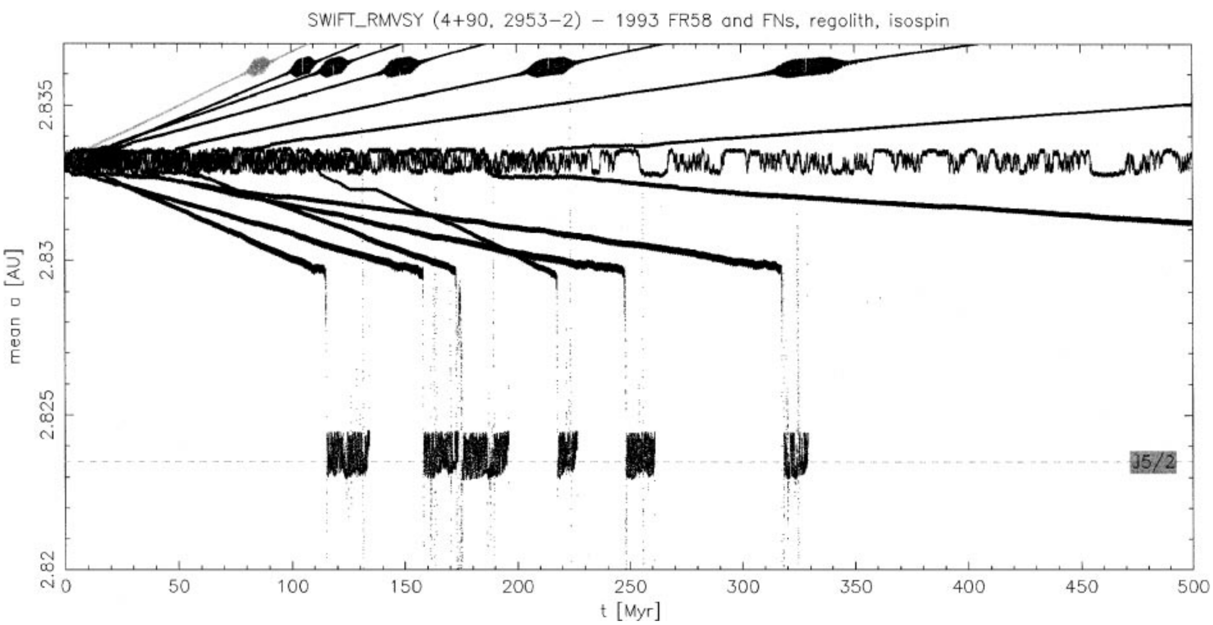

FIG. 7. The same as Fig. 5 but for the asteroid 1993 FR_{58}.

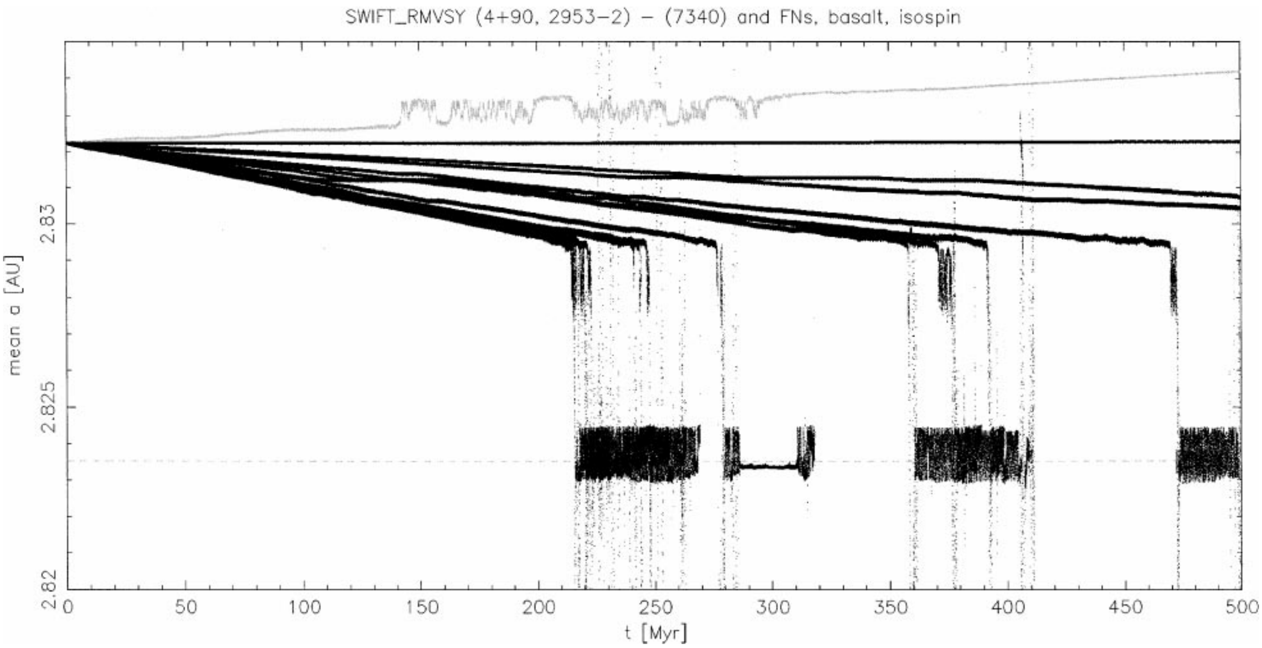

FIG. 8. The same as Fig. 6 but for bodies with a higher value of surface thermal conductivity ($K = 1$ W/m/K). Except for the grey particle (corresponding to zero obliquity), the dominating seasonal contribution from the seasonal variant of the Yarkovsky effect drags the orbits to smaller values of the semimajor axis. The strength of the drift is smaller than in the diurnal-dominated situation in the Fig. 6. Notice a long-lasting capture of the grey particle in the tiny chaotic zone, where the asteroid 1993 FR_{58} is currently located.

the typical residence time is about one Myr. We have verified that the principal mechanism that removes particles from the 5 : 2 resonance is a close encounter with one of the inner solar system planets (mainly Earth and Venus). These findings agree well with recent results of Gladman *et al.* (1997). On the other hand, and what is most important in the context of our work, the general dynamical picture of Vysheslavia itself and of its clones seems to be insensitive to adding the inner solar system planets

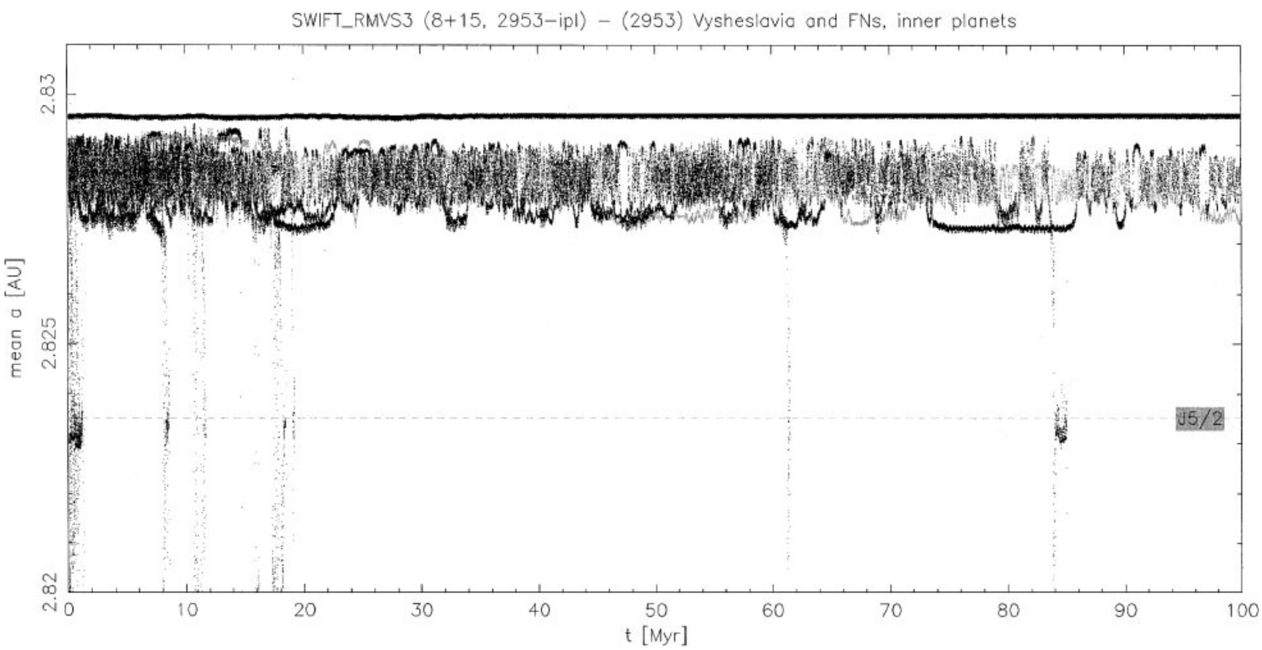

FIG. 9. Mean semimajor axis (in AU) vs. time (in Myr) for Vysheslavia and its 14 fictitious neighbors. As in Fig. 1, the Yarkovsky effect is not included in this simulation, but perturbations due to all the planets (except Pluto) are considered. Note the much shorter lifetime of the objects residing temporarily in the 5 : 2 resonance.

to the model. In particular, Vysheslavia's chaotic zone location is well preserved. This indicates that the chaotic zone, related to the resonance effects due to the outer planets, has been well represented by the four-body problem with appropriately applied barycentric correction. The median time to fall in the 5 : 2 resonance is 10.9 Myr, very close to the value obtained with the perturbing influence of the four outer planets only (Table III). We may thus conclude that adding the inner solar system bodies does not significantly alter our previous results.

2.3.2. The possible role of Ceres and Pallas. The main asteroid belt, where Vysheslavia is located, represents a vast reservoir of much less massive perturbers than the inner planets. However, the possibility of close encounters with these bodies may in part balance the deficiency in their mass as far as the total effect on the Vysheslavia orbit is concerned. Since the most massive asteroids, Ceres and Pallas, might already closely approach Vysheslavia, we decided to perform a test that consists of including these two asteroids and all the planets (except Pluto) in our simulation. Masses and osculating elements of these asteroids were taken from Bowell's catalogue (Bowell *et al.* 1994).

Before commenting on the results of the numerical simulations, let us derive a simple estimate of the expected effect. Let us assume that Vysheslavia is on a circular orbit and a perturber of mass m' (expressed as a fraction of the solar mass) is on a nearby circular orbit, so that the ratio of their semimajor axes $\alpha = a'/a$ is smaller but very close to unity. Then one might estimate a variation δa of the semimajor axis a of Vysheslavia related to the conjunction encounter with the perturber as $(\delta a/a) \simeq 2m'/(1-\alpha)/(1-\alpha^{3/2}) \simeq 2 \times 10^{-6}$ AU (the numerical value pertains to Ceres). Even smaller variations of the semimajor axis are expected due to the interaction with other members of the Koronis family, though the duration of the conjunction is quite different because of their smaller distance. Note, however, that this is just the lowest order-of-magnitude estimate of the effect. When approximating the orbits (of both the perturber and the perturbed asteroid) by precessing ellipses, we might get a better analytical estimate of the effects, but in this case we will instead directly perform a complete numerical simulation. We shall thus see in the following that the gravitational influence of massive asteroids on the orbits of smaller objects (like Vysheslavia) is composed of (i) random, very small-scale fluctuations in the semimajor axis that can accumulate over a longer time, giving rise to a sort of chaotic diffusion (see Fig. 11), and (ii) abrupt changes in the semimajor axis resulting from a close encounter (both Ceres and Pallas eccentricities are large enough to cross the Vysheslavia region). Milani and Farinella (1995) estimated that a close encounter with Ceres resulting in a more than 0.001 AU shift of Vysheslavia's semimajor axis and occurring in the last 100 Myr is not very likely. We shall try to investigate this conjecture by direct numerical integration.

Numerical integration with the massive asteroids that may approach very close to the orbits of integrated objects (e.g, the closest detected approaches to Vysheslavia or to its clones were at about 1.1×10^{-4} AU) is a delicate problem. The close approaches are very short (typically a day), so that a "blind" application of an integrator like `swift` might result in misleading or incorrect conclusions (by definition a close approach to one of the perturbing bodies is a "short-periodic" effect). Either a too long integration timestep from the previous simulations is kept, and then the close approaches are not well represented, or the Hill sphere of the gravitational influence of the massive asteroids was forced to be too large, which means the symplecticity of the swift was largely violated (by the time we did most of the simulations we were not aware of the availability of the symplectic integrators that permit a correct modelling of the close encounters; see e.g., Chambers 1999). Because of this problem, we have integrated the orbit of Vysheslavia and its 14 FNs (and similarly for the asteroids 7340 (1991 UA_2) and 1993 FR_{58}) in the gravitational field of 8 planets (except Pluto), and of Ceres and Pallas by using the `Bulirsh–Stoer` integrator (the Yarkovsky perturbation was not considered in this case). Obviously, tight control of the numerical precision results in large expenses for computation time. The integrations presented in this section took much more CPU time than all the other integrations reported above.

Figure 10 shows the mean semimajor axes of all integrated asteroids (and their FNs) during the 20-Myr time interval. The results are basically similar to those reported in Section 2.3.1, where the perturbations of all 8 planets were considered. However, a closer look at the orbits that are not locked in a seemingly chaotic zone reveals semimajor-axis long-term fluctuations at the level of about $\leq 10^{-3}$ AU. As discussed above, these fluctuations are induced by the gravitational influence of the two massive asteroids considered in the simulation (Ceres and Pallas). We can also see rapid jumps in the semimajor axes of some particles (e.g., FN^0_{-3} of 7340 at about 15.3 Myr) that are triggered by very close encounters with Ceres or Pallas. Notice, however, that none these effects is large enough to place Vysheslavia's neighbor FN^0_{+3}, originally above the chaotic zone, in the chaotic zone. The average time to fall into the 5 : 2 resonance computed for Vysheslavia and its fictitious neighbors is 11.6 Myr, essentially the same amount of time as in the integration without the gravitational influence of Ceres and Pallas.

Because of the somewhat shorter timespan of the previous simulation, we have integrated three FNs, notably FN^0_{+3} of Vysheslavia, FN^0_{-3} of 7340 (1991 UA_2), and FN^0_{+3} of 1993 FR_{58}, over a much longer timespan. Their orbits were again integrated by the `Bulirsh–Stoer` integrator with gravitational effects of 8 planets and the 2 massive asteroids. Resulting evolution of the mean semimajor axes, that now covers 100 Myr, is shown in Fig. 11. Note again that the semimajor axis of Vysheslavia's neighbor FN^0_{+3} fluctuates at the level of 10^{-3} AU, but this is still not enough to get trapped by the chaotic zone. Strictly speaking we cannot exclude the possibility that Vysheslavia has been transferred to its present location by the gravitational influence of Ceres or Pallas, but the fact that we did not observe this occurring in our integration puts a limit on its corresponding probability. There are three interesting features to be noted in Fig. 11: (i) an abrupt change in the semimajor axis of

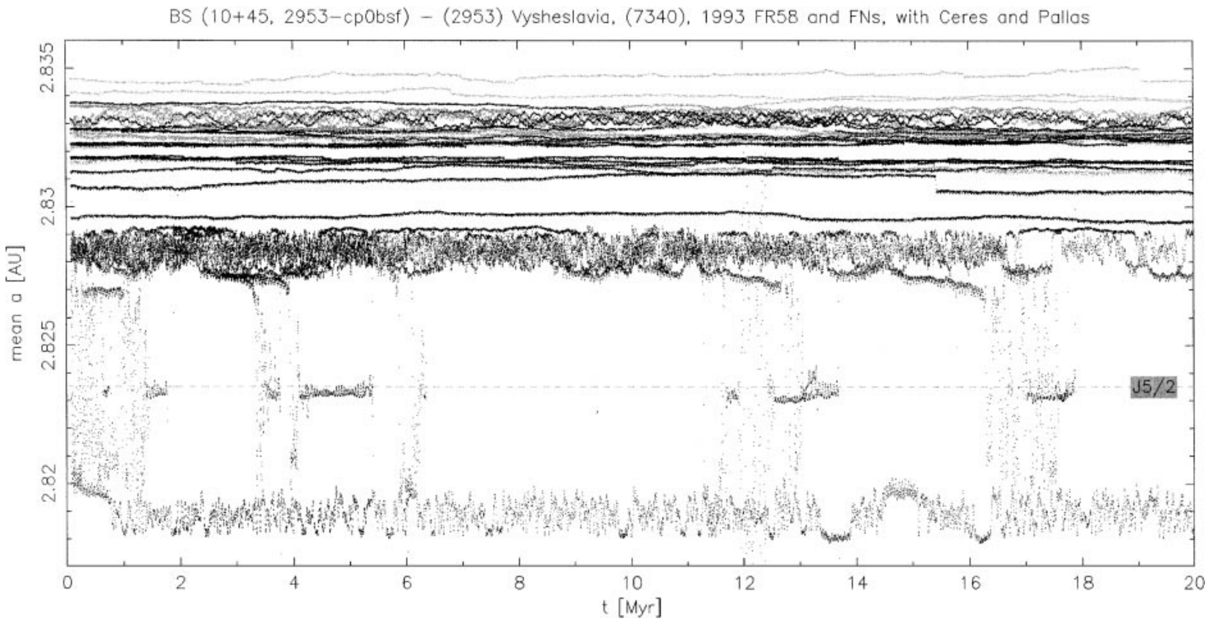

FIG. 10. Mean semimajor axis (in AU) vs. time (in Myr) for Vysheslavia, 7340 (1991 UA_2), 1993 FR_{58}, and their 42 fictitious neighbors defined in Table II (the asteroid 1993 FR_{58} and its clones are grey). Perturbations due to all the planets (except Pluto) and two massive asteroids (Ceres and Pallas) are considered. Note the tiny, long-term fluctuations ($\leq 10^{-3}$ AU) and occasional jumps of the semimajor axes of the previously perfectly stable orbits. Both effects are due to the gravitational influence of Ceres and Pallas. This simulation was performed with the `Bulirsh–Stoer` integrator.

Vysheslavia's FN^0_{+3} at about 5.1 Myr, (ii) a similar change in the semimajor axis of FN^0_{-3} that belongs to 1993 FR_{58}, at about 97.5 Myr, and (iii) the identical effect in the semimajor axis of FN^0_{+3} that belongs to 1993 FR_{58}, at about 68.2 Myr. In all these cases we have detected a very close encounter of the integrated orbit with Ceres. The minimum separation of the two bodies was only 1.1×10^{-4} AU in the last case. Also, due to chaoticity of the orbits, the close encounter of 7340's FN^0_{-3} at

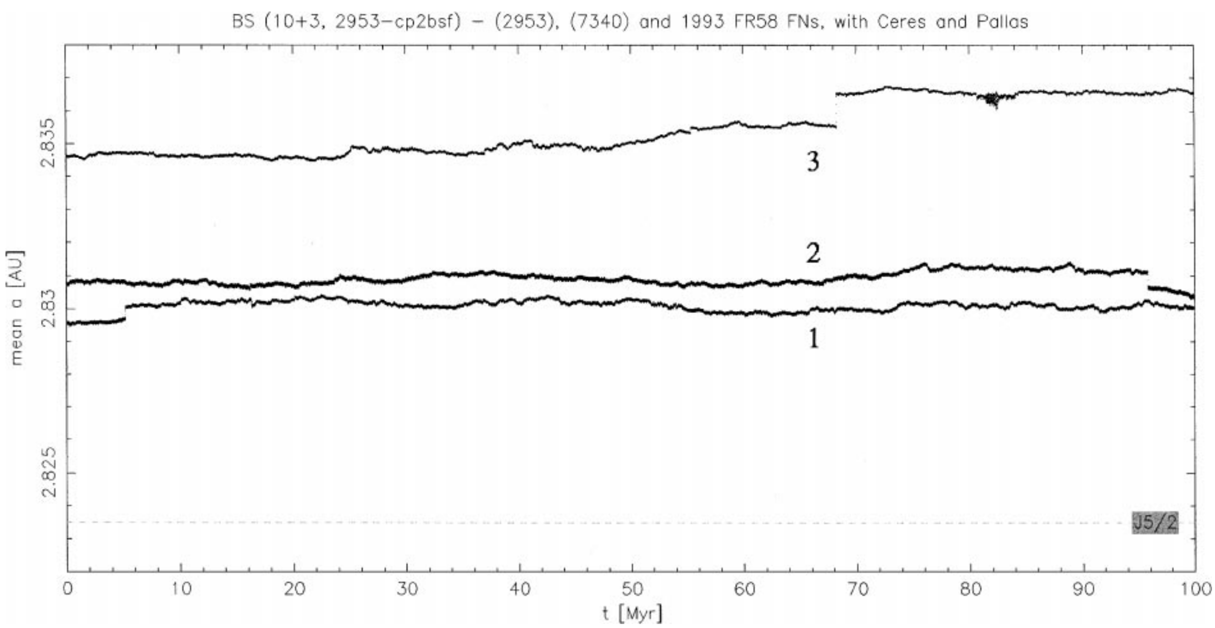

FIG. 11. The same as Fig. 10 but for a longer timespan. Only three bodies are considered: FN^0_{+3} of Vysheslavia—curve 1, FN^0_{-3} of 7340 (1991 UA_2)—curve 2, and FN^0_{+3} of 1993 FR_{58}—curve 3. Note the three deepest close encounters of the integrated particles with Ceres that both resulted in a significant change of the semimajor axis: (i) particle 1 at about 5.1 Myr, (ii) particle 2 at about 97.5 Myr, and (iii) particle 3 at about 68.2 Myr. The closest approach to Ceres in the third case was 1.1×10^{-4} AU only.

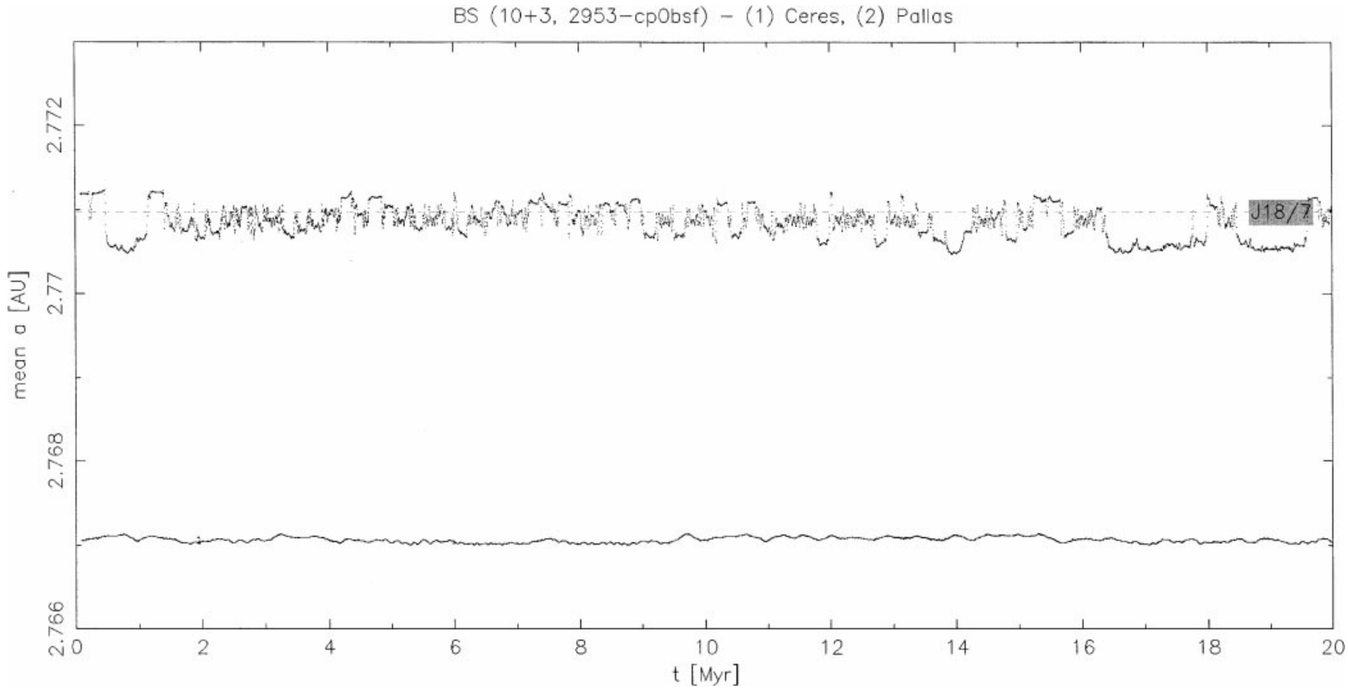

FIG. 12. Long-term evolution of the mean semimajor axes of Ceres and Pallas (time in Myr at the abscissa). Note the difference between the stable behavior of the Ceres semimajor axis (lower curve) and the chaotic fluctuations of the Pallas semimajor axis (upper curve). The latter seems to be locked in a similar chaotic zone (width of about 0.0008 AU) as Vysheslavia. This chaotic region seems to be associated with the high-order 18 : 7 resonance with Jupiter.

approximately 15.3 Myr (see Fig. 10) is not reproduced in this integration. The noticeable ($\simeq$0.0001–0.001 AU) changes of the semimajor axis require very close encounters with a small cross-section, so that two different integrations preserve the statistical characteristics of the encounters only. Note that we have used two different computers (with different Fortran compilers) to perform the simulations in Figs. 10 and 11.

A by-product of our investigation is an interesting finding about the orbit of Pallas. Figure 12 shows the mean semimajor axis of Ceres and Pallas as a function of time in the course of 20 Myr as obtained from the integration that includes all the planets (except Pluto). In contrast to the orbit of Ceres, which appears to be very stable, the semimajor axis of Pallas undergoes chaotic fluctuations on the order of 0.0008 AU. Šidlichovský (1999) pointed out that Pallas is located in the high-order 18 : 7 resonance with Jupiter and this seems responsible for the observed chaos.

2.4. *Evolutionary Implications and Future Work*

There may be several reasons Vysheslavia's orbit is more evolved than the orbits of 7340 (1991 UA_2) and 1993 FR_{58}: (i) it might be originally placed closer to the 5 : 2 resonance (since 7340 (1991 UA_2) and 1993 FR_{58} are smaller object, they might arrive at the 5 : 2 proximity by drifting from a farther original location) and/or (ii) its spin axis orientation and the rotation period results in faster Yarkovsky mobility of the semimajor axis. These are just the most obvious possibilities; future observations may help to evaluate them.

In a forthcoming paper we present an additional, though indirect, argument favoring the Yarkovsky-driven origin of the Vysheslavia orbit. Notably, we have identified about a dozen other asteroids, most probably Koronis members, that are located in or very close to Vysheslavia's chaotic zone. These objects are smaller, typically 4–8.5 km in size, thus representing a more numerous category of family members, which should also have faster Yarkovsky mobility of semimajor axes. It is natural to expect that at any given time some of these bodies will be located in Vysheslavia's chaotic zone. On the other hand, these objects are large enough to place stronger constraints on their own and Vysheslavia's collisional origin (as the possible offspring of the recent, secondary collision in the family), making this hypothesis less probable.

Our conjecture is that other asteroid families located very close to the powerful main-belt resonances might suffer a kind of Yarkovsky erosion similar to that of the Koronis family. Of particular interest could be the Maria and Hestia families that are located very close to the 3 : 1 mean motion jovian resonance, and possibly even the Themis and Hygiea families located close to the 2 : 1 mean motion jovian resonance (e.g., see Zappalà *et al.* 1995, 1997). In fact, the inner part of the Maria family, for example, is even cut by the 3 : 1 resonance, so that a significant number of originally created bodies must have been placed right there or very close to it. A detailed study of the process of possible secondary mass-loss for these families is, however, beyond the scope of this study.

3. CONCLUSIONS

The main results of this paper can be summarized as follows:

- Performing long-term (up to 0.5 Gyr) integrations, we have verified that the Yarkovsky effect might be efficient in altering the positions of small asteroids (km in size) up to 0.01–0.03 AU within their collisional lifetime.
- We have argued that such "small-scale" mobility may eventually drive asteroids onto dynamically unstable or metastable orbits related to some nearby resonances. A significant difference between the dynamical lifetime of the body at the present orbit and the family age (as in the case of Vysheslavia) might be a strong indicator of the evolutionary processes taking place in the family.
- A secondary collision in the family that places the bodies on the dynamically metastable orbits still cannot be ruled out as one of the possible explanations. However, in Vysheslavia's case the probability of such a possibility is low. Similarly, our simulations indicate low probability for gravitational origin of Vysheslavia's orbit due to the influence of massive asteroids (Ceres and Pallas were studied above). Yarkovsky-driven transport is the last and most probable possibility. Though we cannot determine exactly the history of Vysheslavia's orbit, it seems most likely that it has been placed onto its present orbit by a slow drift from its original location in the stable region further from the 5 : 2 resonance.
- Photometric and spectral observations of Vysheslavia (and other small members of the Koronis family located close to the 5 : 2 resonance) are needed to constrain the uncertainties and possibly discriminate between the different hypotheses. In particular, photometric data should yield very useful information about its rotation. The precise value of the rotation period and of the orientation of the polar axis would improve the Yarkovsky-effect modelling, while more reliable information about the rotation state (a possible tumbling) may offer a hint about the age of Vysheslavia. We have mentioned that Šarounová (2000, personal communication) has recently observed Vysheslavia and estimated the rotation period to about 6.29 h. Complementing these observations with other observations at different phase angles might constrain the obliquity of its spin axis (even if determination of the precise spin axis orientation is difficult). Assuming diurnal effects to be the dominant Yarkovsky force, such a constraint may significantly contribute to understanding Vysheslavia's history. In particular, if the inward drift of its semimajor axis is confirmed, the hypothesis of this paper—Yarkovsky-driven origin of Vysheslavia's orbit—would turn to near certitude.
- A detailed study of the Yarkovsky-driven diffusion in the asteroid families and their leakage to the principal resonances in the main asteroid belt appears to be a challenging prospect for future work.

ACKNOWLEDGMENTS

We are grateful to A. Paschke, L. Šarounová, L. Vašta, and the Astroqueyras Society for letting us know about their photometric observations of Vysheslavia prior to publication. We also acknowledge suggestions of the referees (J. N. Spitale and A. Milani) that improved the final version of this paper. Special thanks go to the Observatoire de la Côte d'Azur, Department CERGA, and to the University of Pisa Department of Mathematics, for allowing us to use their computational facilities.

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