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A complete linear model for the Yarkovsky thermal force on spherical asteroid fragments

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Abstract. A linear theory for heat conduction in a spherical, solid and rotating body illuminated by solar radiation is developed. The recoil force due to the thermally re-emitted radiation by the surface of the body is computed, including all the terms depending both on the body's rotation frequency and the mean motion of its revolution about the Sun. The present solution thus overcomes a drawback of the previous approaches, which have been tailored separately either to the diurnal or to the seasonal variant of the so–called Yarkovsky effect, corresponding to different limiting cases of the current theory. We pay a special attention to compute the secular effects on the semimajor axis of the body's orbit about the Sun. The results from the general model coincide with those of the previous approaches to a high level of accuracy, as the relative size of the additional "mixed" terms is smaller than 10^{-3} for plausible parameter choices. This confirms that the use of the simplified formulæ is warranted in the relevant Solar System applications.

Key words: celestial mechanics, stellar dynamics – meteors, meteoroids – minor planets, asteroids

1. Introduction

The so–called Yarkovsky effect, a recoil force due to thermal radiation from anisotropically heated orbiting bodies, has recently attracted a considerable attention in the frame of the studies on the delivery of meteorites and the dynamics of small bodies in the Solar System. Specific issues for which the Yarkovsky effect is probably relevant are: the cosmic–ray exposure ages of stony and iron meteorites, which are much longer than the dynamical lifetimes of particles delivered from the asteroid belt (Farinella et al. 1998; Hartmann et al. 1998; Morbidelli & Gladman 1998); the overabundance of decameter–sized near–Earth objects (Rubincam 1995, 1998; Vokrouhlický & Farinella 1998a); the dynamical evolution of large (> 5 km) main–belt asteroid fragments and their delivery to Mars– and Earth–crossing orbits (Farinella & Vokrouhlický 1999). In all these cases, the Yarkovsky effect plays the role of a dissipative mechanism, resulting into a significant long–term mobility of the orbital semimajor axis and a complex interaction with resonances.

To assess the relevance of the Yarkovsky effect in Solar System dynamics one needs, as a first step, to develop a reliable physical model of the thermal processes occurring within solid, spinning and orbiting bodies. A significant amount of work has been performed on this problem in recent years, after Rubincam (1995) resurrected the interest in the dynamical consequences of these thermal effects. Most importantly, Rubincam (1995, 1998) and Farinella et al. (1998) recognized the existence of two distinct variants of the Yarkovsky effect: a "diurnal" variant depending on the rotation frequency of the body around its instantaneous spin axis ($\omega_{\rm rot}$), and a "seasonal" variant depending on the mean motion frequency of the body around the Sun ($\omega_{\rm rev}$).

Technically speaking, the diurnal variant is obtained when one entirely neglects the orbital motion around the Sun (see e.g. Vokrouhlický 1998a,b), whereas in dealing with the seasonal variant one *a priori* averages all relevant quantities over the (assumedly) fast rotation of the body (e.g. Rubincam 1995, 1998; Vokrouhlický & Farinella 1998b). This classification is meaningful and useful, since the two variants of the Yarkovsky effect result in qualitatively different long–term changes of the semimajor axis. The diurnal version is maximum at zero obliquity and can lead either to semimajor axis decrease or increase, depending on the sense of rotation; on the contrary, the seasonal version is maximum at 90° obliquity and can only result in orbital decay (e.g. Rubincam 1995, 1998; Farinella et al. 1998; Hartmann et al. 1998). At the essence, however, the two variants of the Yarkovsky effect are just two different limiting cases of a single physical mechanism, i.e., the recoil force associated to thermal radiation from a body having an anisotropic temperature distribution on its surface. As their names imply, the diurnal and seasonal variants correspond to different periodicities and geometries of the external illumination on the body's surface. From this perspective, it seems desirable to develop a unified, self–consistent theory for the Yarkovsky effect, including at the same time both the diurnal and the seasonal periodicities, such that the two classical variants can be derived computing suitable mathematical limits.

Although the classical variants of the Yarkovsky effect are present as particular limiting cases, the unified theory inevitably

Send offprint requests to: Prague address will contain additional, "mixed" terms, depending on both the relevant frequencies $\omega_{\rm rot}$ and $\omega_{\rm rev}$. This conclusion holds even in the frame of a linear theory for the temperature changes, such as that developed in the following sections. Thus, the major novelty of this paper consists of the derivation of these "mixed" (or "diurnal–seasonal") terms. Specifically, we shall show that the "diurnal" variant does not exist as an effect depending on the rotation frequency alone, but inevitably contains a linear combination of the two frequencies. As expected, in the limit of a very rapid spin rate this doublet tends to merge into a single line, depending just on $\omega_{\rm rot}$.

Then, we shall assess the contribution of the new terms to the secular changes in the semimajor axis of the body's orbital motion. As noted above, such changes probably play an important role in several problems of astronomical interest, and the quantitative results obtained so far have always been computed as a simple superposition of the diurnal and seasonal effects (e.g. Farinella & Vokrouhlický 1999), neglecting any possible "mixed" effects.

To make the calculations as simple as possible, we shall make three simplifying assumptions: (i) a circular orbit around the Sun; (ii) a spherical shape of the body; and (iii) a commensurability between the rotation and revolution periods. In particular, we shall introduce a parameter $m = \omega_{\rm rot}/\omega_{\rm rev}$, and we shall assume that m is an integer number. However, we stress that while the first two assumptions correspond to physical simplifications, the third one is just a suitable mathematical step to simplify the derivation of our results, and that this assumption can be easily removed by the technique used by Farinella & Vokrouhlický (1996). Therefore, our final results will be valid for any (real) value of the parameter m.

2. Theory

2.1. Formulation of the problem

Hereafter, we use the mathematical approach and the notations introduced by Vokrouhlický (1998a,b). We refer to those papers for a historical background and a more detailed discussion of thermal physics, while here we shall just provide a few general concepts and definitions required to derive a unified solution for the thermal Yarkovsky force on a spherical body.

Since we plan to remain in the framework of a linear theory to describe the thermal response of an orbiting body to external heating, we suppose that the temperature T throughout the body is close to a constant mean value $T = T_{\rm av}$, and therefore write $T = T_{\rm av} + \Delta T$ ($\Delta T \ll T_{\rm av}$). A suitable scaling of the temperature T, as well as other variables, simplifies the mathematical formulation of the problem. The temperature T will be normalized by an auxiliary value $T_\star$ defined by: $\epsilon\sigma T_\star^4 = \alpha\mathcal{E}_\star$. Here, ϵ is the thermal emissivity coefficient of the body, σ the Stefan–Boltzmann constant, α the optical absorption coefficient and $\mathcal{E}_\star$ the solar radiation flux at the mean distance along the orbit. Since we shall restrict our analysis to the case of a circular orbit, the scaled mean temperature reads $T'_{\rm av} = T_{\rm av}/T_\star = 1/\sqrt{2}$ [see Vokrouhlický 1998a; the normalized quantities will be always denoted by a prime]. Similarly, the radial coordinate r measured from the center of the body to its surface (at $r = R$) is to be scaled by the penetration depth of the seasonal thermal wave $l_s = \sqrt{K/\rho C\omega_{\rm rev}}$: $r' = r/l_s$. Here, K is the thermal conductivity, C the thermal capacity, and ρ the density of the material. Finally, the time t will be represented by a complex quantity $\zeta = \exp(i\lambda)$ [$\lambda = \omega_{\rm rev}(t - t_0)$], with t_0 being an arbitrary time origin to be specified below (here $i = \sqrt{-1}$).

The temperature variation $\Delta T' = \Delta T/T_\star$ satisfies the heat diffusion equation (see, e.g., Vokrouhlický 1998a)

$$i\zeta\frac{\partial}{\partial\zeta}\;\Delta T'(r';\theta,\phi;\zeta) = \tag{1}$$
$$= \frac{1}{r'^2}\left\{\frac{\partial}{\partial r'}\left(r'^2\frac{\partial}{\partial r'}\right) + \Lambda\left(\theta,\phi\right)\right\}\Delta T'(r';\theta,\phi;\zeta)\;,$$

with the operator $\Lambda(\theta,\phi)$ given by

$$\Lambda\left(\theta,\phi\right) = \frac{1}{\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2}{\partial\phi^2}\right] \tag{2}$$

(here θ and ϕ are the usual spherical coordinates with the pole defined by the body's rotation axis). Eq. (1) is supplemented by the linearized boundary condition

$$\sqrt{2}\Delta T' + \Theta\left(\frac{\partial\Delta T'}{\partial r'}\right)_{R'} = \Delta\mathcal{E}'\;, \tag{3}$$

with the seasonal thermal parameter Θ defined by

$$\Theta = \frac{\Gamma\sqrt{\omega_{\rm rev}}}{\epsilon\sigma T_\star^3} \tag{4}$$

(here $\Gamma = \sqrt{\rho CK}$ is the thermal inertia). The right–hand side term in Eq. (3) is defined by $\mathcal{E}' = \frac{1}{4} + \Delta\mathcal{E}'$, $\mathcal{E}'_{\rm av} = 1/4$ being the averaged irradiation of the fragment's surface. The temperature variation $\Delta T'$ is a function of the coordinates (r',θ,ϕ) inside the body and of time ζ.

As in Vokrouhlický (1998a), we shall solve for $\Delta T'$ in a rotating, body–fixed reference frame, with the Z-axis coinciding with its spin vector. At the reference time t_0, the X-axis of this system points toward the radiation source (the Sun). After expressing the thermal force vector $\mathbf{f}$ in this reference frame – see Eqs. (14) and (15) below – we will compute its projections in the orbit–related reference system and obtain the force components appearing in the Gauss perturbation equations. The latter operation is simple vectorial algebra.

2.2. The insolation term

As discussed in detail by Vokrouhlický (1998a), a particular attention has to be paid to a suitable development of the irradiation term $\Delta\mathcal{E}'$ on the right–hand side of Eq. (3). In general, we can use a series of spherical functions

$$\Delta\mathcal{E}' = \sum_{n\geq 1}\sum_{k=-n}^{n} a_{nk}(\zeta)\,Y_{nk}(\theta,\phi)\;, \tag{5}$$

where only the dipole ($n = 1$) part

$$a_{10}(\zeta) \;=\; \sqrt{\frac{\pi}{3}}\;\cos\theta_0(\zeta)\;, \tag{6}$$

$$a_{1\pm1}(\zeta) \;=\; \mp\sqrt{\frac{\pi}{6}}\,\sin\theta_0(\zeta)\,e^{\mp i\phi_0} \tag{7}$$

will be relevant. Here θ_0 and ϕ_0 are the solar colatitude (measured from the spin axis) and longitude in the body–fixed reference frame (a mathematical technique allowing one to obtain most elegantly these results has been discussed in Vokrouhlický 1998a). To simplify the following algebra, the origin of the mean longitude λ (i.e. t_0) has been chosen so that

$$\cos\theta_0 = -\sin\gamma\sin\lambda = \frac{i}{2}\sin\gamma\left(\zeta-\zeta^{-1}\right)\;, \tag{8}$$

where γ is the obliquity of the spin axis. Eq. (8) allows us to make immediately an important observation, namely that the Fourier spectrum of the $a_{10}(\zeta)$ coefficient contains only the revolution frequency.

On the other hand, the Fourier development for the tesseral coefficients $a_{1\pm1}(\zeta)$ is more complicated. It contains a linear combination of the rotation and revolution frequencies, since

$$\sin\theta_0\,e^{\pm i\phi_0} = \sin^2\frac{\gamma}{2}\,\zeta^{\mp(m+1)} + \cos^2\frac{\gamma}{2}\,\zeta^{\mp(m-1)}\;. \tag{9}$$

As an alternative way of obtaining these results, one may use the insolation function development of Rubincam [1994; Eq. (6)].

2.3. Linear solution and thermal force evaluation

The linearity of the system (1) – (3) and the development (5) of the insolation term allows us to make a suitable decomposition of the temperature $\Delta T'$ into multipole components

$$\Delta T'(r';\theta,\phi;\zeta) = \sum_{n\geq1}\sum_{k=-n}^{n} t'_{nk}(r';\zeta)\,Y_{nk}(\theta,\phi)\;. \tag{10}$$

The coefficients $t'_{nk}(r';\zeta)$, weighing the different multipole terms, satisfy the following system of decoupled differential equations

$$\begin{aligned} i\zeta\frac{\partial}{\partial\zeta}\quad t'_{nk}(r';\zeta) &= \\ &= \frac{1}{r'^2}\left\{\frac{\partial}{\partial r'}\left(r'^2\frac{\partial}{\partial r'}\right) - n\,(n+1)\right\}\,t'_{nk}(r';\zeta)\;, \end{aligned} \tag{11}$$

with the boundary constraints

$$\sqrt{2}\,t'_{nk}(R';\zeta) + \Theta\left(\frac{\partial t'_{nk}}{\partial r'}\right)_{(R';\zeta)} = a_{nk}(\zeta) \tag{12}$$

at the surface of the body. The regularity of t'_{nk} at the central position $r'=0$ is also assumed. In the next sections we shall obtain a general solution of Eqs. (11) and (12) for the dipole ($n=1$) part of development (5).

When determining the recoil force due to thermal radiation, we assume – in agreement with the boundary condition (3) – that the isotropic Lambert's law holds as far as the directional characteristics of the emission are concerned. Linearizing the fourth power of the surface temperature as before, we obtain

$$\mathbf{f}(\zeta) = -\frac{2\sqrt{2}}{3\pi}\alpha\Phi\int d\Omega\,\Delta T'(R';\theta,\phi;\zeta)\,\mathbf{n}\;, \tag{13}$$

which gives the thermal recoil force per unit of mass of the body. Here, $\Phi = (\mathcal{E}_\star\pi R^2/mc)$ is the usual radiation force factor: m is the body's mass, c the velocity of light and $\mathbf{n}$ the unit vector normal to the surface. Given the multipole development (10) of the temperature variation $\Delta T'$, we easily obtain the following formulæ for the thermal force components

$$f_X(\zeta) + i f_Y(\zeta) \;=\; -\frac{8}{3\sqrt{3\pi}}\,\alpha\Phi\,t'_{1-1}(R';\zeta)\;, \tag{14}$$

$$f_Z(\zeta) \;=\; -\frac{4}{3}\sqrt{\frac{2}{3\pi}}\,\alpha\Phi\,t'_{10}(R';\zeta)\;, \tag{15}$$

confirming that only the dipole part ($n=1$) of the temperature development (10) is relevant at the level of a linear theory. The force components (14) and (15) are given in the co–rotating body–fixed reference frame defined in Sect. 2.1.

2.4. Solution for the seasonal component

Hereafter, we give a solution for the "zonal" part $t'_{10}(r';\zeta)$ of the temperature distribution, which yields the along–spin component of the thermal force [see Eq. (15)]. Given the simplicity of the corresponding $a_{10}(\zeta)$ coefficient from Eq. (8), we may assume: $t'_{10}(r';\zeta) = \kappa_+(r')\zeta + \kappa_-(r')\zeta^{-1}$. The radial functions $\kappa_\pm$ satisfy the equation

$$\left[\frac{d}{dr'}\left(r'^2\frac{d}{dr'}\right) - \left(2\pm ir'^2\right)\right]\kappa_\pm(r') = 0\;. \tag{16}$$

whose solution is given by

$$\kappa_\pm(r') = c^\pm j_1(\sqrt{\mp i}r')\;. \tag{17}$$

Here, $j_1(z)$ is the spherical Bessel function of order 1. Determining the proportionality factors $c^\pm$ in Eq. (17) by the surface boundary constraint (12), we obtain the following expression for the zonal dipole coefficient $t'_{10}(r';\zeta)$ at the surface of the body ($r'=R'$):

$$t'_{10}(R';\zeta) = -\sqrt{\frac{\pi}{6}}\,\sin\gamma\,\frac{E_{R'}\sin(\lambda+\delta_{R'})}{1+\chi}\;. \tag{18}$$

Following the notations of Vokrouhlický (1998a), we introduce the amplitude $E_{R'}$ and the phase $\delta_{R'}$ by

$$E_{R'}\exp\left(i\delta_{R'}\right) = \frac{A(x)+iB(x)}{C(x)+iD(x)}\;, \tag{19}$$

with $x=\sqrt{2}R'$. The auxiliary functions $A(x)$, $B(x)$, $C(x)$, $D(x)$ read

$$A(x) \;=\; -(x+2) - e^x\left[(x-2)\cos x - x\sin x\right]\;, \tag{20}$$

$$B(x) \;=\; -x - e^x\left[x\cos x + (x-2)\sin x\right]\;, \tag{21}$$

$$C(x) \;=\; A(x) + \frac{\chi}{1+\chi}\times \{3\,(x+2) + e^x\left[3\,(x-2)\cos x + x\,(x-3)\sin x\right]\}\;, \tag{22}$$

$$D(x) \;=\; B(x) + \frac{\chi}{1+\chi}\times \{x\,(x+3) - e^x\left[x\,(x-3)\cos x - 3\,(x-2)\sin x\right]\}\;, \tag{23}$$

with the parameter χ in Eqs. (18), (22) and (23) defined by: $\chi = \Theta/(\sqrt{2}R')$. Note that the same quantity had been called λ by Vokrouhlický (1998a).

As we are mainly interested in the mean rate of change (da/dt) of the orbital semimajor axis a due to Yarkovsky effects, we look for the perturbations caused by the along–spin thermal force component f_Z in the along–track direction $-T_1$. Assuming a quasi–circular orbit we have

$$T_1 = f_Z \sin\gamma \cos\lambda + \mathcal{O}(e) \; . \tag{24}$$

Averaging over one revolution of the body around the Sun (i.e. one cycle of the mean longitude λ) we finally obtain

$$\left(\frac{da}{dt}\right)_s = \frac{4\alpha}{9} \frac{\Phi}{\omega_{\rm rev}} \frac{E_{R'} \sin\delta_{R'}}{1+\chi} \sin^2\gamma \; . \tag{25}$$

This formula coincides exactly with the semimajor axis decay rate due to the seasonal variant of the Yarkovsky effect (see e.g. Rubincam 1998; Farinella et al. 1998).

2.5. *Solution for the diurnal/mixed components*

Next, we determine the "tesseral" coefficients $t'_{1\pm1}(r';\zeta)$ of the temperature development (10), which are related to the out–of–axis thermal force components (f_X, f_Y) [see Eq. (14)]. Given the Fourier expansion of the insolation coefficients $a_{1\pm1}(\zeta)$ from (7) and (9) we may expect

$$t'_{1\pm1}(r';\zeta) = \tau_1^{\pm}(r')\,\zeta^{\pm(m+1)} + \tau_{-1}^{\pm}(r')\,\zeta^{\pm(m-1)} \; , \tag{26}$$

and we find again from Eq. (11) that the radial amplitudes $\tau_{\pm1}^{\pm}(r')$ satisfy a system of decoupled homogeneous spherical Bessel equations. Their solution reads

$$\tau_1^{\pm}(r') \;=\; c_1^{\pm} \;\; j_1\left[\sqrt{\mp i\,(m+1)}r'\right] \; , \tag{27}$$

$$\tau_{-1}^{\pm}(r') \;=\; c_{-1}^{\pm}\, j_1\left[\sqrt{\mp i\,(m-1)}r'\right] \; . \tag{28}$$

So far, we have been keeping the normalization of the radial coordinate r by the penetration depth l_s of the seasonal thermal wave. However, a normalization by the penetration depth $l_d = l_s/\sqrt{m}$ of the diurnal wave is more suitable now, and will be used hereafter in this section. Thus

$$\tau_1^{\pm}(r') \;=\; c_1^{\pm} \;\; j_1\left[\sqrt{\mp i\,(1+1/m)}r'\right] \; , \tag{29}$$

$$\tau_{-1}^{\pm}(r') \;=\; c_{-1}^{\pm}\, j_1\left[\sqrt{\mp i\,(1-1/m)}r'\right] \; . \tag{30}$$

After deriving the constant factors $c_{\pm1}^{\pm}$ from the boundary conditions we obtain

$$f_X + i f_Y = -\frac{4}{9}\frac{\alpha\Phi}{1+\chi}\Big\{\sin^2\frac{\gamma}{2}\, E_{R'_+} \exp(-i\delta^n_{R'_+})\,\zeta^{-1} + \cos^2\frac{\gamma}{2}\, E_{R'_-} \exp(-i\delta^n_{R'_-})\,\zeta\Big\}\,\zeta^{-m} \; . \tag{31}$$

Here we defined $R'_{\pm} \;=\; \sqrt{1 \pm 1/m}\,R'$, while the remaining quantities are the same we used previously. Note that, like the insolation coefficients $a_{1\pm1}(\zeta)$, the "diurnal" force components (31) also depend on a linear combination of the rotation and revolution frequencies, but not on the rotation frequency itself. This statement is obviously "coordinate dependent". To make a more obvious link to the results of Vokrouhlický (1998a; Eq. 30), one may transform (f_X, f_Y) force components to the body–centered frame with the axis z along its spin axis and the axis x so that the local direction to the Sun lies in the xz–plane (this system has been used by Vokrouhlický, 1998a). Denoting the equatorial Yarkovsky force components in this reference system by we obtain

$$f_x + i f_y = \frac{f_X + i f_Y}{\sin\theta_0}\left(\sin^2\frac{\gamma}{2}\,\zeta + \cos^2\frac{\gamma}{2}\,\zeta^{-1}\right)\zeta^m \; . \tag{32}$$

Fourier development of (32) contains infinite series of spectral lines depending on both rotation and revolution frequencies. In what follows, we use (31) because of its simplicity.

As in the previous section, we are primarily interested in the contribution of the (f_X, f_Y) force components to the along–track perturbation on a quasi–circular orbit. Simple algebra yields

$$T_2 \;=\; \frac{1}{2}\,(f_Y - i f_X)\left(\sin^2\frac{\gamma}{2}\,\zeta^{m+1} - \cos^2\frac{\gamma}{2}\,\zeta^{m-1}\right) \;+\; \text{C.C.} + \mathcal{O}(e) \tag{33}$$

for the along–track perturbation force; C.C. means the complex conjugate quantity of the previous term. Taking the average of T_2 over one revolution around the Sun we obtain the mean rate of change of the semimajor axis

$$\left(\frac{da}{dt}\right)_d \;=\; -\frac{8\alpha}{9\omega_{\rm rev}}\,\frac{\Phi}{1+\chi}\Big\{\cos^4\frac{\gamma}{2}\,E_{R'_-}\sin\delta_{R'_-} - \sin^4\frac{\gamma}{2}\,E_{R'_+}\sin\delta_{R'_+}\Big\} \, . \tag{34}$$

where the index d reminds us that we are dealing with the diurnal components (f_X, f_Y) of the thermal force. Interestingly, Eq. (34) shows that the diurnal effect on the semimajor axis is not simply proportional to the cosine of the obliquity – as most commonly used – but depends on γ in a more complicated way. However, the classical $\cos\gamma$ result can be recovered realizing that in the typical astronomical applications $\omega_{\rm rot}/\omega_{\rm rev} \;=\; m \;\gg\; 1$. Then, to a high accuracy we can set $R'_{\pm} \;\simeq\; R'$ and $\cos^4\gamma/2 - \sin^4\gamma/2 \;=\; \cos\gamma$. Then, Eq. (34) becomes identical with the classical result (e.g. Vokrouhlický 1998a), confirming that in the fast rotation limit ($m \to \infty$) the diurnal variant of the Yarkovsky effect is naturally decoupled from its seasonal counterpart. It is easy to check that in all astronomical applications listed in Sect. 1 the exact result (34) differs from the classical formula by less than one part in 10^3.

Another consequence of Eq. (34) is that the diurnal effect on the semimajor axis does not vanish exactly at 90° obliquity. This asymmetry implies that the corresponding semimajor axis drift does not average out to zero when an isotropic distribution of spin axes is assumed (e.g., due to frequent and random impact reorientation events). Denoting by angled brackets such an average over all the spin orientations, we obtain

$$\left\langle\left(\frac{da}{dt}\right)_d\right\rangle \;=\; \frac{8\alpha}{27\omega_{\rm rev}}\,\frac{\Phi}{1+\chi}$$

$$\times \left\{ E_{R'_+} \sin \delta_{R'_+} - E_{R'_-} \sin \delta_{R'_-} \right\} . \qquad (35)$$

However, unless the m parameter is unrealistically small, collisional events will anyway tend to spin up the rotation rate besides reorienting the spin axis, the long–term average (35) of the diurnal semimajor axis effect is very small. Quantitatively, one can easily show that $\langle (da/dt)_d \rangle \propto 1/m$ at large m.

3. Conclusions

The main results of this paper can be summarized as follows:

– We have developed a self–consistent, unified linear theory for the thermal Yarkovsky effects on a spherical, rotating body which orbits around a radiation source. The solution contains consistently all terms that depend on both the rotation and the revolution frequency and their mutual combinations. In the limit of a rapidly rotating body the two classical variants of the thermal effects – the so–called diurnal and the seasonal effects previously modeled in a separate fashion – are recovered at the same time.
– Besides rederiving the previously known results for the seasonal and diurnal variants of the Yarkovsky effect, we have computed and clarified the contribution of the "mixed" terms to the mean semimajor axis rate of an orbiting body.
– We have found that in most relevant astronomical applications, such as the motion of asteroidal fragments in the main asteroid belt, the relative contribution of the newly derived mixed terms to the Yarkovsky semimajor axis perturbations is less than one part in 10^3. This is a negative result, but it is important to justify a number of recent studies taking into account only the limit variants of the Yarkovsky effect, namely the diurnal and seasonal effects, in dealing with the meteorite delivery issue. Only in the case of very slowly rotating bodies orbiting in the inner Solar System (a possible case would be that of Mercurian ejecta escaped from the planet's gravity field), the more exact solution derived in this paper might be used to improve the accuracy of the results.

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