

## SPIN VECTOR AND SHAPE OF (6070) RHEINLAND AND THEIR IMPLICATIONS

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### ABSTRACT

Main belt asteroids (6070) Rheinland and (54827) 2001 NQ8 belong to a small population of couples of bodies that reside in very similar heliocentric orbits. Vokrouhlický & Nesvorný promoted the term “asteroid pairs,” pointing out their common origin within the past tens to hundreds of kyr. Previous attempts to reconstruct the initial configuration of Rheinland and 2001 NQ8 at the time of their separation have led to the prediction that Rheinland’s rotation should be retrograde. Here, we report extensive photometric observations of this asteroid and use the light curve inversion technique to directly determine its rotation state and shape. We confirm the retrograde sense of rotation of Rheinland, with obliquity value constrained to be  $\geq 140^\circ$ . The ecliptic longitude of the pole position is not well constrained as yet. The asymmetric behavior of Rheinland’s light curve reflects a sharp, near-planar edge in our convex shape representation of this asteroid. Our calibrated observations in the red filter also allow us to determine  $H_R = 13.68 \pm 0.05$  and  $G = 0.31 \pm 0.05$  values of the  $H-G$  system. With the characteristic color index  $V - R = 0.49 \pm 0.05$  for S-type asteroids, we thus obtain  $H = 14.17 \pm 0.07$  for the absolute magnitude of (6070) Rheinland. This is a significantly larger value than previously obtained from analysis of astrometric survey observations. We next use the obliquity constraint for Rheinland to eliminate some degree of uncertainty in the past propagation of its orbit. This is because the sign of the past secular change of its semimajor axis due to the Yarkovsky effect is now constrained. The determination of the rotation state of the secondary component, asteroid (54827) 2001 NQ8, is the key element in further constraining the age of the pair and its formation process.

*Key words:* minor planets, asteroids: general

*Online-only material:* color figure

### 1. INTRODUCTION

Pairs of asteroids residing in very similar heliocentric orbits were recently discovered in the Hungaria population and in the main belt (e.g., Vokrouhlický & Nesvorný 2008; Pravec & Vokrouhlický 2009; Milani et al. 2010). The orbits of components in a pair, often too similar to be a random fluke in the background population of asteroids, suggest a common origin. Indeed, by backward integration of paired asteroids’ orbits, we were able to identify, in most cases, specific epochs in the past tens to hundreds of kyr when the two components become very close to each other. These close encounters were interpreted as formation events of the pairs during which the two components gently separated from a common parent body.

Asteroid pairs thus share some fundamental properties with related asteroid families, the similarity being most apparent for very young families (e.g., Nesvorný et al. 2006; Nesvorný & Vokrouhlický 2006; Vokrouhlický & Nesvorný 2011); notably, members in both pairs and families arise as fragments from a disintegrated parent asteroid. However, it has been unclear whether they also share a common formation process. Indeed, while the larger asteroid families are obviously of collisional origin, Vokrouhlický & Nesvorný (2008) discussed several other putative formation processes for the asteroid pairs. The hunt

for the formation process of asteroid pairs motivated Pravec et al. (2010) to conduct photometric observations of the primary (larger) components in numerous pairs. Their main results can be summarized as follows: (1) there is a strong correlation between the rotation period of the primary component and the mass ratio of the two asteroids in the pair, and (2) there is a lack of pairs with a mass ratio of the two asteroids greater than  $\approx 0.2$ . The asymptotic behavior of (1) above is as follows: in pairs where one component is much smaller than the other, the primaries systematically rotate very fast (near the rotation fission barrier observed for solitary asteroids; e.g., Pravec et al. 2002), whereas in pairs that have a smaller mass ratio between the larger and smaller components, the primaries systematically rotate very slow. These observations convincingly demonstrate that most of the asteroid pairs were formed by rotational fission rather than catastrophic (collisional) breakup of the parent body (cf. Pravec et al. 2010). The YORP effect<sup>10</sup> has been suggested as the underlying physical mechanism that brought the parent body’s rotation to the fission limit.

To further characterize the principal formation process of asteroid pairs, it is important to both (1) continue observations

<sup>10</sup> The YORP effect is due to torques of scattered sunlight on an asteroid’s surface, and torques due to the thermal radiation of the body itself (e.g., Bottke et al. 2002, 2006).

of parameters of the whole population, and (2) characterize selected pairs as precisely as possible. This work focuses on the second objective. Vokrouhlický & Nesvorný (2008) have already recognized that the pair of asteroids (6070) Rheinland and (54827) 2001 NQ8 is somewhat exceptional among other known pairs since its age can be determined very precisely. This is because it is young, only  $\simeq 17$  kyr, and the two asteroids are large enough that effects of both dynamical chaos and thermal forces are minimized in their past orbital evolution. Vokrouhlický & Nesvorný (2009) extended and substantiated previous work by also taking into account mutual gravitational forces of the two components in the initial phase of their separation. A statistical analysis of the angle between the angular momenta of the heliocentric orbital motion of Rheinland and the mutual motion of the two components at their separation allowed them to assert that Rheinland’s rotation should be preferentially retrograde rather than prograde. In this paper, we probe this conjecture by directly determining Rheinland’s pole orientation (Sections 2 and 3). Using this information, we revisit the determination of this pair’s age by backward tracking its components’ orbits into the past (Section 4).

## 2. OBSERVATIONS

Previous photometry of Rheinland, from its favorable opposition in 2009, has been reported in the supplementary materials of Pravec et al. (2010). In this paper, we report additional observations from three oppositions in 2008, 2009, and 2010–2011. Altogether we thus present 34 light curves whose observational details, such as the aspect data, heliocentric and observer distances, and observing stations, are given in Table 1. More detailed information about the telescopes and data-reduction procedures can be found in the supplementary materials of Pravec et al. (2010).

The data from 2008 are limited, yet they are important for our modeling because they offer a new viewing geometry and help to constrain the precise value of the sidereal rotation period. The data from the 2009 opposition are very numerous, reach up to  $28^\circ$  phase angles before and after opposition, and cover an interval of four months. This is because during the opposition in 2009 September the asteroid was close to perihelion of its orbit and thus was quite bright, up to a magnitude of 15 in the visible band. The data from 2010 to 2011 opposition are fewer because of fainter brightness, but they still cover an interval of nearly four months as well. They are less symmetrically distributed about the opposition in 2011 March, with fewer observations before and more observations after the opposition. The sufficiently long periods of time covered by observations in 2009 and 2010–2011 allow an unambiguous link of the data and provide a unique solution for the rotation period. During the 2009 and 2010–2011 oppositions, the geocentric ecliptic latitudes of the asteroid were different, which provides complementary aspects of view. However, due to a small inclination of Rheinland’s orbit with respect to the ecliptic plane, the difference of the observations’ latitudes was still rather small. As a result, the determination of Rheinland’s rotation pole longitude is problematic and is more uncertain (Section 3).

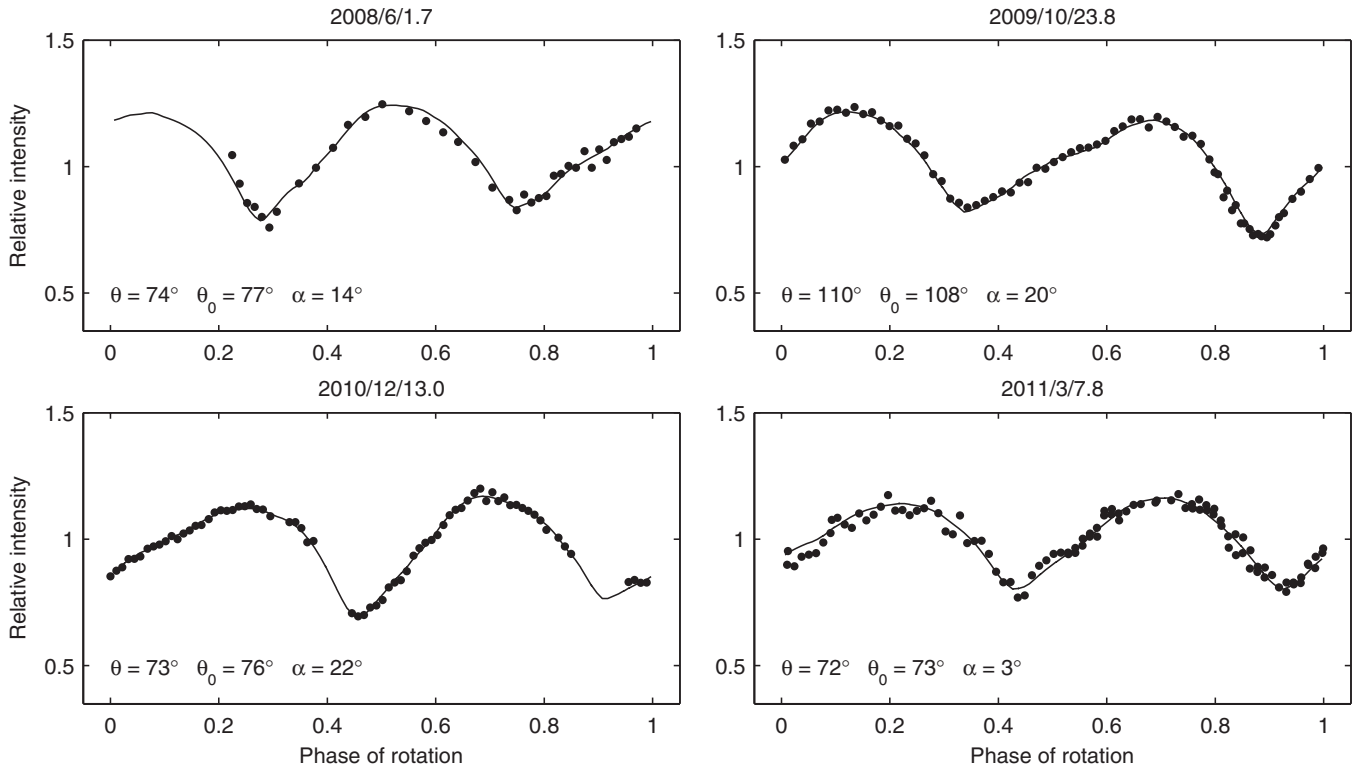
Most of the data are on relative magnitude scales, either in clear or  $R$  filters, but the three nights taken from the Ondřejov Observatory in 2011 February and March were absolutely calibrated in the Cousins  $R$  system using Landolt standard stars. Using the parameterization of the phase function as described by the  $H-G$  system (e.g., Howell et al. 1989), we derived the best-fit values for the absolute  $R$  magnitude  $H_R = 13.68$  and

**Table 1**  
Aspect Data for Observations of (6070) Rheinland

Date	$r$ (AU)	$\Delta$ (AU)	$\alpha$ (deg)	$\lambda$ (deg)	$\beta$ (deg)	Obs.
2008 05 14.9	2.826	1.860	7.5	213.2	3.3	Kh
2008 05 31.7	2.809	1.958	13.5	210.3	2.9	Ma
2008 06 01.7	2.808	1.965	13.8	210.2	2.9	Ma
2009 07 22.0	1.981	1.267	26.4	359.1	-4.6	W1
2009 07 24.0	1.978	1.246	25.9	359.4	-4.7	W1
2009 07 25.0	1.976	1.237	25.7	359.6	-4.7	W1
2009 07 26.0	1.975	1.226	25.5	359.8	-4.7	W1
2009 07 27.0	1.973	1.217	25.3	359.9	-4.8	Kh
2009 07 28.0	1.972	1.208	25.0	0.0	-4.8	Kh
2009 07 32.0	1.965	1.170	24.0	0.5	-5.0	Kh
2009 08 01.9	1.964	1.161	23.7	0.6	-5.1	Kh
2009 08 17.9	1.941	1.032	18.1	1.1	-5.8	W1
2009 08 19.0	1.940	1.024	17.6	1.0	-5.8	Kh
2009 09 20.9	1.905	0.904	3.7	355.7	-6.6	W1
2009 10 20.8	1.888	0.994	18.4	351.1	-5.7	W1
2009 10 23.8	1.887	1.012	19.7	351.0	-5.6	W1
2009 11 17.8	1.886	1.209	27.6	353.4	-4.4	Si
2009 11 18.7	1.887	1.217	27.8	353.6	-4.3	Si
2009 11 20.8	1.887	1.237	28.2	354.1	-4.2	W1
2009 11 22.7	1.888	1.255	28.5	354.5	-4.1	W1
2010 12 13.0	2.621	2.359	22.0	166.5	3.0	Ma
2010 12 17.0	2.629	2.311	21.8	167.2	3.1	Ma
2011 01 08.9	2.670	2.051	18.8	169.1	3.7	Ma
2011 01 09.9	2.672	2.040	18.6	169.1	3.8	Ma
2011 02 24.5	2.744	1.761	3.0	162.2	4.7	Oey
2011 02 26.5	2.747	1.761	2.3	161.7	4.7	Oey
2011 02 27.0	2.747	1.761	2.2	161.5	4.7	Ond
2011 03 01.9	2.752	1.763	1.7	160.8	4.8	Mo
2011 03 03.9	2.754	1.765	1.9	160.4	4.8	W1
2011 03 07.8	2.759	1.774	3.1	159.4	4.8	W2
2011 03 08.8	2.761	1.778	3.6	159.0	4.8	Ond
2011 03 28.9	2.787	1.898	11.3	154.8	4.6	Ond
2011 04 01.8	2.791	1.931	12.6	154.4	4.5	W1
2011 04 02.0	2.792	1.932	12.6	154.4	4.5	W2

**Notes.** The whole data set of Rheinland’s observations used in our work. All but those from the Wise Observatory in 2009, which were already reported in the supplementary materials of Pravec et al. (2010), are new data. The table gives Rheinland’s distance from the Sun  $r$  and from the Earth  $\Delta$ , the solar phase angle  $\alpha$ , the geocentric ecliptic coordinates of the asteroid ( $\lambda$ ,  $\beta$ ), and the observatory (Mo: Modra Observatory, 0.6 m; Ma: Maidanak Observatory, 1.5 m; Oey: Leura Observatory, 0.35 m; W1: Wise Observatory, 0.46 m; W2: Wise Observatory, 1 m; Ond: Ondřejov Observatory, 0.65 m; Kh: Kharkiv Observatory, 0.7 m; Si: Simeiz Observatory, 1 m).

the slope parameter  $G = 0.31$ . Their formal errors, estimated accounting for uncertainties of the absolute calibrations, are 0.02 and 0.03, respectively. A systematic error of absolute magnitude estimated using the  $H-G$  function can be  $\sim 0.05$  mag (see Harris 1991); we adopt this larger uncertainty for our estimated  $H_R$ . The absolute magnitude  $H_R$  is that of the mean value over the light curve cycle. Assuming  $V - R = 0.49 \pm 0.05$ , which is the mean color index for S-type asteroids (e.g., Shevchenko & Lupishko 1998) that predominate in the inner main belt where Rheinland is located, we estimated its absolute  $V$  magnitude  $H = 14.17 \pm 0.07$ . Interestingly, this value is significantly larger than  $H = 13.6$  given by the Minor Planet Center database or  $H = 13.7$  given by the AstDyS database (both use data from astrometric surveys). This example shows the importance of dedicated and accurate photometry in specific projects like the analysis of asteroid pairs.



**Figure 1.** Sample of Rheinland’s photometric data (symbols) fitted with synthetic light curves based on the convex shape model (solid line). We used the formally best-fit model with pole orientation  $(\lambda, \beta) = (4^\circ, -76^\circ)$  in ecliptic longitude and latitude, and sidereal rotation period  $P = 4.27371$  hr. The viewing and illumination geometry is given by the aspect angle  $\theta$ , the solar aspect angle  $\theta_0$ , and the solar phase angle  $\alpha$ .

### 3. POLE AND SHAPE OF RHEINLAND

We used the light curve inversion method of Kaasalainen & Torppa (2001) and Kaasalainen et al. (2001) to derive Rheinland’s shape, sidereal rotation period, and spin axis direction from the available data described in Section 2.<sup>11</sup> We assume that the body rotates about the shortest axis of the inertia tensor which is fixed in the inertial space. This is because (1) Fourier analysis of the individual light curves from different nights was sufficiently well fitted with a single rotation period and its overtones (due to irregular shape),<sup>12</sup> and (2) gravitational and radiative torques can change the spin state only on much longer timespans than the four years between the first and last observations. Fourier fits of the individual light curves were also used to estimate the statistical uncertainty of the individual measurements, a task which is characteristically murky for asteroid photometry. This is because the number of systematic error sources may prevent the assignment of a clean, Gaussian-type uncertainty to the measurements. Still, we are able to discriminate between data with a very low scatter of the neighboring measurements and data with a large scatter of the neighboring measurements, and

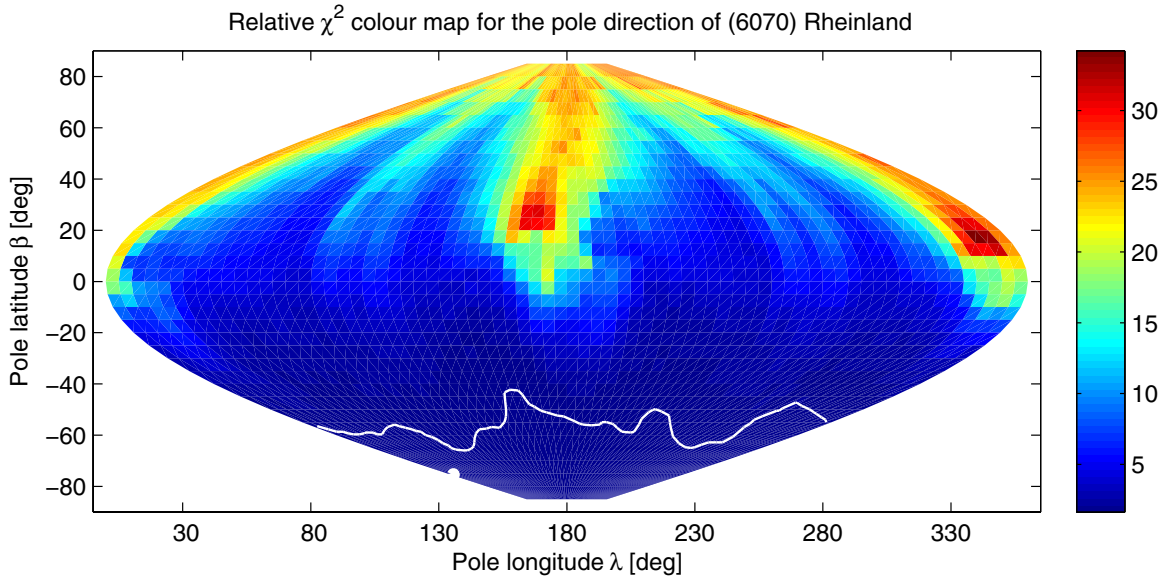
to assign appropriate relative weights to the data. We also assume a convex shape represented with a polyhedron of a certain number (typically hundreds to thousands) of surface facets whose areas are given by the exponential representation described in Kaasalainen & Torppa (2001). We only consider a combination of the Lommel-Seeliger and Lambert scattering of the sunlight on the surface of the asteroid. This method seeks to adjust free parameters in order to minimize a  $\chi^2$ -type target function.<sup>13</sup>

Our best-fit solution has a sidereal rotation period of  $P = 4.27371$  hr and a rotation pole at  $(\lambda, \beta) = (4^\circ, -76^\circ)$ , where  $\lambda$  and  $\beta$  are ecliptic longitude and latitude, respectively. Figure 1 shows a sample of light curve data compared to the model. The pole position is, however, not strongly constrained, and using the longitude–latitude parameterization we cannot simply assign some formal uncertainties. Rather, we show in Figure 2 a whole-sky map of the  $\chi^2$  values for individually best-fitted shape models. Since the  $\chi^2$  values were normalized by the number of degrees of freedom, the solutions with  $\chi^2 \simeq 1$  would formally match the data in a statistical sense. However, we recall that the photometric observation uncertainties may not, strictly speaking, obey the Gaussian statistics and also that systematic and modeling errors are important. For these reasons, the globally best-fit solution has  $\chi^2 = 1.6$ . To make the best-fit solution statistically acceptable, we would have to increase the formal errors of the measurements by about 25%. The  $\chi^2$ -isocontour shown in Figure 2 corresponds to solutions

<sup>11</sup> The whole data set of observations, parameters of the shape model, and further information are available from the DAMIT database at <http://astro.troja.mff.cuni.cz/projects/asteroids3D/web.php> (see also Āurech et al. 2010).

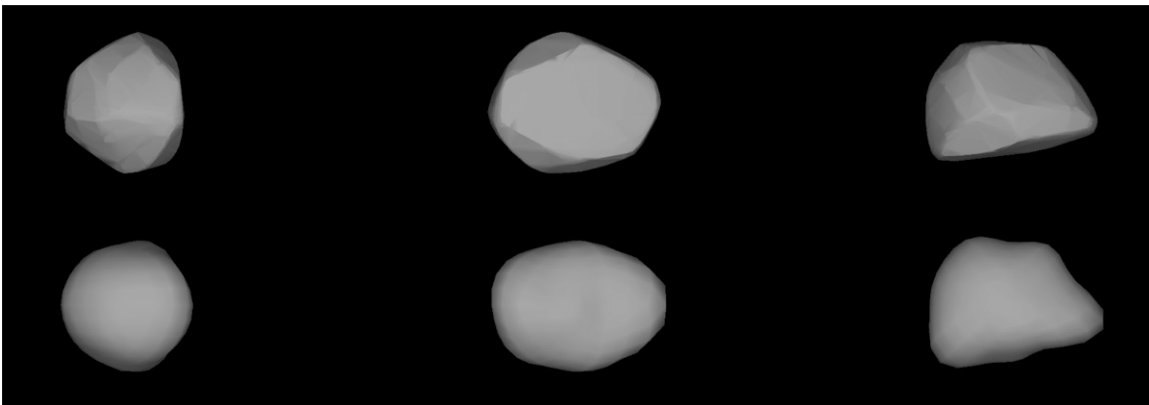
<sup>12</sup> In fact, our observations confirm that (6070) Rheinland is very close to the principal axis rotation mode, which is by itself an interesting result. Note that a characteristic timescale to damp a tumbling state is about 1 Myr for this body (see, e.g., Harris 1994), while the age of the Rheinland–2001 NQ8 pair is much younger (Section 4). This implies that the disruption process that has led to this pair formation was very gentle and did not excite Rheinland’s rotation. Actually, the same conclusion holds also for many primaries in sub-Myr–old pairs analyzed by Pravec et al. (2010).

<sup>13</sup> We have  $\chi^2 = 1/(N - M) \sum_{i=1}^N (O - C)_i^2 / \sigma_i^2$ , where  $N$  is the total number of observations and  $M$  is the number of solved parameters of the model,  $\sigma_i$  is their estimated uncertainty from the analysis of observational scatter about the Fourier representation of the individual light curves, and  $(O - C)_i$  is the difference between the observed and computed brightnesses. For relative photometry, the light curves can be arbitrarily shifted on the magnitude scale.



**Figure 2.** Statistical quality of Rheinland’s pole solutions shown in a sinusoidal projection of the sky in ecliptic coordinates. The grade of shading and the scale bar on the right indicate the  $\chi^2$  value normalized by the number of observations. The globally best-fit solution at  $(\lambda, \beta) = (4^\circ, -76^\circ)$  (full circle) has  $\chi^2 = 1.6$ . The solid line, which delimits solutions with 10% larger  $\chi^2$  value than the best-fit solution, represents our region of admissible solutions (see the main text for details).

(A color version of this figure is available in the online journal.)



**Figure 3.** Shape model for (6070) Rheinland from the light curve inversion analysis. We show two variants of the formally best-fit model: (1) a convex model in the top panels and (2) a non-convex model in the bottom panels. This latter is, however, not unique, and we give it as an example only. The three views are from the equatorial level (left and center) and the pole-on (right).

with a 10% larger  $\chi^2$  value than the global minimum (i.e.,  $\chi^2 \simeq 1.8$ ), which we still consider admissible. Because in our case  $N \simeq 1750$  and the number of parameters  $M \simeq 100$ , the number of degrees of freedom is  $\nu = N - M \simeq 1650$  and the 10% increase of  $\chi^2$  corresponds to about a  $3\sigma$  interval of the  $\chi^2$  distribution with  $\nu$  degrees of freedom.<sup>14</sup> We consider solutions with  $\chi^2 > 1.8$  values to be inadmissible as, indeed, they show too large inconsistencies between the observed and computed magnitudes. Adopting this approach for estimating the uncertainty of our model, we may conclude that the ecliptic longitude of Rheinland’s pole is not yet well constrained, but the ecliptic latitude must be smaller than  $\simeq -50^\circ$ . With only a very small inclination of the orbit with respect to the ecliptic plane (its proper value is  $\simeq 2:18$ ), our result thus implies that the obliquity  $\varepsilon$  of Rheinland’s pole  $\geq 140^\circ$ , with a best-fit solution value of  $\simeq 165^\circ$ . Rotation-period solutions of Rheinland within the admissible zone differ by at most  $\simeq 2 \times 10^{-5}$  hr. We can

thus consider this value a realistic uncertainty for the sidereal rotation period for (6070) Rheinland.

The best-fit shape of Rheinland is shown in Figure 3. The convex representation with 2038 surface facets is shown in the top panels. Panels at the bottom show, for the sake of interest, a non-convex model that has basically the same  $\chi^2$  value as the convex shape solution. In general, the photometry of main belt asteroids, such as Rheinland, cannot unambiguously reveal non-convex features of their surfaces (e.g., Āurech & Kaasalainen 2003). The left and right views in Figure 3 indicate that our shape model of Rheinland has a sharp, planar-like edge. While the light curve data set is still not very abundant and our shape modeling may thus have its limitations, we note that this feature is correlated with the observed steep light curve decreases (see, e.g., near the phase 0.8 in the top right panel in Figure 1) and cannot be entirely artificial. It is tempting to hypothesize that this feature may correspond to the surface zone where the secondary component 2001 NQ8 separated from the parent body of this pair. Further photometric observations of Rheinland are important not only to decrease the persisting uncertainty in

<sup>14</sup> The  $\chi^2$  distribution with  $\nu$  degrees of freedom has a mean of  $\nu$  and a variance of  $2\nu$  (e.g., Press et al. 2007).

**Table 2**  
Osculating Orbital Elements, Their Uncertainties, and Other Parameters of the Asteroid Pair (6070) Rheinland and (54827) 2001 NQ8

Asteroid		$a$ (AU)	$h$	$k$	$p$	$q$	$\lambda$ (deg)	$H$ (mag)
6070	Rheinland	2.388143165	0.06019115	0.20141467	0.02717789	0.00285789	138.859782	14.17
54827	2001 NQ8	2.388531447	0.06005468	0.20149015	0.02716790	0.00285259	174.303041	15.2
Uncertainty ( $\delta a, \delta h, \delta k, \delta p, \delta q, \delta \lambda, \delta H$ )								
6070	Rheinland	1.9e-8	7.0e-8	9.0e-8	6.7e-8	8.1e-8	1.0e-5	0.07
54827	2001 NQ8	4.5e-8	1.0e-7	1.3e-7	9.0e-8	1.1e-7	1.7e-5	0.5?

**Notes.** Osculating orbital elements and their uncertainty are given for epoch MJD 55600 provided by the `OrbFit9` software (<http://adams.dm.unipi.it/~orbmain/orbfit/>). We use a heliocentric equinoctial system of non-singular elements as of 2011 May:  $a$  is the semimajor axis,  $(h, k) = e(\sin \varpi, \cos \varpi)$ , where  $e$  is the eccentricity and  $\varpi$  is the longitude of perihelion,  $(p, q) = \tan(i/2)(\sin \Omega, \cos \Omega)$ , where  $i$  is the inclination and  $\Omega$  is the longitude of node, and  $\lambda = \varpi + M$  is the mean longitude in orbit ( $M$  is the mean anomaly). The default reference system is that of mean ecliptic of J2000. In the case of the primary component, (6070) Rheinland, we use the absolute magnitude  $H$  value determined in Section 2. In the case of the secondary component, (54827) 2001 NQ8, we adopted the absolute magnitude  $H$  value given by the Minor Planet Center.

the pole’s position, but also to confirm this interesting surface feature. Unfortunately, the next favorable opposition that will provide novel viewing geometry of the asteroid, and when the target will be bright enough, starts only in 2013 November and lasts until 2014 January.

#### 4. IMPLICATIONS AND DISCUSSION

The constraint obtained above of the pole orientation for (6070) Rheinland may help us to refine the determination of its age using backward integration of the two components’ orbits in this pair. This is because the known obliquity importantly constrains the value of Yarkovsky effect, one of the two factors that limit our ability of an accurate (deterministic) past orbital reconstruction.

##### 4.1. Backward Orbital Integrations

Detailed description of the age determination of a given pair of asteroids using backward integration of their orbits was given by Vokrouhlický & Nesvorný (2008, 2009). Here, we only outline the main features of the approach, especially where relevant to the findings in this paper.

The currently best-fit osculating orbits of both (6070) Rheinland (primary) and (54827) 2001 NQ8 (secondary), derived from available astrometric observations, are given in Table 2. These data were taken from the `AstDyS` database provided by the University of Pisa (see <http://newton.dm.unipi.it/orbfit/>). Both orbits are fairly well constrained at comparable levels, reflecting that both asteroids have been observed over many oppositions, and hundreds of astrometric measurements are available for each of them. Table 2 gives information about the uncertainty of the six orbital osculating elements  $\mathbf{E}$ , but the complete solution obviously also provides the full covariance matrix  $\Sigma$  of the orbital fit from which mutual correlations can be derived. While these correlations are only moderately significant, with the largest correlation of  $\sim 80\%$  between the semimajor axis and longitude in the orbit solutions, it is important to take them into account. Based on this information, we construct the probability density distribution  $p(\mathbf{E}) \propto \exp[-\frac{1}{2}\Delta\mathbf{E} \cdot \Sigma \cdot \Delta\mathbf{E}]$  (e.g., Milani & Gronchi 2010), where  $\Delta\mathbf{E} = \mathbf{E} - \mathbf{E}^*$  with  $\mathbf{E}^*$  being the best-fit orbital values given in Table 2. All solutions  $\mathbf{E}$  with high-enough values of  $p(\mathbf{E}) \geq C$ , where  $C$  is related to a given confidence level, are statistically equivalent and thus we cannot consider  $\mathbf{E}^*$  as the only orbital realization of either primary or secondary components in our pair of asteroids. Choosing a number of orbits that will represent each of the asteroids in our

numerical simulation, we used  $p(\mathbf{E})$  to determine their initial orbital values  $\mathbf{E}$ . We call these different initial orbital realizations “geometrical clones.” The geometrical clones occupy a six-dimensional ellipsoid in the  $\mathbf{E}$ -space, or—after an appropriate transformation—a six-dimensional ellipsoid region in the Cartesian space of heliocentric positions and velocities. When restricted to a three-dimensional space of heliocentric positions, the geometrical clones occupy a three-dimensional ellipsoid region with the longest axes approximately 200 km and 400 km, respectively, for the primary and secondary components in the Rheinland–2001 NQ8 pair. This shows how tightly constrained both orbits are at the initial epoch. For the sake of comparison with our convergence efforts described below, we note that the size of both uncertainty ellipsoids today is smaller than the radius of the Hill sphere of influence of the primary (Rheinland) component (approximately 1000 km).

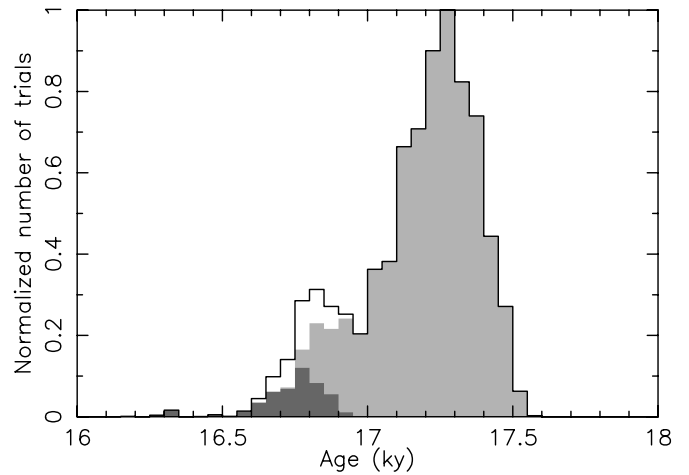
When propagated backward in time, the region occupied by geometric clones extends. In the case of the Rheinland–2001 NQ8 pair, and over the relevant  $\simeq 17.2$  kyr timescale of its age, this extension is basically a simple stretching in the along-track direction by the Keplerian shear of initial orbits with slightly different values of the semimajor axis (in angular terms, the uncertainty translates to about  $\pm 0:02$  uncertainty in the longitude of orbit). This is because the orbits are not affected by any of the major resonances. So while the short axes of the uncertainty ellipsoid only slightly increase with respect to their initial sizes, the long axis stretches to about  $3 \times 10^5$  km some  $\simeq 17.2$  kyr ago. This represents about 300 times the radius of the Hill sphere of influence of Rheinland.

This uncertainty is very small and would have allowed an even more precise age determination of the pair if there was not uncertainty for the second source in the past ephemerides for both components. This latter effect is due to uncertainty in the dynamical model, in particular parameters that influence the strength and direction of the thermal accelerations known as the Yarkovsky effect (Bottke et al. 2002, 2006). The main orbital perturbation by the Yarkovsky effect is a secular change in the semimajor axis, whose magnitude and sign depend on the asteroid’s size, surface thermal inertia, and rotation state. While the asteroid’s size can be roughly estimated from the absolute magnitude and the assumed value of geometric albedo, the surface thermal inertia and rotation state are a priori unknown from astrometric observations. Thermal inertia influences only the magnitude of the effect to a factor which is typically not more than  $\sim 5$  (e.g., Vokrouhlický et al. 2000); however, the spin axis obliquity value determines the overall sign of

the semimajor axis drift: for prograde-rotating asteroids the semimajor axis increases in time, whereas for the retrograde-rotating asteroids it decreases in time. As a result, having been able to constrain Rheinland’s obliquity value, we remove a significant degree of uncertainty in its past orbital evolution. As described in Vokrouhlický et al. (2000), the semimajor axis’s secular change due to the Yarkovsky effect ( $da/dt$ ) directly propagates into a quadratic perturbation in the longitude of an orbit. The Yarkovsky effect thus adds an additional component to the orbital stretching in the long-track direction, and over the  $\simeq 17.2$  kyr timescale it becomes more important than the effect of the initial orbit uncertainty. Using Equation (30) in Vokrouhlický et al. (2000), we obtain  $\pm(0^{\circ}.6-0^{\circ}.7)$  the uncertainty of the Rheinland’s orbit’s longitude  $\simeq 17.2$  kyr ago.<sup>15</sup> This is  $\simeq 30$  times more than the spread of geometrical clones at the same time. Because the Yarkovsky effect magnitude is indirectly proportional to the asteroid’s size, the along-track uncertainty is even larger for the secondary component (54827) 2001 NQ8, for which it amounts to  $\pm(1^{\circ}.1-1^{\circ}.3)$  uncertainty in the orbit’s longitude. This is again an effect  $\simeq 30$  times larger than that produced by the uncertainty of the initial orbital data for this asteroid.

We model the influence of the unconstrained Yarkovsky effect by assigning to each geometric clone a spectrum of Yarkovsky accelerations. We call these different orbital variants “Yarkovsky clones.” To simplify computations, we represent the Yarkovsky acceleration by an empirical transverse acceleration with a magnitude determined by the modeled rate  $da/dt$  of the secular change in the semimajor axis (e.g., Vokrouhlický & Nesvorný 2008, 2009). We used the SWIFT\_MVS numerical integrator for orbit propagation to the past (e.g., Levison & Duncan 1994) with a fixed time step of five days. Perturbations due to all planets, whose initial data at MJD 55600 were taken from the JPL DE405 ephemerides, are included. The empirical formulation of the thermal forces, as described above, has been added to the code. We propagated 20 geometrical and 30 Yarkovsky clones for both primary and secondary components in the Rheinland–2001 NQ8 pair, altogether 600 clones for each asteroid, and examined online their mutual distances every 0.25 yr during the orbital propagation. Because we had preliminary knowledge of the age for this pair, we integrated orbits of all clones to 20 kyr in the past. Velocity components of the initial data, both planets and asteroid clones, were reversed, and the integration time step was positive. With that setting, the true drift rate values of the semimajor axis are reversed. Therefore, while the obliquity  $\geq 140^{\circ}$  for (6070) Rheinland implies a negative value for  $da/dt$ , we assigned formally positive  $da/dt$  values to the Yarkovsky clones of this asteroid in our backward integration (to prevent confusion, however, we use the true  $da/dt$  values in what follows). Because of the unknown value of the surface thermal inertia of Rheinland, we conservatively considered all values of  $da/dt$  between  $-5.3 \times 10^{-5}$  AU Myr $^{-1}$  and 0 (appropriate for this asteroid size; Bottke et al. 2002, 2006). In the case of the secondary component, (54827) 2001 NQ8, we took Yarkovsky clones with both positive and negative  $da/dt$  values. For the

<sup>15</sup> We used  $\sim 3.9$  km size estimate from the absolute magnitude determined in Section 2 and assumed geometric albedo  $\simeq 0.25$ , and thermal inertia  $\sim 200$  J m $^{-2}$  s $^{-0.5}$  K $^{-1}$ , an appropriate mean value for small asteroids of Rheinland’s size (e.g., Delbò et al. 2007). Since we consider an albedo value near the upper limit of the S-type class asteroids of Rheinland’s absolute magnitude, the obtained size is rather underestimated. As in Vokrouhlický & Nesvorný (2008, 2009) we adopt this conservative approach in order to not exclude any possible Yarkovsky drift rates of the semimajor axis from our analysis.



**Figure 4.** Distribution of the number of trials that resulted in a satisfactory convergence solution of (6070) Rheinland and (54827) 2001 NQ8 orbits: time in the past at the abscissa (in kyr) and a normalized histogram of converging solutions in 50 yr bins at the ordinate. We used pairs of clones that approached closer than 750 km and had a relative velocity smaller than  $2$  m s $^{-1}$ . All clones of (6070) Rheinland had negative secular drift in the semimajor axis due to the Yarkovsky effect in agreement with the pole solution from Section 3. The clones of (54827) 2001 NQ8 had both positive and negative drifts in the semimajor axis. The open histogram shown by the enclosing solid line corresponds to all cases; its maximum also serves as the normalization. The light gray histogram corresponds to the cases where the (54827) 2001 NQ8 clones had negative drift in the semimajor axis, while the dark gray histogram corresponds to the cases where the (54827) 2001 NQ8 clones had positive drift in the semimajor axis. The mean value and formal standard deviation of the distributions are  $17.2 \pm 0.2$  kyr in the first case and  $16.75 \pm 0.15$  kyr in the second case (the realistic uncertainty might be slightly larger due to the non-Gaussian nature of distribution functions).

sake of the more detailed analysis below, we actually ran two simulations, first with 30 clones of 2001 NQ8 and  $da/dt$  positive values, and second with 30 clones of 2001 NQ8 and  $da/dt$  negative values. The maximum  $|da/dt|$  value in this case was  $10^{-4}$  AU Myr $^{-1}$ , because the secondary component in the pair has about half the size of the primary.

As described above, some 17 kyr ago the regions of uncertainty in the past ephemerides occupied by the geometric and Yarkovsky clones of both Rheinland–2001 NQ8 pair components resemble very elongated ellipsoids in Cartesian space. Their long axes are  $\sim 4000$  times for Rheinland, resp.  $\sim 15,000$  times for 2001 NQ8, the estimated Hill sphere of influence of Rheinland, which is the quantitative measure of the orbital convergence (see, e.g., Vokrouhlický & Nesvorný 2009). Henceforth only a fraction of propagated clones result in a successful convergence in our numerical experiment. In practice, every 0.25 yr step in our propagation we compute the relative distance and velocity of each Rheinland clone and each 2001 NQ8 clone. We consider the configuration to be convergent when the clone distance is less than 75% of the instantaneous Hill sphere of Rheinland (typically  $\simeq 750$  km) and their relative velocity is less than  $\simeq 2$  m s $^{-1}$  (i.e., the estimated escape velocity from Rheinland). Examining these convergent cases not only provides a constraint of the age for this pair, but it may also provide additional information such as preference between the Yarkovsky clones of the secondary component, 2001 NQ8, with positive or negative  $da/dt$  values.

Figure 4 shows the results of our backward tracking of clones for both primary and secondary components in the Rheinland–2001 NQ8 pair. The light gray histogram corresponds to the run where the Yarkovsky clones of 2001 NQ8 had  $da/dt < 0$ , i.e., the same sign as those of Rheinland. The

black histogram corresponds to the run where the Yarkovsky clones of 2001 NQ8 had  $da/dt > 0$ , i.e., the opposite sign as those of Rheinland. The former case thus means the rotation of 2001 NQ8 has the same (retrograde) sense as that of Rheinland, while the latter case implies the opposite. There are about 15 times more successful convergence solutions in the former case than in the latter. The mean and the standard deviation values of the age estimates are  $17.2 \pm 0.2$  kyr for the former case and  $16.75 \pm 0.15$  kyr for the latter case. As it has been suggested above, not all combinations of clones provide convergent configurations: at best, we had an  $\sim 10^{-3}$  fraction of success. Since the long axes of the ellipsoids occupied by clones have been estimated to  $\sim 4000$ , resp.  $\sim 7500$ , Hill spheres of Rheinland,<sup>16</sup> while the short axes are comparable to the Hill sphere of Rheinland, the  $\sim 10^{-3}$  success rate for convergence implies a very small tilt between the long axes of the uncertainty ellipsoids of the Rheinland and 2001 NQ8 clones. Indeed, our convergent solutions were always characterized with a very small relative velocity of the order of  $10\text{--}30$  cm s<sup>-1</sup>, implying very similar orbits (see also Vokrouhlický & Nesvorný 2008, 2009).

#### 4.2. Rotation State of (54827) 2001 NQ8 and Formation Scenario

While we obtained some convergent solutions for the opposite rotation sense of the secondary component 2001 NQ8 as compared to Rheinland, we had an order-of-magnitude more solutions for the same sense of rotation of both components in the pair. If we were to attribute a purely statistical meaning to this difference, we would conclude that the case of parallel spin orientations of both components in the Rheinland–2001 NQ8 pair is a more likely case. Obviously, such a conclusion is problematic because so far we obtain a convergence solution for both spin orientations of 2001 NQ8. It thus appears that the determination of the rotation state for 2001 NQ8 is the key element for both a better determination of this pair's age and also for constraining the formation process. Note, for instance, that the  $\simeq 5.8764$  hr rotation period of the secondary by itself favors “a prompt ejection scenario” as opposed to “a destabilization of a binary scenario” (see Pravec et al. 2010), but knowing the pole orientation of 2001 NQ8 would provide much more complete information. The analysis would be eased if it were indeed near-to-parallel with the pole of Rheinland, as hinted here, because the spin–orbit secular resonances do not affect the retrograde rotation states (e.g., Vokrouhlický et al. 2006).

#### 4.3. Future Fate of (6070) Rheinland

While the solution of the rotation state and shape of Rheinland in Section 3 is still very limited, we may use it to estimate the value of a secular change in its rotation rate  $\nu = d\omega/dt$  due to the YORP effect. One should take this exercise as an example of interest rather than a true prediction, since the YORP effect has been shown to eventually depend on many unknown or inaccurately known parameters such as the small-scale structures of the asteroid shape (e.g., Statler 2009; Breiter et al. 2009) or inhomogeneities in the density distribution (e.g., Scheeres & Gaskell 2008). Taking thus the best-fit solution for Rheinland's shape and rotation state from Section 3, we obtain  $\nu \simeq 10^{-9}$  rad d<sup>-2</sup>. In terms of magnitude, this is about the expected value for an asteroid of its size and heliocentric

distance if we appropriately scale the directly detected YORP values for (54509) YORP (e.g., Lowry et al. 2007; Taylor et al. 2007), (1862) Apollo (e.g., Kaasalainen et al. 2007), or (1620) Geographos (e.g., Āurech et al. 2008). The positive sign of  $\nu$  implies that the rotation rate of Rheinland is accelerated by the YORP torques, and in  $\sim(50\text{--}100)$  Myr it may bring its rotation state to the fission limit. Assuming the Rheinland–2001 NQ8 pair was actually born by rotational fission of a precursor asteroid, this would have been at least the second such event for the same body. While future improved shape solutions for Rheinland, from larger observation data sets, may modify our result, we consider this to be an example of a process that may actually be frequent for small asteroids in the main belt: a sequence of fission events driven by YORP torques that continually erode the body by mass shedding and producing either paired secondaries or binary systems. We note that the estimated timescale above is quite a bit shorter than the collisional lifetime of Rheinland, some  $\sim 1$  Gyr according to Bottke et al. (2005). Unfortunately, the small value of  $\nu$  means that we will not be able to directly measure the YORP effect for this asteroid any time soon.<sup>17</sup> One can easily estimate that at least four to five decades with suitably distributed data are necessary for this task.

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<sup>16</sup> The smaller value of the long axis for clones of 2001 NQ8, as compared to that given above, is because we propagate cases for positive and negative Yarkovsky drift rates in two different simulations.

<sup>17</sup> On the other hand, the long timescale for YORP acceleration of Rheinland's rotation rate is fortunate, since the currently observed rotation rate of Rheinland is the same as at the moment of separation. Therefore, the observed rotation periods of primaries directly probe the separation process of components in asteroid pairs (Pravec et al. 2010).

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