Improved constraint on the α_1 PPN parameter from lunar motion

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Some theories of gravity, alternative to general relativity, introduce a preferred rest frame for local gravitational physics. At the post-Newtonian level, these preferred-frame effects in Lagrangian-based theories are described by two phenomenological parameters α_1 and α_2, of which only the latter is strongly constrained today. Following previous theoretical suggestions of Nordtvedt, Damour, and Vokrouhlický, we show that the analysis of the lunar motion provides an improved bound of the former parameter, notably $\alpha_1=(-8\pm9)\times10^{-5}$ realistic error [and $\alpha_1=(-8\pm4)\times10^{-5}$ (90% C.L.)]. [S0556-2821(96)50222-5]

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I. INTRODUCTION

Modern unification theories suggest that the (long-range gravitational) interaction between macroscopic bodies may be mediated not only by the metric tensor field, but also by other scalar, vector, or tensor fields. In the theories that contain vector or other tensor fields, in addition to the metric field, distribution of matter in the Universe typically selects a preferred rest frame for local gravitational physics. At the post-Newtonian level, appropriate for dynamics of the solar system bodies, such preferred-frame effects are phenomenologically describable by two parameters α_1 and α_2 [1–3]. While the latter of the two parameters has been tightly constrained by noting the close alignment of the solar spin axis with the angular momentum vector of the whole solar system [4], the former is constrained relatively weakly. The best current solar system value $\alpha_1=(2.1\pm3.1)\times10^{-4}$ (90% C.L.) has been reported in Ref. [5], while analysis of the binary pulsar data yields a slightly better value of $|\hat{\alpha}_1|<1.7\times10^{-4}$ (90% C.L.) [6].[1] These constraints are only slightly better than the corresponding limits on the much more fundamental parameters β and γ [7,8].

This situation motivated several suggestions for experiments aiming to obtain better limits on α_1. For instance, Damour and Esposito-Farèse [10] considered motion of artificial Earth satellites on well-tuned orbits as possible probes of the α_1 parameter. However, despite a network of singularly sensitive satellite orbits, frozen with respect to fixed space, it seems that none of the currently laser-tracked objects suit the task [with regard to the Laser Geodynamics Satellite (LAGEOS), the geodynamics satellite with the currently best-measured orbit, see Ref. [11]].

Another promising candidate of a well-tracked Earth satellite is the Moon. Nordtvedt considered this possibility soon after the lunar laser ranging (LLR) project became operational and the parametrized-post Newtonian (PPN) framework classifying gravitational theories was formulated [12]. Recently, Damour and Vokrouhlický [13] reexamined the problem by using a more involved analytical method (Hill-Brown technique) and confirmed that the lunar ranging data may include interesting information on the α_1 parameter. The latter work is the prime motivation for our Rapid Communication, in which we aim to use the lunar data to obtain a new bound on the α_1 PPN parameter.

II. THE METHOD AND DATA SET

Because we use primarily lunar ranging data, it is appropriate to focus on the dynamical influence of the preferred-frame effects on the three-body system consisting of the Moon (hereafter indexed by 1), the Earth (indexed by 2), and the Sun (indexed by 3). The associated Lagrangian that enters the gravitational dynamics of N-bodies, reads [10] $(A,B=1,2,3)$

$$L_{\alpha_1}=-\frac{\alpha_1}{4}\sum_{A\neq B}\frac{GM_AM_B}{c^2r_{AB}}(\mathbf{v}_A^0\cdot\mathbf{v}_B^0), \tag{2.1}$$

where M_A is the mass of body A, $\mathbf{r}_A$ its position coordinate vector in the solar system center-of-mass frame $(r_{AB}\equiv|\mathbf{r}_A-\mathbf{r}_B|)$, and $\mathbf{v}_A^0$ its "cosmic velocity," which splits into the velocity $\mathbf{v}_A$ of the body relative to solar system center of mass, and the global motion $\mathbf{w}$ of the solar system with respect to the gravitationally preferred frame. A standard assumption [3] links the latter frame to the rest system of the cosmic microwave radiation. In this case $w\equiv|\mathbf{w}|=368.9\pm2.5$ km/s in the direction $(\alpha,\delta)=(168^0\pm0.07^0, -7.23^0\pm0.07^0)$ [14,15].

[1] Note, however, that the strong field parameter $\hat{\alpha}_1$ can be expressed as a linear function of its weak field counterpart α_1, discussed in this paper, and an additional parameter α_1' [9,6]. Combining the solar system and binary pulsar tests of the preferred-frame effects, qualitatively independent, thus yields the possibility to constrain the two parameters α_1 and α_1'.

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Because of the smallness of the investigated effects (see the existing limits on α_1 from Sec. I), we adopt the following simplifications: (i) the solar motion with respect to the solar system center of mass is neglected (hence, $\mathbf{r}_3 \equiv 0$, $\mathbf{v}_3^0 \equiv \mathbf{w}$), (ii) in deriving formulas for the ''α_1-accelerations,'' resulting from the Lagrangian (2.1), we substitute only the leading Newtonian parts for the accelerations (neglecting tidal influence of the solar gravitational field in the Earth-Moon vicinity). The geocentric coordinate position of the Moon is denoted by $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and the corresponding velocity $\mathbf{v} = d\mathbf{r}/dt$. The barycentric accelerations corresponding to the (hypothetical) preferred-frame effect modification of the solar system dynamics of the Earth and Moon then read

$$\left(\frac{d^2\mathbf{r}_1}{dt^2}\right)_{\alpha_1} = \frac{\alpha_1}{2}\frac{GM_2}{c^2r^2}\left[(\mathbf{v}_1^0\cdot\mathbf{v}_2^0)\mathbf{n} + r\frac{d\mathbf{v}_2}{dt} - (\mathbf{n}\cdot\mathbf{v})\mathbf{v}_2^0\right] + \frac{\alpha_1}{2}\frac{GM_3}{c^2r_1^2}[(\mathbf{v}_1^0\cdot\mathbf{w})\mathbf{n}_1 - (\mathbf{n}_1\cdot\mathbf{v}_1)\mathbf{w}], \tag{2.2a}$$

$$\left(\frac{d^2\mathbf{r}_2}{dt^2}\right)_{\alpha_1} = -\frac{\alpha_1}{2}\frac{GM_1}{c^2r^2}\left[(\mathbf{v}_1^0\cdot\mathbf{v}_2^0)\mathbf{n} - r\frac{d\mathbf{v}_1}{dt} + (\mathbf{n}\cdot\mathbf{v})\mathbf{v}_1^0\right] + \frac{\alpha_1}{2}\frac{GM_3}{c^2r_2^2}[(\mathbf{v}_2^0\cdot\mathbf{w})\mathbf{n}_2 - (\mathbf{n}_2\cdot\mathbf{v}_2)\mathbf{w}], \tag{2.2b}$$

Here, we denote $\mathbf{n} = \mathbf{r}/r$, $\mathbf{n}_1 = \mathbf{r}_1/r_1$, $\mathbf{n}_2 = \mathbf{r}_2/r_2$, and $d\mathbf{v}_1/dt$, $d\mathbf{v}_2/dt$ indicate the Newtonian accelerations. One must be careful when using the previous formulas. For instance, it is necessary to subtract the w^2 part from the first terms in square brackets, it being simply a recalibration of the local gravitational coupling factor G.

The above formulation of the preferred-frame perturbation has been introduced into our numerical model for integration of the orbits of Moon, Earth, and Sun, as well as the other bodies of the solar system (including several asteroids). The force terms (2.2) generate an α_1 ''partial,'' which as a correction to the nominal orbits allows a least squares fit of α_1 along with other LLR model parameters.

As an alternative method, and cross-check, to the fully numerical procedure described above, we used the analytical formulas for the radial orbit perturbation derived in Refs. [12,13]. They yield an estimate of the amplitudes for selected principal spectral lines, namely those with frequencies n (sidereal period), n' (annual period), and $n-2n'$. In the course of our work we found that this set of analytic terms is not sufficiently complete for obtaining comparably precise results as the direct numerical treatment. ''Missing'' terms are due to eccentricity of the lunar orbit (or, in the terminology of Refs. [13,18], to coupling between free and forced perturbations of the lunar motion). This problem has been addressed by Nordtvedt, using procedures analogous to those employed in Ref. [16], and he has shown that the annual lunar perturbation produces, apart from the principal effect mentioned above, important sidebands at frequencies (n_0-n') and (n_0+n') (n_0 being anomalistic frequency). The resulting oscillations of the lunar range are given by

$$\delta r \simeq -\alpha_1\frac{wV}{2c^2}re(1+2n/n')\cos[(n_0-n')(t-t_0)-\phi_-], \tag{2.3a}$$

$$\delta r \simeq -\alpha_1\frac{wV}{2c^2}re(1-2n/n')\cos[(n_0+n')(t-t_0)-\phi_+], \tag{2.3b}$$

with V expressing mean velocity of the Earth-Moon center-of-mass motion around the Sun. Phases $\phi_\pm$ are given by addition and subtraction of a longitude angle of perigee occurrence ϕ_p and a longitude angle ϕ_a of $\mathbf{w}$ measured in ecliptic from the lunar (and solar) position at time t_0 corresponding to an arbitrary new-moon phase. Expressed in physical quantities related to the lunar orbit, we obtain amplitudes for the terms (2.3) of about 38 α_1 and 31 α_1 meters, respectively, which is comparable to the direct effects. Principal effects, supplemented by terms (2.3), are used for the analytical estimate of α_1. It should be noted that there exist a number of additional eccentricity and inclination induced sidebands with smaller amplitude. Although they are of less interest for analytical understanding of the results, they may help in decorrelating different types of preferred direction effects (see below).

The data set involved in our analysis consist of the LLR measurements collected by the McDonald Observatory (USA), Haleakala station (USA), and OCA/CERGA (France) in the period March 1970 to February 1996. Dates of operation of the three stations and discussion of the continually increasing data precision can be found in Refs. [7,8]. Apart from estimating the α_1 parameter our code solves for about 160 parameters, such as body initial positions and velocities, the Earth station and lunar retroreflector locations, the Earth rotation parameters, geophysical and selenophysical constants, other PPN parameters etc. General information can be found in Refs. [19–21], some previous estimates of the relativistic parameters are in Ref. [22].

III. RESULTS AND DISCUSSION

Along with the α_1 parameter, we estimate a minimal set of other PPN parameters, namely Eddington's β and γ. Time variation of the gravitational constant $\dot{G}/G$ has been discarded from our tests because most physically well-motivated gravitation theories with appropriately constrained parameter values for β, γ, and α_1 have negligibly small value of $\dot{G}/G$. de Sitter precession has been included in our analysis.

The Eddington parameters do not show significant sensitivity to the frequencies listed in the previous section [23,17,8], and their fit is well decorrelated from the estimation of α_1. The following realistic errors[2] have been ob-

[2]The precisions with which γ and β are explicitly determined from LLR data cannot be understood theoretically at this time. This has also been noted by Williams *et al.* [8], who obtain LLR fits for γ and β comparable to our fits. We present these results in hope of stimulating interest in this problem.

tained: $\beta - 1 = (0.9 \pm 5.0) \times 10^{-3}$, $\gamma - 1 = (-0.8 \pm 6.0) \times 10^{-3}$, and an error of 0.15 milliarcseconds per year for de Sitter precession of the lunar orbit. For α_1 we obtained an estimate $\alpha_1 = (-8 \pm 4) \times 10^{-5}$ (90% C.L.).

An important part of this work consists of estimating the ''realistic'' error of the α_1 parameter fit. Although the nature of an error estimation in case of very complex problems (such as that of lunar orbit determination) is to some degree subjective, we tried to follow some systematic steps. First, we checked whether presence of the α_1 parameter in other sectors of the gravity theory could modify the results. We considered two such known contributions: (i) modification of the gravitational to inertial mass ratio of bodies such as the Earth and Moon in proportion to α_1 [12,9], (ii) an annual term in relativistic geodetic precession [12,3,10]. Neither of these effects altered estimation of α_1.

There are a number of candidates among nonrelativistic or geometrical effects that may show aliasing with the α_1 signal. Covariance analysis represents an approximate measure of this. The largest correlation with other LLR parameters is smaller than 0.4 (e.g., tidal lag angle and velocity of the Earth about the Sun). Such a low correlation coefficient means that α_1 can be well determined within the LLR model. However, our LLR model is only complete up to a certain level. Some physical effects are just modeled with an accuracy corresponding to about 1 cm in the Earth-Moon distance, e.g., solid Earth tides, atmospheric correction, Earth rotation. Other effects are not considered at all, e.g., ocean loading, atmospheric loading. These error sources can lead to signals that influence the determination of α_1. To check the magnitude (worst case) of these effects, we analyzed the post-fit residuals, and obtained values between 0.5 cm and 0.8 cm for the amplitudes of annual, semiannual, and sidereal periods. Therefore a value of 8×10^{-5} has been added to the formal α_1 error in consideration of the influence of unmodeled or insufficiently modeled effects in our computer program. A further error may be introduced by using quantities in the LLR analysis that are of limited accuracy but cannot be improved during the fit, e.g., nutation coefficients, coefficients of the lunar gravity field of higher degree and order, or the solar mass. For such error sources we have added a value of about 4×10^{-5} to the error of α_1.

In conclusion, we arrive at $(8 \pm 9) \times 10^{-5}$ for a realistic error bar of the α_1 PPN parameter.

Analytical estimation of the preferred frame effects mentioned in the previous section offers the possibility to cross-check numerical results given above. Five principal spectral lines [three terms from Ref. [13] and those from Eq. (2.3)] have been introduced in our program and led to the following estimate of α_1: $\alpha_1 = (-9 \pm 6) \times 10^{-5}$ (90% C.L.). The weak disagreement between the numerically and analytically estimated values of α_1 probably results from the rich spectrum of the complete preferred-frame perturbation.

Particular care is paid to possible correlation between the preferred-frame lunar orbit perturbation with that due to cosmic polarization [24,25,13]. The latter acts principally at the sidereal period, competing at this frequency with the preferred-frame effects, but having negligible sidebands. The phases of the two effects at this frequency are close, because the apex direction of the solar system motion with respect to the cosmic microwave rest frame is nearly perpendicular to the direction towards the galactic center. However, simultaneous solution for α_1 and a (hypothetical) differential cosmic acceleration g_c of the Earth and Moon suggests satisfactory decorrelation of the two effects [correlation coefficient of about 0.2]. The rich spectrum of the preferred frame perturbations of the lunar orbit, discussed previously, appears to be the essential factor in this respect.

In conclusion we note that the LLR data analysis contains high-quality information about possible anisotropy of the ''gravitational constant.'' Exploiting this possibility, we obtained here a new limit on the α_1 parameter. Nordtvedt [16] has recently suggested that an interesting LLR measurement bound can be obtained for the α_2 parameter as well. This task remains for the future.

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