ALBEDO PERTURBATION MODELS : GENERAL FORMALISM AND APPLICATIONS TO LAGEOS

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Abstract. The force due to radiation pressure on a satellite of arbitrary shape is written in a general form within a formalism similar to that used in the theory of radiative transfer in atmospheres. Then the corresponding integrals are evaluated for the simple case of a spherically symmetric satellite, and applied to model the perturbation due to the Earth–reflected radiation flux on LAGEOS. For this purpose, the optical behaviour of the Earth's surface and atmosphere is described as a combination of Lambertian diffusion (continents), partial specular reflection consistent with Fresnel law (oceans) and anisotropic diffusion according to Chandrasekhar's radiative transfer theory (clouds). The in–plane Gauss components T and S vs. mean anomaly are computed for a simple orbital geometry and for different models of the Earth's optical properties. A sensitive dependence is found on the assumed cloud distribution, with significant perturbations possibly arising from oceanic specular reflection when the satellite is close to the Earth's shadow boundaries.

Key words: Artificial satellites – non-gravitational perturbations – radiation mechanisms.

1. Introduction

Non–gravitational perturbations on artificial satellites have been an active research subject for celestial mechanicians in the 50s and the early 60s, at the beginning of the space age; later on, the widespread use of purely numerical models for predicting and fitting the orbital evolution of satellites and probes decreased the interest in more general and/or abstract approaches. However, starting in the late 70s the situation has changed again. The main reason has been the availability of very accurate tracking techniques, specifically designed nearly spherical and "passive" satellites and drag–free technology — to be applied in particular to space geodesy and geodynamics, but also to other purposes, such as general relativity tests. In this context, new physical mechanisms have been analyzed and new formalisms and algorithms have been developed, both for predicting the instantaneous value of very tiny non–gravitational forces, and for modelling their long–term orbital effects (for a general review, see Milani *et al.*, 1987).

One of the most complex such forces is that due to radiation pressure resulting from light rediffused or reflected from the Earth's surface and atmosphere — commonly referred to as *albedo force* or *albedo perturbation.* Though the corresponding force is typically at least one order of magnitude smaller than that due to solar radiation pressure, the model uncertainties are normally higher, due to the

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difficulties involved in modelling the momentum flux associated with radiation for a source such as the Earth, geometrically extended and having a complex and variable optical behaviour. Often in the past simplifying approximations have been used — e.g., uniform albedo, purely diffusive behaviour, etc. — which neglect some critical features of the process resulting into detectable orbital perturbations. For this reason, in the last decade a significant number of papers have been devoted to discuss more general or refined treatments of the albedo force, with applications to spacecraft as diverse as LAGEOS, ERS–1, Magellan and MACEK (Anselmo *et al.*, 1983; Barlier *et al.*, 1986; Rubincam and Weiss, 1986; Rubincam *et al.*, 1987; Borderies, 1990; Borderies and Longaretti, 1990; Lucchesi and Farinella, 1992; Vokrouhlický and Sehnal, 1992a, 1992b). Still, however, these models are typically based on "paradigms" on the relevant physical mechanisms which limit their suitability in a more general framework.

The main purposes of this paper are the following : (i) to present very general formulae, inspired by the treatment of radiative transfer in planetary and stellar atmospheres, that give the force due to radiation pressure on a satellite of arbitrary shape moving within the radiative field of an arbitrary source, and that can then be specialized to deal with the simple case of a spherically symmetric satellite (Section 2); (ii) to discuss a physical model of the optical behaviour of the Earth's surface and atmosphere, which is relatively simple from a physical point of view but at the same time realistic enough to incorporate some subtle critical effects (Section 3); (iii) to apply these formulae and models to LAGEOS, a satellite for which the assumption about spherical symmetry provides a good approximation of its real structure and whose laser–tracked orbit has shown residual unmodelled accelerations probably caused for a significant part by the albedo force (Section 4).

2. Acceleration of a Satellite Plunged into a Radiative Field

In this section we will formulate in a general manner the mathematical tools needed for the derivation of the perturbative acceleration of a satellite, arising from the interaction with a radiative field. Of course, this is not the first attempt to deal with this problem (for another recent treatment see e.g. Borderies and Longaretti, 1990), but our aim is to present the theory in a different formalism. Our approach — which was already partially used by Vokrouhlický and Sehnal, 1992a,b) is inspired by the techniques widely used in the radiative transfer theory applied to planetary and stellar atmospheres (see e.g. Mihalas, 1978).

Our basic physical quantity is *the radiative intensity* I, which from the macroscopic point of view fully describes the radiative field interacting with the satellite. More precisely, this is true provided one neglects the effects of polarization, which are negligible as far as the radiative acceleration of satellites is concerned. Moreover, we use the frequency integrated radiative intensity, as we neglect any frequency dependence of all the relevant processes. This is actually a strong approximation, which *a priori* rules out the possibility of taking into account energy transfers due to frequency–dependent processes in the Earth's surface and atmosphere and thus, for instance, of describing by a unified theory the albedo effect (due to visible light) and the perturbations due to the Earth's thermal radiation. This can be justified *a posteriori* by considering that the dynamical effects of the infrared radiation field are very different, owing to its nearly–isotropic geometry with no significant day–night asymmetry (see e.g. Rubincam, 1987).

Many previous papers devoted to this subject use from the very beginning the *radiative flux*, which in our approach is a quantity derived from the radiative intensity (see Eq. (1) below). In our opinion, a clear formulation of the basic equations using as a fundamental quantity the radiative intensity is useful, in particular when deriving more complicated formulae (e.g. in the case of a complex satellite geometry or of a radiative field without any symmetry). The formulation of Borderies and Longaretti (1990) is close to our approach (note that they speak about *the radiance* rather than about radiative intensity; we adopt the latter terminology which is closer to that of the classical radiative transfer theory). They concentrate mainly on some subtle methods aiming at integrating their formulae analytically as far as possible (for another impressive analytical work see Rubincam and Weiss, 1986). Our goal here is different — to present a formulation (i) as compact and clear as possible; (ii) putting no restriction on the involved radiative field; and (iii) allowing to incorporate directly the results of the radiative transfer theory of the stellar/planetary atmospheres. Of course, when the equations have to be applied to specific concrete situations, we will need to adopt suitable numerical integration methods; on the other hand, the analytical formalisms such as those quoted above can be applied only after strongly restricting the generality of the model (e.g., by assuming purely diffusive light scattering by the Earth).

Let us assume that the satellite surface is a two–dimensional surface $\mathcal{S}$ parameterized by any two variables μ, φ (for instance, for a spherical satellite we can use $\mu = \cos\vartheta$, ϑ, φ being some spherical angles). Moreover, we assume that the satellite is plunged into a radiative field $\mathcal{R}$ parameterized by another pair of variables ν, ϕ (for instance, specifying a direction; in any specific situation both sets of parameters together with the corresponding reference frame will be explicitly given). Under this assumptions, the perturbative acceleration is given by the following formula

$$\mathbf{a} = \frac{1}{mc} \int_{\mathcal{S}(\mu,\varphi)} dA(\mu,\varphi) \left[(\mathbf{E} + \rho(\mu,\varphi)\mathbf{R}(\mathbf{N}(\mu,\varphi))) \cdot \mathbf{J}_1 + \frac{2}{3}\delta(\mu,\varphi)\mathbf{J}_2 \right] , \quad (1)$$

where

$$\mathbf{J}_1 = -\int_{\mathcal{R}(\nu,\phi)} \Pi_{\mathbf{n}(\nu,\phi)}(\mathbf{N}(\mu,\varphi))\, I(\nu,\phi;\mu,\varphi)\, \tilde{\Theta}(\nu,\phi;\mu,\varphi)\, d\nu d\phi \, ,$$

$$\mathbf{J}_2 = -\int_{\mathcal{R}(\nu,\phi)} \Pi_{\mathbf{N}(\mu,\varphi)}(\mathbf{n}(\nu,\phi))\, I(\nu,\phi;\mu,\varphi)\, \tilde{\Theta}(\nu,\phi;\mu,\varphi)\, d\nu d\phi \, ,$$

$$\mathbf{R}(\mathbf{a}) = 2\Pi_{\mathbf{a}} - \mathbf{E} \quad , \quad \tilde{\Theta}(\nu,\phi;\mu,\varphi) = \Theta(-\mathbf{N}(\mu,\varphi)\cdot\mathbf{n}(\nu,\phi)) \; .$$

Here $dA(\mu,\varphi)$ is the satellite surface element, $\mathbf{E}$ is the unit matrix of dimension 3, Π is a projection operator defined by : $\Pi_{\mathbf{a}}(\mathbf{b}) = (\mathbf{a}\cdot\mathbf{b})\,\mathbf{a}$, $\Theta(x)$ is the Heaviside step function, c is the velocity of light and m is the satellite mass, $\mathbf{N}(\mu,\varphi)$ is the surface normal and $\mathbf{n}(\nu,\phi)$ the unit vector in the the direction of a chosen ray. The three terms in Eq. (1) describe respectively the linear momentum of radiation removed from the incident beam (both by absorption and scattering), specular reflection (phenomenologically described by the coefficient $\rho(\mu,\varphi)$) and diffusive scattering by the satellite's surface (phenomenologically described by the coefficient $\delta(\mu,\varphi)$). We stress that this decomposition of the interaction of radiation with the satellite surface is just another *paradigmatic* assumption of the radiation pressure model, whose validity may become questionable under specific circumstances. Let us also note, that we take into account only processes connected with direct momentum exchange between radiation and satellite, disregarding the emission of *thermal* radiation from it. The latter effect is due to the true absorption (i.e. contingent on energy exchange between the radiation and satellite). Although the two effects are physically related, their consequences are different : while the reflected radiation acts *immediately* and (roughly speaking) in a direction opposite to that of the source, the thermal re-emission of the absorbed radiation is normally subjected to some time delay (the so–called *thermal inertia*, which for a spinning satellite may show up as a phase delay) and also depends on other parameters characterizing the structure and orientation of the satellite itself. This effect has been studied in detail for LAGEOS (see Rubincam, 1987; Afonso *et al.*, 1989; Farinella *et al.*, 1990; Scharroo *et al.*, 1991) and we will not deal with it further in the present paper.

It is to be stressed that formula (1) does not take into account possible role of the photon multiple reflections on the satellite surface. As for the specular reflection, this rather complicated task was treated by tricky numerical models (e.g. Renard and Koeck, 1989). However, keeping in mind the LAGEOS application, the satellite shape is simple enough (spherical in a good approximation) so that formula (1) is sufficiently general. Moreover, since no shadowing of one satellite part by another can occur, we suppress the satellite surface element $(\mu,\ \varphi)$ dependence of the radiative intensity I (see formulas (2-5)).

We do not give here a full formal proof of Eq. (1), for which we refer to Vokrouhlický, 1991. The essential hints can also be found in Vokrouhlický and Sehnal, 1992a, who also give a proof of the full equivalence with the formulae of Borderies (1990). Instead, we will apply Eq. (1) to evaluate the total acceleration in a simple particular case, that of a spherical satellite with constant optical parameters $(\rho,\ \delta)$. Eq. (1) involves in fact a double integration (say, "over the satellite surface" and "over the radiation field"). We shall show that in the case of a spherical satellite the integral over the satellite surface can be evaluated analytically, leaving just one integral in the final formula. The introduction of a spherical angle parameterization of the satellite surface is advantageous in this case, though we shall fix later on the reference frame (in fact we will profit from this freedom to simplify the computations).

For practical reasons let us treat the three terms in Eq. (1) separately. The first one (E in the integrand brackets) is to be treated in the following way :[1]

$$\begin{aligned}
\mathbf{a}_A &= -\frac{R_{\mathcal{L}}^2}{mc}\int_{\mathcal{S}(\mu,\varphi)} d\mu d\varphi \int_{\mathcal{R}(\nu,\phi)} d\nu d\phi \, (\mathbf{n}(\nu,\phi)\cdot\mathbf{N}(\mu,\varphi))\,\mathbf{n}(\nu,\phi) I(\nu,\phi) \\
&\qquad \tilde{\Theta}(\nu,\phi;\mu,\varphi) \\
&= -\frac{R_{\mathcal{L}}^2}{mc}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi \, I(\nu,\phi)\,\mathbf{n}(\nu,\phi) \int_{\mathcal{S}(\mu,\varphi)} d\mu d\varphi \, (\mathbf{n}(\nu,\phi)\cdot\mathbf{N}(\mu,\varphi)) \\
&\qquad \tilde{\Theta}(\nu,\phi;\mu,\varphi) \\
&= \frac{R_{\mathcal{L}}^2}{mc}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi \, I(\nu,\phi)\,\mathbf{n}(\nu,\phi) \int_{\mu=0}^{1}\int_{\varphi=0}^{2\pi} d\mu d\varphi \mu \\
&= \frac{A_\perp}{mc}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi \, I(\mathbf{n}(\nu,\phi))\,\mathbf{n}(\nu,\phi) \, . \qquad (2)
\end{aligned}$$

where $R_{\mathcal{L}}$ and $A_\perp$ are the satellite radius and cross section, respectively. Note that we have fixed the local satellite frame (in which the spherical angles ϑ, φ are defined) in such a way to simplify the expression in the second line of Eq. (2), thus resulting in the third line. The quantities ν, ϕ parameterizing the radiative field (up to now unspecified, but normally also spherical angles in possibly another local frame) specify 'one ray' in it. The second line in Eq. (2) clearly shows that for every such ray we have to fulfill the integration over the satellite surface $\mathcal{S}$. In each of these cases we have rotated the satellite local frame in such a way that its z–axis coincides with the direction of the chosen ray. Thus, for each ray (ν, ϕ) we use a different satellite frame (and therefore different spherical angles ϑ, φ). This trick is allowed by the isotropy of satellite surface with respect to the parameters ρ, δ.

In the same manner we can treat the other two terms in Eq. (1). The second one, corresponding to the specularly reflected radiation, becomes

$$\begin{aligned}
\mathbf{a}_R &= -\frac{R_{\mathcal{L}}^2}{mc}\int_{\mathcal{S}(\mu,\varphi)} d\mu d\varphi \int_{\mathcal{R}(\nu,\phi)} d\nu d\phi \, (\mathbf{n}(\nu,\phi)\cdot\mathbf{N}(\mu,\varphi)) \; I(\nu,\phi) \\
&\qquad (\mathbf{R}(\mathbf{n}(\mu,\varphi))\cdot\mathbf{n}(\nu,\phi)) \; \tilde{\Theta}(\nu,\phi;\mu,\varphi) \\
&= -\frac{R_{\mathcal{L}}^2}{mc}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi I(\nu,\phi) \int_{\mathcal{S}(\mu,\varphi)} d\mu d\varphi \, (\mathbf{n}(\nu,\phi)\cdot\mathbf{N}(\mu,\varphi)) \\
&\qquad \left(2\Pi_{\mathbf{N}(\mu,\varphi)}(\mathbf{n}(\nu,\phi)) - \mathbf{n}(\nu,\phi)\right) \tilde{\Theta}(\nu,\phi;\mu,\varphi) \\
&= \frac{R_{\mathcal{L}}^2}{mc}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi I(\nu,\phi)\mathbf{n}(\nu,\phi) \int_{\mu=0}^{1}\int_{\varphi=0}^{2\pi} d\mu d\varphi \mu (2\mu^2 - 1) = 0 \, . \qquad (3)
\end{aligned}$$

[1] Note, that we implicitly neglect variations of the radiative field at different part of the satellite. Instead, we work with radiative field $I(\nu,\phi)$, which results from averaging over its surface. The same applies for formulas (3) and (4).

This result is a classical one, and in fact has its counterpart in the same result for the elastic specular reflection of the neutral atmosphere molecules in the theory of the atmospheric drag. Of course any, but very particular cases, deviation from the spherical shape or from the isotropy of the specular reflection coefficient ρ would lead to a nonzero result.

As for the third term (diffusively scattered radiation from the satellite surface), we have

$$\begin{aligned}
\mathbf{a}_D &= -\frac{2\delta R_{\mathcal{L}}^2}{3cm}\int_{\mathcal{S}(\mu,\varphi)} d\mu d\varphi \int_{\mathcal{R}(\nu,\phi)} d\nu d\phi\, (\mathbf{n}(\nu,\phi)\cdot\mathbf{N}(\mu,\varphi))\, \mathbf{N}(\mu,\varphi)\, I(\nu,\phi) \\
&\qquad \tilde{\Theta}(\nu,\phi;\mu,\varphi) \\
&= -\frac{2\delta R_{\mathcal{L}}^2}{3cm}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi\, I(\nu,\phi) \int_{\mathcal{S}(\mu,\varphi)} d\mu d\varphi\, (\mathbf{n}(\nu,\phi)\cdot\mathbf{N}(\mu,\varphi))\, \mathbf{N}(\mu,\varphi) \\
&\qquad \tilde{\Theta}(\nu,\phi;\mu,\varphi) \\
&= \frac{2\delta R_{\mathcal{L}}^2}{3cm}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi I(\nu,\phi)\mathbf{n}(\nu,\phi) \int_{\mu=0}^{1}\int_{\varphi=0}^{2\pi} d\mu d\varphi \mu^2 \\
&= \frac{4\delta A_\perp}{9cm}\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi I(\mathbf{n}(\nu,\phi))\mathbf{n}(\nu,\phi)\,. \qquad (4)
\end{aligned}$$

Here note one subtle point – the arguments of the vectorial quantity have changed proceeding from the second line to the third. This is also luckily due to the spherical symmetry of the satellite surface (roughly speaking, this means that the integral of the normal vectors to the sphere over one hemisphere is parallel to its axis). In the case of nonspherical shapes, the result would thus be different.

The resulting perturbing acceleration is $\mathbf{a} = \mathbf{a}_A + \mathbf{a}_R + \mathbf{a}_D$, hence in total

$$\begin{aligned}
\mathbf{a} &= \frac{A_\perp}{mc}\left(1+\frac{4}{9}\delta\right)\int_{\mathcal{R}(\nu,\phi)} d\nu d\phi \mathbf{n}(\nu,\phi) I(\mathbf{n}(\nu,\phi)) = \frac{A_\perp}{mc}\left(1+\frac{4}{9}\delta\right)\mathbf{F} \\
&= \left(\begin{array}{c}\text{satellite}\\ \text{cross section}\end{array}\right)\left(\begin{array}{c}\text{some coefficient characterizing the inte}-\\ \text{raction of light with the satellite surface}\end{array}\right) \\
&\quad \left(\begin{array}{c}\text{radiative flux, i.e. energy transferred}\\ \text{through the perpendicular area unit}\end{array}\right)\left(\begin{array}{c}\text{velocity}\\ \text{of light}\end{array}\right)^{-1}\left(\begin{array}{c}\text{satellite}\\ \text{mass}\end{array}\right)^{-1} \qquad (5)
\end{aligned}$$

The second and third lines give a clear "physical interpretation" of the relevant terms (notice that the product of the two terms in third line yields the momentum flux, namely the amount of linear momentum transferred through a perpendicular unit area). This result of course is not new; see e.g. Aksenov, 1977, Milani *et al.*, 1987 (pp. 74–75), Borderies and Longaretti, 1990 (where one has to substitute $\nu \to \frac{1}{3}\delta$); here it is just an example of the application of the more general Eq. (1).

The result given by Eq. (5) in general still holds for nearly–spherical satellites with *the quasi–isotropic* surface optical properties. This — admittedly, somewhat vague — notion is inspired by LAGEOS–type satellites, whose surface is covered by a large number of retroreflectors. We call the surface of a nearly–spherical satellite to be quasi–isotropic, if, roughly speaking, "it looks the same from all the directions to a high accuracy". For instance, let us imagine a parallel beam of radiation impinging on LAGEOS. Obviously, if the satellite is illuminated along the axis of any retroreflector the radiative force will be somewhat different with respect to the case of irradiation out of the retroreflector axis. But the difference will be very small, provided the retroreflector has a small size compared with the satellite. The quasi–isotropy assumption just neglects this small difference. More precisely, we have to assume that the results of the satellite surface integration along the different rays of the radiative field (as explained in the paragraph following Eq. (2)) in the formulae (3), (4) and (5) are the same, or at least that the differences are negligible.

Notice that the peculiar optical behaviour of the retroreflectors — perfect back-scattering of the impinging radiation — causes their contribution to the momentum transfer equation to be just twice that of the absorption term (2), so that for a nearly–spherical and quasi–isotropic satellite whose surface is covered by retroreflectors for a fraction f of the total area, the factor $\left(1 + \frac{4}{9}\delta\right)$ in Eq. (5) needs just to be replaced by $\left[1 + f + \frac{4}{9}\delta(1 - f)\right]$. For LAGEOS, the effective value of f is much lower than the "geometrical" one (≈ 0.4), because the corner–cube retroreflectors backscatter light only within some restricted incidence cone about their axis. More in general, the classical factor $\left(1 + \frac{4}{9}\delta\right)$ will be replaced by a coefficient $\mathcal{C}_R(\rho, \delta)$, where the assumption of quasi–isotropy assures that $\mathcal{C}_R(\rho, \delta)$ is a function of the surface coefficients ρ, δ only and not e.g. of the orientation of the satellite rotation axis. The acceleration is thus simply rescaled if, for instance, the optical parameters are not constant in time. For LAGEOS, $\mathcal{C}_R$ has been treated as a constant solve–for parameter in the orbit determination process, and its value has been found to be about 1.2.

Of course, if the quasi–isotropy assumption fails by a significant amount, the perturbative acceleration will be given by an expression more complicated than (5). Interestingly, the possibility of a sizeable difference between the optical properties of two different hemispheres of LAGEOS has been recently proposed by Scharroo *et al.* (1991) as a way of explaining some unmodelled peaks in LAGEOS' residual along–track acceleration. In the present context, however, we shall stick to the quasi–isotropy assumption.

Finally, note that the character of the radiative field is not limited in any way in the previous discussion. It may be either the nearly parallel direct solar radiation or the geometrically more complex radiative field coming from the Earth's surface and atmosphere. In what follows, we shall apply Eq. (5) to the latter case. In the next section we shall describe a model of this radiative field as well as its (ν, ϕ) parameterization in Eq. (5).

3. Models of the Earth's Radiative Field

We shall now apply the preceding formulae to the particular case when the radiative field is originated by the reflection/diffusion of the sunlight from the Earth — the so–called *albedo effect*.

The Earth is assumed to be spherical. No general *a priori* assumption is made on the optical characteristics of the Earth surface, but they are just subject to a set of different model assumptions, with the aim of studying their influence on the resulting orbital perturbations of LAGEOS. In particular, we are going to consider the specific role of the scattering of light from the cloud coverage. Actually, one should always speak about the reflection/diffusion from the Earth–atmosphere system rather than from the Earth surface only.

Also, we assume that the Earth–scattered radiation does not interact with any exoatmospheric near–Earth medium, so that the intensity of the radiation is constant along the photon rays out of the atmosphere (Mihalas, 1978). This assumption is essential to obtain a geometrical link between the Earth's optical models (treated basically in the Earth surface/upper atmosphere local frames) and the radiative field which appears in the satellite acceleration formulae (1) or (5). Recall that in the previous Section we considered processes occurring in a satellite–fixed local frame, while throughout this section we will formulate the Earth's optical models in the Earth surface/atmosphere local frames. The previous assumption bridges the gap between them.

Similarly to Vokrouhlický and Sehnal (1992b), we will consider three different *modes* of reflection/diffusion from the Earth's surface/atmosphere. They are :

- diffusion of light from the continents, which will be assumed to occur in a simple isotropic way (*Lambert diffusion*) with a multiplying albedo value developed in spherical harmonics of the geographical latitude and longitude (as given by Sehnal, 1979);
- reflection/diffusion from the oceans : here together with a diffusive behaviour part of the light is assumed to be reflected in a specular way;
- scattering of light from compact atmosphere formations (which we will simply name *clouds*), treated according to Chandrasekhar's solutions of the radiative transfer equations for a planar atmosphere. The detailed assumptions will be specified in what follows.

We introduce the spherical angles ϑ, φ (and also $\mu = \cos\vartheta$) in the local Earth surface/atmosphere frame (these angles should not be confused with those used in the previous section). The z–axis of this local frame coincides with the outer normal of the surface element. The orientation of the other two axes does not appear in the formulae and consequently it is irrelevant. We reserve $\mu_\odot$, $\varphi_\odot$ for the Solar direction, while μ, φ specify the satellite direction in this system (see Fig. 1).

Using these notations, one can write the intensity of the isotropically diffused

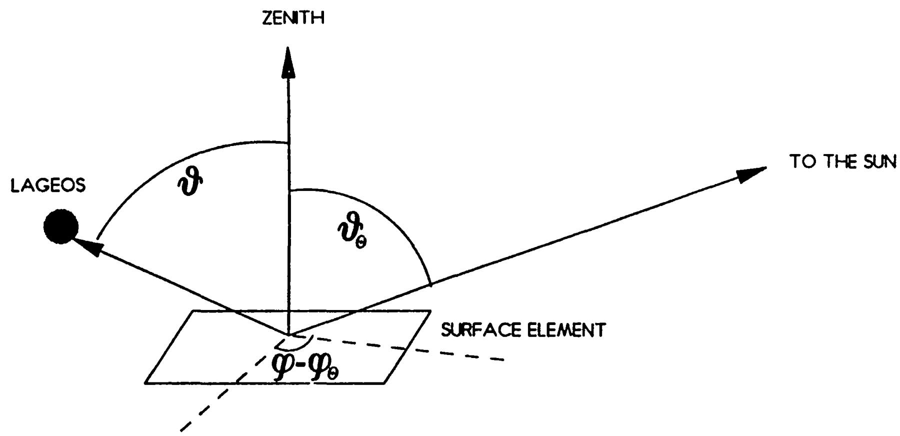

Fig. 1. The spherical coordinate system centered at an Earth surface element. The angle ϑ is measures the distance from the zenith direction of the chosen element. The origin of the φ coordinate is irrelevant in our models.

radiation according to Lambert's law as :

$$I_{\rm dif}^{\rm (cont)} = \frac{1}{\pi}\Phi_0 \mathcal{A}\mu_\odot \; , \tag{6}$$

where Φ_0 is the flux (W/m^2) of the incoming solar radiation and $\mathcal{A}$ is *the albedo* coefficient. The $\mu_\odot$ factor is just due to geometrical reasons, namely is proportional to the cross–section with respect to the solar rays. Some criticisms to this purely geometrical approach can be found as early as in Levin, 1962, whose arguments were explicitly quoted by Vokrouhlický and Sehnal (1992b). Later on, the deviations from linearity with respect to $\mu_\odot$ were referred to as dependence of the albedo $\mathcal{A}$ on $\mu_\odot$ (see e.g. Lála et al., 1978; Rubincam et al., 1987; Knocke, 1989; and from an observational point of view Stephens et al., 1981). We feel that in spite of these studies, which usually empirically fit some sets of observed data, the theoretical understanding of this phenomenon is not satisfactory. Both light scattering in the atmosphere and details of the diffusion mechanisms on the Earth's solid surface (e.g., small–scale mutual shadowing) are plausible causes for a $\mu_\odot$ dependence of the albedo, and more in general for a dependence on all the involved angular parameters $(\mu_\odot, \varphi_\odot \; ; \mu, \varphi)$. But little is known about the separate importance of each of these effects and their detailed physics. In view of this, we have decided just to use the normal albedo model as given by Sehnal (1979). The albedo $\mathcal{A}$ is here a general function of the geographical latitude and longitude (expanded in spherical harmonics), but is constant with respect to the solar zenith angle $\mu_\odot$. In fact, it may be understood as an *averaged* value over $\mu_\odot$.

As for the oceans, we assume that besides Lambertian diffusion their surface can cause specular reflection of light. Following Barlier *et al.* (1986), we then decompose the intensity of the scattered radiation in two parts :

$$
\begin{aligned}
I_{\rm dif}^{(\rm oc)} &= \frac{1}{\pi}\Phi_0 \mathcal{A}\mu_\odot(1-\mathcal{R}(\mu_\odot)) \, , \\
I_{\rm spec}^{(\rm oc)} &= \Phi_0 \mathcal{A}\mathcal{R}(\mu_\odot)\psi_{\rm sc}(\mu_\odot)\delta(\mu-\mu_\odot)\delta(\phi-\phi_\odot+\pi) \, ,
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
\psi_{sc}(\vartheta_\odot,\alpha) &= \frac{\alpha^2 \sin\vartheta_\odot \cos\vartheta_\odot}{\sin\beta\,(2\cos(2\vartheta_\odot-\beta)-\alpha\cos\vartheta_\odot)} \, , \\
\alpha &= R_\oplus/r \, ,
\end{aligned} \tag{8}
$$

r being the current satellite geocentric distance and the angle β fulfills the equation

$$\alpha \, \sin\vartheta_\odot \; = \sin(2\vartheta_\odot-\beta) \, .$$

Note that, in agreement with the definition of $\mathcal{A}$ as a fraction of incident sunlight which is not absorbed, but either diffused or reflected, the factor $\mathcal{A}$ appears also in the intensity of the specularly reflected component. Barlier *et al.* (1986; see their Eq. (10))on the contrary included $\mathcal{A}$ only in the intensity of the diffused component, thus implicitly adopting a different definition of albedo and amplifying the effects of radiation pressure from specularly reflected sunlight when compared with the present work.

For the derivation of Eq. (8) see Barlier *et al.*, 1986, where this factor is denoted as $J(\theta)$ (it actually simulates the divergence of the reflected radiation field due to the fact that the assumed mirror surface is spherical). The δ function in Eq. (7) is an obvious consequence of the specular reflection assumption. Of course, this is a somewhat idealized model : small scale wave phenomena on the ocean surfaces will actually cause some more divergence of the reflected light (as with the "sword of the sun" observed from a beach at sunset). A phenomenological model of this beam divergence can be found in Rubincam et al., 1987, who instead of the δ function adopt an exponential behaviour, and in Lucchesi and Farinella, 1992, who assume that the light intensity is uniformly distributed inside a narrow cone centered at $\mu = \mu_\odot$. The latter paper did show that the resulting perturbations depend weakly on the aperture of the reflection cone (or the corresponding exponential decay coefficient) provided it is less than several degrees. Therefore, here we shall stick to the simpler specular reflection assumption.

The "partitioning function" $\mathcal{R}$ in Eq. (7) is derived from the Fresnel reflection law (see Barlier *et al.*, 1986). The implicit assumptions here include a sufficiently short wavelength of the radiation and a negligible polarization (for a detailed treatment consult e.g. Jackson, 1975, and Ditchburn, 1976). Note that the Fresnel function $\mathcal{R}$ fulfills the following relation

$$(1-\mathcal{R}(\mu_\odot))_{\mu_\odot\simeq 0} \simeq \mu_\odot + \mathcal{O}(\mu_\odot^2) \, ,$$

which results into a rapid increasing importance of the specularly reflected part of radiation with respect to the diffusively reflected part when the satellite is close to the Earth shadow's boundary, namely it "sees" a narrow Earth crescent. This is due to the fact that when $\mu_\odot \simeq 0$, $I^{(oc)}_{\rm dif} \propto \mu_\odot^2$ but $I^{(oc)}_{\rm spec} \propto \mu_\odot$ only. The corresponding sudden "kick" due to the specularly reflected radiation is in a way the result of extreme assumption, as the decomposition of the scattered radiation in the two parts is not unique (see Vokrouhlický and Sehnal, 1992b) and not all such decompositions give this feature. It should be noted that in accepting this model we just follow another common argument made in recent literature on LAGEOS perturbations : that of testing the effects of extreme assumptions whenever a reliable physical understanding is missing. In Sec. 4 we shall comment on some consequences of this assumption when describing the results of our model.

As for the scattering of light from the clouds, we shall use the model developed by Chandrasekhar (see e.g. Chandrasekhar, 1950; van de Hulst, 1980). The intensity of the scattered radiation is given by

$$\begin{aligned} I^{(cl)}_{\rm scat} = & \frac{1}{\pi}\frac{\omega_0\mu_\odot\Phi_0}{\mu+\mu_\odot}\big(H(\mu;\omega_0,x)H(\mu_\odot;\omega_0,x)\Psi(\mu,\mu_\odot;\omega_0,x) - \\ & x\sqrt{(1-\mu^2)(1-\mu_\odot{}^2)}H^{(1)}(\mu;\omega_0,x)H^{(1)}(\mu_\odot;\omega_0,x)\cos(\varphi-\varphi_\odot)\big)\,, \end{aligned} \tag{9}$$

where

$$\Psi(\mu,\mu_\odot;\omega_0,x) = 1 - c(\omega_0)(\mu+\mu_\odot) - x(1-\omega_0)\mu\mu_\odot\;.$$

These formulae solve the radiative transfer problem in a planar atmosphere with infinite optical thickness and scattering centers characterized by the phase function

$$p(\Theta) = \omega_0(1 + x\cos\Theta),\quad 0 \le \omega_0 \le 1,\quad -1 \le x \le 1\,, \tag{10}$$

where $H(\mu)$ and $H^{(1)}(\mu)$ are the Chandrasekhar H-functions and the associated μ–moments. They are defined as solutions of integral equations, but several numerical techniques for their evaluation are known (van de Hulst, 1980). Θ is the angle between the incident and the scattered ray at one scattering center in the atmosphere, and the phase function $p(\Theta)$ weighs the scattering probability at this angle.

The solution (9) is two–parametric, with the parameters ω_0 and x controlling physically different effects :

– ω_0 can just be interpreted as the albedo of the scattering centers in the atmosphere (not to be confused with the global atmospheric albedo $\mathcal{A}_{eff}$; their relation is discussed in Vokrouhlický and Sehnal, 1992b, e.g. in the case $x = 0$ one gets $\mathcal{A}_{eff} = \frac{4\omega_0}{(1+\omega_0)^2}$, while in the case $x \neq 0$ no such simple relation exists),
– the parameter x is related to the asymmetry of the light scattering from separate centers in the atmosphere : $x = 0$ implies symmetry, while $x = 1$ and $x = -1$ correspond to the extreme forward– and back–scattering cases (see Eq. (10)).

Since we are interested in testing models with local asymmetry of the cloud scattering behaviour (as it will be explained later), we will often put $x = 1$ (note however, that due to simplicity of the phase function (10), we cannot reach "the extreme asymmetry" as described by the asymmetry parameter g defined e.g in van de Hulst, 1980; for a review of more realistic phase functions see the brilliant book van de Hulst, 1980 and references in).

As with many theoretical models, that given by Eq. (9) suffers from several oversimplifications when compared to reality, e.g. :

- infinite optical thickness of the atmosphere (i.e., clouds);
- simplified phase function;
- both vertical and horizontal homogeneity of the atmosphere.

These simplifying assumptions are shortly commented in Vokrouhlický and Sehnal, 1992b, while more detailed discussions can be found in the specialized literature (Chandrasekhar, 1950; Irvine, 1975; van de Hulst, 1980). Here we are not going to include possible generalizations according any of the items listed above. But we stress that, as far as the first two items are concerned, these generalizations can be introduced if the comparison with the data shows that they are necessary to obtain a better fit.

We also note that all the modes of the light scattering/diffusion/reflection lead to formulae for the resulting radiative intensity depending only on the angle $\varphi - \varphi_\odot$ and not on φ, $\varphi_\odot$ independently (this property justifies the choice not to define the orientation of the local x–, y–axes, as noted earlier). This is well understandable, since the (x, y) local plane was chosen as tangent to the Earth's surface/atmosphere. We do not have any reason to distinguish symmetric directions with respect to the incident solar radiation. As far as we know, also all the models inferred from empirical data (e.g. Taylor and Stowe, 1984; Rubincam *et al.*, 1987) have the same property.

We now turn to description of the geographical features of the Earth's surface/atmosphere that we have used in the computation of the integrals in Eq.(5). In all our models, we used a realistic distribution of the continents, modelled by a $2^o \times 2^o$ grid in latitude and longitude. Then the final Earth *mask* giving the morphology of the different optical modes is created by covering the continental grid with an atmospheric grid, which gives the assumed cloud distribution. Of course, the latter is rapidly variable, and a realistic description could only be derived *a posteriori* from the available knowledge of global meteorological processes. This is well beyond the scopes of this work. Here we shall just distinguish among several possible idealized cases :

- the atmospheric grid is empty (we shall refer to this model as the *no cloud model*);
- the atmospheric grid is full (*full cloud model*);
- the atmospheric grid is empty on the Southern hemisphere and full on the Northern one (*hemisphere model*);

- the atmospheric grid is full in a belt surrounding the equator up to $\pm 10°$ in geographical latitude and empty elsewhere (*cloud belt model*);
- 60% of the atmospheric grid has been filled at random (*stochastic cloud model*).

4. Albedo Perturbations on LAGEOS

In this section we will discuss the perturbative accelerations resulting from the different models introduced earlier, and how they vary along one satellite revolution around the Earth. For the sake of simplicity we shall use a simplified orbit geometry, as already done by Rubincam *et al.* (1987) and Lucchesi and Farinella (1992) : LAGEOS' orbit is assumed to be polar ($I = 90°$; the real orbit has an inclination of about 110°) and the Sun lies in the LAGEOS orbit ascending node. We keep the true LAGEOS eccentricity ($e = 0.00444$) and longitude of perigee at epoch May 15, 1976 ($\omega = 235.35°$, see Barlier *et al.*, 1986), which determines the origin of anomalies. This configuration will allow us to estimate the maximum amplitude of the components of the albedo effect perturbative accelerations in the radial (S) and in–plane transverse (T) directions. These two components are important because they can be used to derive both the short– and the long–term perturbations on semimajor axis and mean anomaly; notice, however, that the assumed geometry minimizes the out–of–plane component W, which is important if one aims at estimating the inclination and node perturbations.

The most time–consuming part of the numerical treatment of the problem is the integration over the radiative field contributing to the radiative flux at the satellite. In order not to lose the detailed information on the Earth's surface/atmosphere optical behaviour, for each satellite position along the orbit we divided the visible Earth cap into a grid of 3025 surface elements, chosen in such a way as to be of equal area. This latter requirement implies that the grid is somewhat denser near the edge of the visible cap, and this may be important to properly model the acceleration when the satellite is close to the Earth's shadow boundary and just "sees" a thin illuminated Earth crescent. A similar method was used by Lucchesi and Farinella (1992), who showed that at least ≈ 100 surface elements are required for an accurate calculation; in our case the presence of specular reflection makes this requirement even more stringent. Similar accuracy problems have been taken into account in choosing the step size along the orbit; we have used 360 steps per revolution, i.e. 1° mean anomaly steps.

The results of several models are presented in Fig. 2, where the transverse component T is given as function of the mean anomaly M (whose origin is taken in such a way that at the subsolar point $M = 100°$). The curve(s) labeled 1 correspond to the no cloud model. More precisely, the solid curve 1a corresponds to the no cloud model as described in the Sec. 3, which takes into account the real distribution of continents, while the dashed curves 1b and 1c "ignore" the distribution of the continents : curve 1b corresponds to the ideal case when the

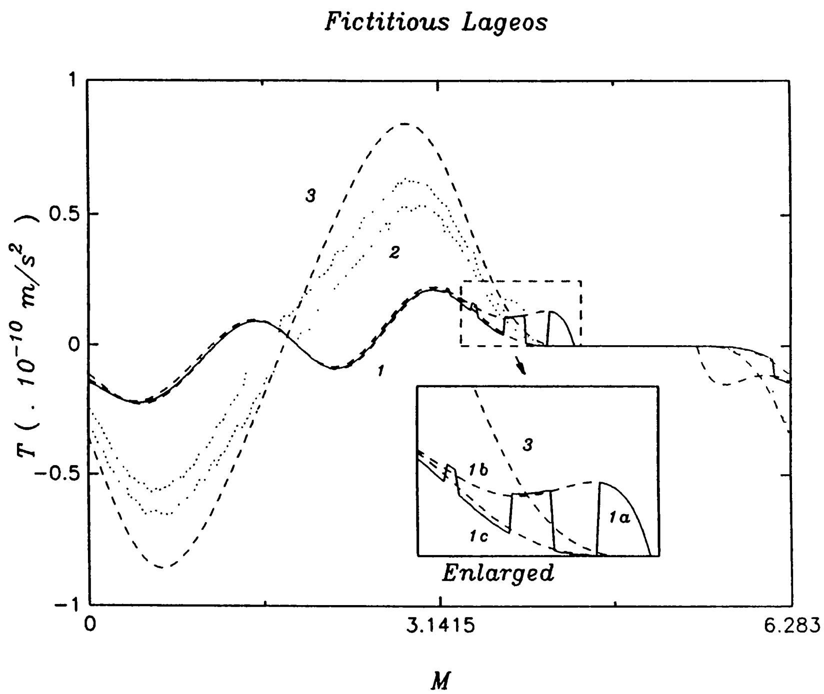

Fig. 2. The transverse component T of the albedo acceleration as a function of the mean anomaly M over one LAGEOS revolution, for the orbit geometry described in the text. Different *no cloud* models have been used to derive curves 1 : the (1a) model includes the real distribution of the continents on the Earth surface, the (1b) model considers an idealized Earth totally covered by the oceans, and the (1c) model assumes only diffusive (continental) reflection from the whole Earth surface. Note the essential role of the specular reflection effects when the satellite is close to the boundaries of the Earth's shadow. The dotted curves 2 bound the results of 50 different runs of the *stochastic* cloud model. Curves 3 correspond to the full cloud model. the adopted cloud parameters in both cases are : $\omega_0 = 1,\ x = 1$.

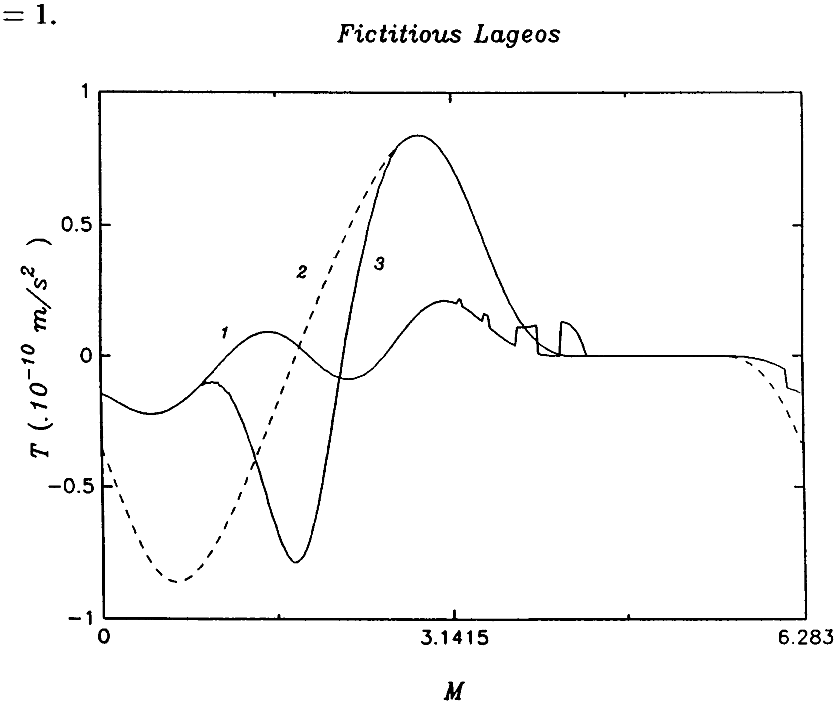

Fig. 3. The same as in Fig. 2, but comparing results of the *no cloud* model (curve 1), the *hemisphere* model (curve 2), and the *full cloud* model (curve 3). Cloud model parameters are kept as follows : $\omega_0 = 1,\ x = 1$.

whole Earth is covered by an ocean with the optical behaviour described in Sec. 3, and curve 1a corresponds to the ideal case when the whole Earth surface is covered by a continent with diffusive reflection. As expected, the largest differences occur near the satellite entry to/exit from the Earth shadow. Curve 1c shows there a smooth decrease, while curve 1b exhibits clearly the specular reflection "flash", as discussed earlier. The no cloud model 1a, combining 1b and 1c according to the actual distribution of the continents, shows sudden jumps due to interchanging of the two "pure" modes of reflection (due to the chosen orbit geometry, in the enlarged window in Fig. 2 the zone of the Earth possibly relevant for reflection is the Arctic). Such structures have a significant influence on the averaged value $< T >$ over an entire orbit, which is important to yield long–term perturbations. Such a potentially important role of specular reflection from relatively small zones of the Earth's surface was already pointed out by Barlier *et al.* (1986). It may be noted that our assumption that the Arctic polar cap behaves as an ocean is somewhat artificial; however, according to the data reported by Taylor and Stowe (1984), snow–covered surfaces do generate some specular reflection features, albeit less marked than liquid water, and thus may be seen as "intermediate" between the oceans and the continents.

Curve 3 corresponds to the full cloud model with parameters ($\omega_0 = 1,\ x = 1$). As no specular reflection occurs in this case, the curve is smooth. Actually, as noted already by Vokrouhlický and Sehnal (1992b), this case closely resembles the simple one with just a diffusion with a constant effective albedo $\mathcal{A}_{eff} = 1$ (see Lucchesi and Farinella, 1992, Fig. 5a, where the value $\mathcal{A}_{eff} = 0.2$ was used); only the anisotropy of the local scattering behaviour slightly disturbs the diffusion pattern. The amplitude of curve 1 is thus consistent with an average value of the albedo for the Earth's surface of approximately 0.3.

The region bounded by the dotted curves 2 corresponds to the stochastic cloud model; in this case, there is no unique curve, because the filling of the atmosphere grid is made at random. Therefore, we performed 50 runs of the program computing 50 different cloud distributions (always with a filling fraction of 60%), and then plotted the maximum and minimum values of the along-track accelerations. The typical difference of $10^{-11}\ m/s$ gives an idea of the consequences of the variability in the real cloud distribution (note in particular that the occurrence of the "Arctic reflection flash" is subject to the local cloud coverage). A similar method was used the case of the lower–altitude satellite ERS–1 by Vokrouhlický and Sehnal (1992b). An interesting observation can be done by comparing the ERS–1 results with those in Fig. 2 — the three curves 1–3 in Fig. 2 are well separated, as the region between the dotted curves 2 does not span the whole "gap" between curves 1 and 3, while the opposite was true for ERS–1. The reason has to do both with the simpler satellite geometry and with the higher orbit of LAGEOS.

In Fig. 3 we have replotted the mean anomaly dependence of the T acceleration component in the case of the no cloud model (curve 1), and the full cloud model (dashed curve 2), but we have added the curve (3) corresponding to the hemisphere model. As expected, the hemisphere model signal coincides with either one of the two previous cases when the satellite is over the poles (as its "visible cap" of the Earth behaves according to only one mode of reflection), while differs from them in the transient region, when the satellite is over the equator.

Curves 3 in Fig. 4 compare with the no cloud (curve 1) and full cloud (curve 2) models the results of the cloud belt model. The dashed and dotted curves allow an estimate of the effects of changing the x isotropy parameter : dashed curves correspond to the $x = 1$ choice, dotted curves to the $x = 0$ one. No qualitative new feature appears, though there is a significant quantitative difference. However, this is not a general result, due to the assumed symmetrical orientation of the orbital plane with respect to the Sun. Curves 3a–c also show the dependence of the results on the effective cloud albedo : 3a corresponds to $\mathcal{A}_{eff} = 1$, 3b to $\mathcal{A}_{eff} = 0.8$, and 3c to $\mathcal{A}_{eff} = 0.6$. A decreasing effective albedo of the clouds clearly diminishes their role.

Other remarks follow from the inspection of the radial component of the perturbative acceleration. Its dependence on the mean anomaly is shown in Fig. 5 in the case for the some models discussed in the previous paragraphs. Curve 1 corresponds to the no cloud model, curves 2 to the full cloud model (with different values of x) and curves 3 to the cloud belt model (with different values of both x and $\mathcal{A}_{eff}$). Similar graphs can be plotted for the other models. The observed differences are clearly of the order of several times 10^{-10} m/s^2. As the real cloud distribution is highly variable and indeed unknown when analyzing the real tracking data (unless a special effort is made to recover global meteorological data), one can expect an intrinsic uncertainty on the magnitude of the instantaneous albedo acceleration for LAGEOS of the order of 10^{-10} m/s^2. The corresponding error in LAGEOS' position predictions (or determinations) over a fraction of one orbital period (≈ 4 hr) is of the order of some mm. This is not much less than the current satellite laser–tracking capabilities, thus providing some motivation for modelling the albedo effect even when short orbital arcs are analyzed (see e.g. Milani *et al.*, 1992).

We do not discuss in detail here the $< T >$ values arising from averaging the resulting values of T over one orbital revolution, contrary to Rubincam *et al.*, 1987 and Farinella and Lucchesi, 1992. Although these $< T >$ values may provide some rough estimates of the long–term effects associated with the different models (see some remarks on this in Sec. 5), in most cases they sensitively depend on the assumed orientation of the satellite orbit, the Sun and Greenwich meridian. In view of this, we believe that using just *one* particular choice of the above parameters may mislead the interpretation. Consider for instance the dominant effect of specular reflection from the oceans when the satellite is close to the Earth's shadow boundaries. As it was pointed out earlier, any asymmetry caused in these zones by the actual occurrence of continents and/or clouds (not reflecting specularly) versus oceans may become a strong contributor to a nonzero $< T >$ value. Moreover, the albedo distribution model shows a significant longitude dependence with the

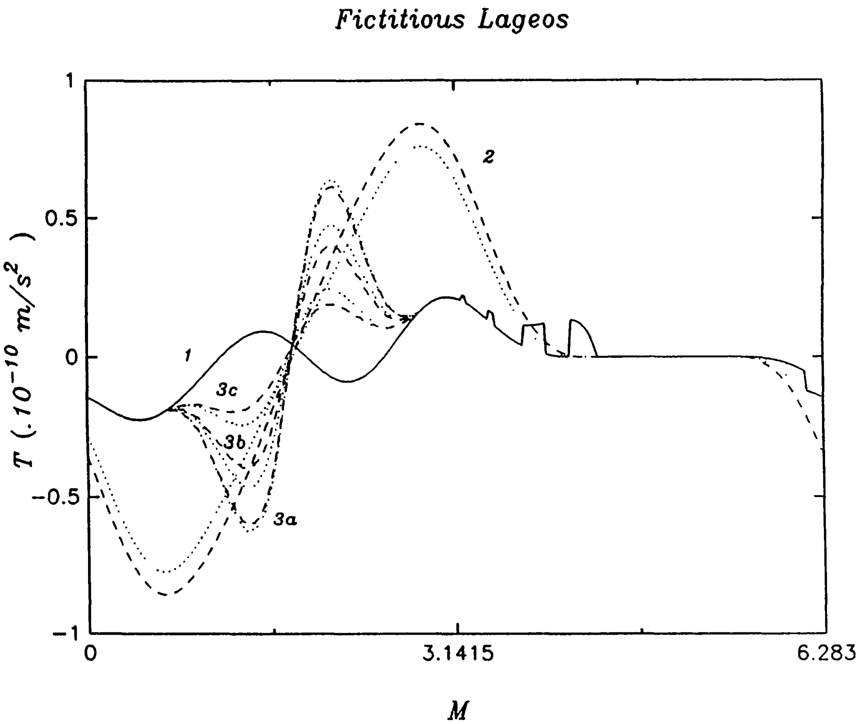

Fig. 4. Here the results of the *no cloud* model (solid curve 1) and of the *full cloud* models (curves 2) are compared with those of the *cloud belt* model (curves 3). Dashed curves correspond to strongly anisotropic light scattering from the atmosphere centers ($x = 1$), dotted curves to symmetric scattering ($x = 0$). The dependence on the effective cloud albedo is tested as well : curves (3a) correspond to $\mathcal{A}_{eff} = 1$, curves (3b) to $\mathcal{A}_{eff} = 0.8$, and curves (3c) to $\mathcal{A}_{eff} = 0.6$.

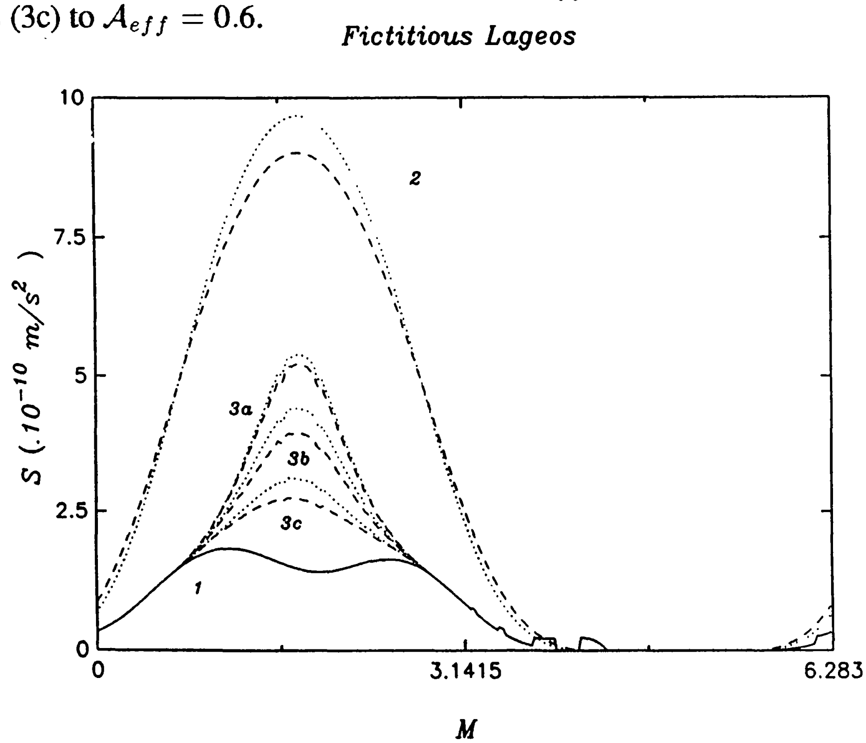

Fig. 5. The same as in Fig. 4, but for the radial component S of the albedo acceleration.

presence of localized structures (see Sehnal, 1979, Figs. 1 and 2), whose position could also somewhat affect the shape of the T curves. Thus the real value of $< T >$ may change in a substantial way even in successive revolutions. As a consequence, a reliable estimate of the long–term albedo effects requires a long time span integration (many thousands of revolutions) of the perturbation equations, followed by a Fourier transform analysis to point out periodicities.

It is also important to note that the previous discussion applies only to the intervals when LAGEOS' orbit crosses the Earth's shadow. Out of these periods the significant "kicks" caused by specularly reflected radiation are suppressed, since the Fresnel function $\mathcal{R}(\mu)$ undergoes a rapid decrease to very small values for non–tangential illumination of the reflecting surface elements. Actually, the LAGEOS orbital residuals show sharp peaks and dips correlated with the Earth's shadow crossing periods. While other effects may be responsible for this correlation (Afonso *et al.*, 1989; Scharroo *et al.*, 1991), we plan to carry out a detailed study of the long–term albedo effects in a forthcoming paper.

5. Conclusions

The main results obtained in this paper can be summarized as follows.

1. We have provided general integral formulae (1), similar to the ones appearing in the context of radiative transfer theory, that express the perturbing acceleration due to radiation pressure for a satellite of arbitrary shape moving in an arbitrary radiation field. We have then shown that they can applied in a straightforward way to the simple case of a nearly spherical, quasi–isotropic satellite such as LAGEOS.
2. In order to study the albedo orbital perturbations on LAGEOS, we have discussed in some detail the available models for the optical behaviour of the Earth's surface, including reflection from the oceans and possibly anisotropic scattering from clouds.
3. The Gauss components T and S vs. mean anomaly have been numerically computed for a simple orbital geometry (90° inclination, Sun at an equinox and in the orbital plane) and for different models of the cloud geographical distribution and optical properties. A sensitive dependence has been found for both components, with significant along–track perturbations possibly arising from oceanic specular reflection when the satellite is close to the Earth's shadow boundaries. This may contribute to the observed eclipse–correlated spikes in the unmodelled T acceleration residuals found from LAGEOS orbital determination.
4. The radial S component shows differences of the order of several times $10^{-10}\ m/s^2$ when different cloud models are used, implying short–term errors (when arcs of a fraction of an orbital period are considered) in LAGEOS' position predictions/determinations of the order of some mm.

5. In spite of the *caveats* mentioned in Sec. 4, it can be noted that the difference in the orbit–averaged $< T >$ values between the no cloud model and the hemisphere model is of the order $10^{-12}\ m/s^2$. This is consistent with the findings of Lucchesi and Farinella (1992), who used somewhat different optical models and another numerical algorithm. As discussed by these authors, such a small along–track acceleration may cause on the long term comparatively large errors in LAGEOS' orbital predictions (or determinations). Moreover, this value is of just the same order of magnitude as those corresponding to other non–gravitational perturbations, including both drag–like forces and radiative effects (see the papers referred in Sec. 1). All these effects are intrinsically complex and uncertain, and the resulting long–term perturbations are probably mixed up in the observed acceleration residuals. As a consequence, we feel that it is dangerous to try to fit these residuals by semi–empirical models including a number of different mechanisms at the same time, as the modeling errors can cause large biases and/or aliasing effects. Our plan is thus to further study the albedo effect, investigating the long–term perturbations associated with the different models introduced in this paper.

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