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Specular reflection of sunlight from wavy ocean surfaces and the albedo effect on satellite orbits

II. LAGEOS long-term albedo perturbations reassessed

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Abstract. We reanalyse the role of radiation pressure by sunlight specularly reflected from the Earth's ocean surface in generating long–term perturbations on the orbit of the laser–tracked satellite LAGEOS, by using the new theory for light reflection from a "rough" (or "wavy") surface developed in the previous paper of this series (Vokrouhlický & Farinella 1994a). The results of the more accurate method based on the application of geometrical optics to a distribution of surface elements with different orientations are compared to those based on a simple redefinition of the Fresnel reflection coefficient. In agreement with the estimates based on the dependence of the perturbing acceleration upon the satellite's mean anomaly, we show that the "roughness" of the ocean surface and the resulting finite–lobe reflection pattern cause a 10–40% decrease of the magnitude of the long–term perturbations of LAGEOS' semimajor axis with respect to the ideal, mirror–like reflection case. However, the resulting perturbations are still large enough that radiation pressure by specularly reflected sunlight provides a major contribution to the observed LAGEOS along–track residuals. We also confirm that the secular effect on the nodal longitude of LAGEOS does not exceed 1 milliarcsec/year, namely a few percent of the general–relativistic Lense–Thirring precession, and that long–periodic effects on the inclination can reach about 3 milliarcsec/year. In order to model in a reliable way all these effects, extensive data on the optical properties of ocean surfaces and the global cloud coverage would be required.

Key words: celestial mechanics – artificial satellites, space probes – radiation mechanisms: miscellaneous

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1. Introduction

Investigating the various non-gravitational mechanisms which affect artificial satellite orbits has been an active research subject since the beginning of the space age. In the last 15 years, the availability of very accurate laser tracking data for the passive geodynamics satellite LAGEOS (more recently joined by other satellites of the same type) has given rise to the challenging task of understanding and modelling the long-periodic residuals of the orbital elements inferred from observations with respect to those computed from standard dynamical models, which include all the main gravitational and non–gravitational perturbing forces. A number of additional subtle mechanisms, mostly related to the interaction of the satellite with electromagnetic radiation, have been discussed over the years as possible sources for the residuals, in particular those in semimajor axis, which result into an accumulating drift of the satellite's along–track position (see e.g. Milani et al. 1987). Although nowadays there is a wide consensus that a mixture of such mechanisms is very likely to be responsible for the observed residuals, still there are problems in assessing the relative weights of the different effects (Vokrouhlický & Farinella 1994b).

Among these effects, the radiation pressure from sunlight diffused/reflected by the Earth (the so–called *albedo effect*) has been the subject of continuing modelling efforts. The diffusive and specular parts of the Earth–reflected sunlight were studied in the context of the LAGEOS residuals problem either separately (e.g. Anselmo et al. 1983; Barlier et al. 1986) or in combination (e.g. Vokrouhlický et al. 1993b). Actually, it was realized soon (Barlier et al. 1986) that the diffusive part cannot contribute to the long–term residuals of LAGEOS' semimajor axis but with effects of first order in the orbital eccentricity. Since for LAGEOS this parameter is quite small (e = 0.004), radiation pressure from Earth–diffused sunlight does not appear as a plausible candidate mechanism for generating the observed residuals.

On the other hand, Anselmo et al. (1983) had discussed some general reasons why the albedo effect should result into a nonzero one–revolution average of the transverse component of the perturbing force, such as required to yield secular or long–periodic semimajor axis perturbations. The basic idea was that owing to latitudinal and seasonal asymmetries in the optical properties of the Earth's surface/atmosphere, during each revolution the satellite would undergo two opposite, but unbalanced "kicks" in the along–track direction. This phenomenological approach was sufficiently general to cover also other possible mechanisms for generating the LAGEOS residuals, such as solar radiation pressure during Earth penumbra passages (see Vokrouhlický et al. 1994a).

Barlier et al. (1986) and Vokrouhlický et al. (1993b) have translated this general idea into detailed physical models of the sunlight reflection process – first pointing out, as mentioned above, that the purely diffusive part of Earth–reflected radiation must play a minor role (independently of the longitudinal and latitudinal dependence of the albedo coefficient); and second, remarking that in general both a local anisotropy of the reflection lobe and a global asymmetry in the distribution of the different reflection modes† is required to yield long–term semimajor axis perturbations. Most real local reflection modes (in contrast with the commonly used, but highly idealized, Lambertian isotropic mode) show a strong anisotropy (see e.g. the photometric data analysed by Taylor & Stowe 1984; Ahmad & Deering 1992; Perovich 1994). A global inhomogeneity is clearly associated to the complex distribution of continental masses and oceans on the Earth's surface, and also to the variable distribution of optically thick cloud formations in the atmosphere. Both processes – anisotropic specular reflection from oceanic surfaces and anisotropic rediffusion of sunlight from clouds – have been examined as possible sources for LAGEOS' long–term residuals in Vokrouhlický et al. (1993b). The results suggested that they can well give rise to effects of significant magnitude when compared to the data, although this mechanism alone does not provide a full explanation for the time dependence of the residuals.

A questionable assumption we made in this work was that of describing the optical behaviour of oceans through an "ideal" specular reflection mode, namely treating the sea surface as a smooth spherical mirror with a reflection coefficient given by Fresnel's law. V. Slabinski (1993, private communication) suggested to us that possibly a more realistic reflection mode, taking into account the "rough" or "wavy" geometry of the sea surface, might result into a significant decrease of the perturbations on LAGEOS' orbit (see discussion in Sect. 1 of Vokrouhlický & Farinella 1994a). Thus, we decided to develop a realistic, statistical theory of sunlight reflection from a "rough" ocean surface, and to assess its implications for LAGEOS' orbital evolution. The theory has been worked out and presented in detail in Vokrouhlický & Farinella 1994a (hereinafter referred to as paper I). The results discussed in paper I on the dependence of the perturbing force components on LAGEOS' mean anomaly for some specific orientations of the orbit suggested that a moderate decrease of the long–term semimajor axis effects (up to 40%, and depending on the detailed model assumptions) would be generated by the finite–lobe pattern characterizing the reflection mode from a "rough" water surface.

Here, we are going to deal with the same problem by carrying out *ad hoc* numerical simulations of LAGEOS' long–term orbital evolution under the influence of ocean–reflected radiation pressure, according to the theory developed in paper I. Thus, we will better assess the extent to which the long–term semimajor axis effects depend on the assumed reflection mode, and compare the amplitude of the calculated residuals with the observed ones. Also, we will obtain some quantitative estimates on the long–term behaviour of other orbital elements, such as the nodal longitude and the inclination, under the same perturbing force.

The remainder of this paper is organized as follows. In Sect. 2, we provide some background information about specular reflection models used in LAGEOS orbit analysis, and in particular we summarize the main assumptions made in paper I. In Sect. 3, we present the results of our long–term simulations for the orbital perturbations on LAGEOS, using different models and/or parameter choices. In Sect. 4, we summarize the main results of this work and reassess in this light the importance of the albedo effect in generating the observed LAGEOS residuals.

2. Background on specular reflection models

Let us start with some general remarks on the oceanic reflection mode and a short summary of the theory developed in paper I (we shall use here the same notations adopted there).

Whereas for the continental parts of the Earth's surface a diffusive, isotropical ("Lambertian") reflection mode is a good approximation, for the oceans it is clear that such an approximation would fail to account for a basic feature of the real optical properties, that is the strong dependence of the reflected intensity on the orientation of the incoming sunlight. On the other hand, a purely specular ("mirror–like") reflection mode would also neglect the finite–lobe pattern of the reflected intensity, which does not vanish for an extended angular domain and not only for a single direction as it is the case for a smooth mirror. This can be clearly seen in the photometric maps of the reflection patterns presented by Taylor & Stowe 1984.

Two approaches are possible to model this kind of behaviour:

(i) defining a non–isotropic reflection law with no partitioning between the specular and diffusive parts (e.g. Rubincam et al. 1987);
(ii) treating separately the two parts, and specifying how they are partitioned (e.g. Vokrouhlický et al. 1993a,b).

† By reflection modes we mean the different physical processes associated with light reflection from different types of Earth surface/atmosphere elements, resulting into different "reflection lobes", namely angular dependences of the reflected light intensity; see Vokrouhlický et al. 1993a,b.

Approach (ii) is somewhat artificial, but is more consistent with the development of a physical theory based on the laws of optics, and not just on a purely mathematical formulation. Also, it has the advantage that the diffusive part of the reflected light can be neglected in the current context, as it does not contribute long–term semimajor axis effects. Therefore, we chose this approach in our work.

In paper I, we developed a simple theory for specular reflection from the wavy ocean surface, resulting into the formation of a finite reflection lobe. We started with a statistical description of the instantaneous orientation of the infinitesimal elements of the sea surface, namely by defining a distribution function $\hat{p}(\alpha)$ for the deflection angle α of their local normal with respect to the vertical direction. This distribution can be inferred from the data, or from some model for the geometry of surface waves. In what follows, we will use two different assumptions (*model 1* and *model 3* in paper I, respectively: a uniform distribution up to an upper cutoff for α, and an empirical distribution function derived from the work of Cox & Munk (1954). These authors used airplane–based photography of the solar glitter on the sea surface to fit the parameters of a suitably chosen Gram–Charlier distribution. Like in paper I, we will fix the Cox & Munk model parameters in such a way that the distribution of the orientations of the surface elements is symmetrical around the local vertical, i.e. no dependence upon the azimuthal angle β is present (a similar simplifying assumption has been made by other authors, e.g. Hennings et al. 1994). Also, a cutoff value for α is set at $\alpha_\star = 25°$.

As discussed in paper I, there are now two ways of taking into account the "roughness" of the reflecting surface in calculating the reflected light intensity:

(i) a simplified method is that of keeping the smooth–mirror assumption that for each solar ray reflection occurs from a single, well–defined point of the Earth surface, but substituting the classical Fresnel function valid for reflection from a smooth surface with an effective Fresnel coefficient, obtained by averaging over all the orientations of the reflecting surface elements;

(ii) a more complicated method is based on the construction of the local reflected radiative field at the satellite position by accounting for the geometry of all the individual light rays reflected from surface elements with different orientations, and for the resulting extended "solar image" (the $\mathscr{H}$ domain in the terminology of paper I) seen on the ocean surface from the satellite.

Of course, the latter approach is physically sounder and more accurate, whereas the former one is much more efficient from the computational point of view. In this work, we have used both methods and we are going to compare the corresponding results.

To account in a realistic way for the reflection process, a detailed "mask" of non–reflective continents must be superimposed to the reflective spherical Earth, as in Barlier et al. (1986) and Vokrouhlický et al. (1993b). Finally, the global curvature of the Earth surface has to be taken into account, as it causes a significant "dilution" of the specularly reflected radiative field coming from an extended light source. This can be done by including in the expression for the reflected intensity a suitable "dilution factor" (Barlier et al. 1986; Vokrouhlický et al. 1994b).

3. Long–term orbital perturbations on LAGEOS

The nature and purpose of this work do not require a very accurate definition of LAGEOS' orbit. Therefore, we are going to use the Keplerian orbital elements given by Barlier et al. (1986), with the additional assumption that the longitude of the ascending node and the argument of perigee undergo a secular drift due to the zonal geopotential harmonics, according to the formulae given by Brouwer & Clemence (1961) up to the order $\mathscr{O}(J_2^2, J_4)$, and with a consistent rescaling of the orbital mean motion. The motion of the Sun is represented by a Keplerian approximation with appropriate initial conditions, corresponding to the same epoch used for LAGEOS' initial elements (see Barlier et al. 1986; Vokroulický et al. 1993b). To obtain accurate values for the one–revolution averages yielding the long–term perturbations on the elements, we have used 400 steps per LAGEOS revolution. We have checked that this is enough to keep random numerical errors below the few percent level.

First, we present the results of two models based the idea of taking into account the sea surface roughness just by replacing the "ideal", mirror–like Fresnel function in the expression for the radiative intensity of the reflected light beam (as adopted by Vokrouhlický et al. 1993b, and whose results are shown in Fig. 3 of that paper and reproduced in Fig. 1a here) with the "effective", or "averaged", counterpart defined by Eq. (8) in paper I. To derive this, one has to specify the distribution law for the orientation of the sea surface elements. We used either a uniform distribution function $\hat{p}(\alpha)$ = constant, up to cutoff values $\alpha = \alpha_\star = 5°$ (Fig. 1b) or $\alpha = \alpha_\star = 20°$ (Fig. 1c), or an empirical distribution derived from Cox' and Munk's (1954) data as described in paper I (Fig. 1d).

In all the four plots, the lower, thicker curves (label 1) correspond to the average of the transverse T component of the perturbing acceleration – which gives rise to long–term semimajor axis changes – over one revolution; this average varies with time due to the changing orientation of the orbit with respect to both the Earth and the Sun. These curves include strong short–periodic components, because during each revolution (LAGEOS' mean motion is about 6.386 rev/day) the satellite is illuminated by different parts of the Earth surface, with a rapidly variable distribution of non–reflective continents and reflective oceans. These short–periodic components can be removed through a low–pass Fourier filtering procedure (Ferraz–Mello 1981; see also discussion in Barlier et al. 1986). The resulting upper curves (label 2) include only the major long–periodic lines of the "signal" contained in the lower curves, and better show its long–term behaviour.

Comparing Figs. 1a–d, it is interesting to note that the basic behaviour of the $\langle T \rangle$ vs. time curves – e.g., the position of the main maxima and minima – is unchanged. The amplitude of the

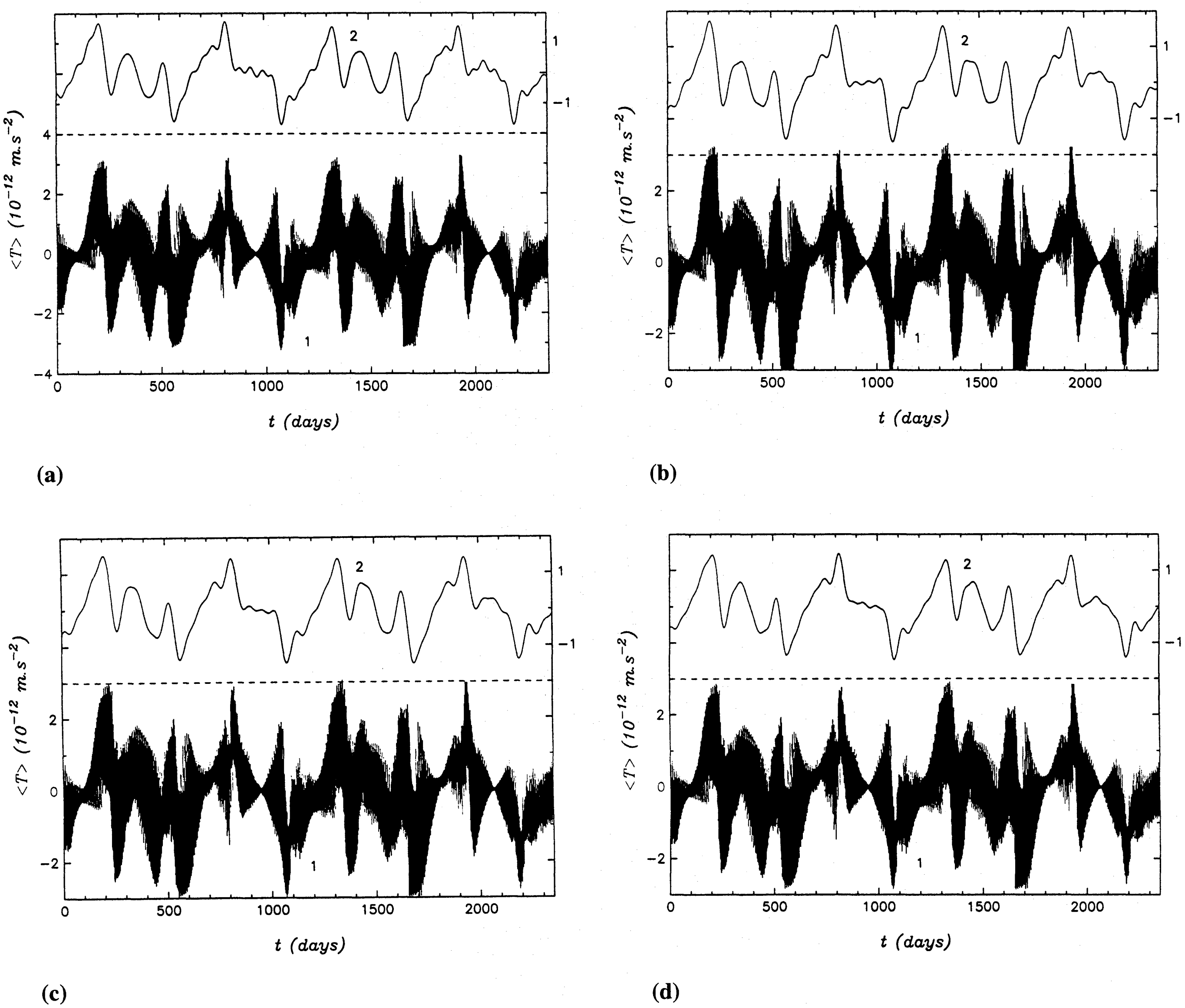
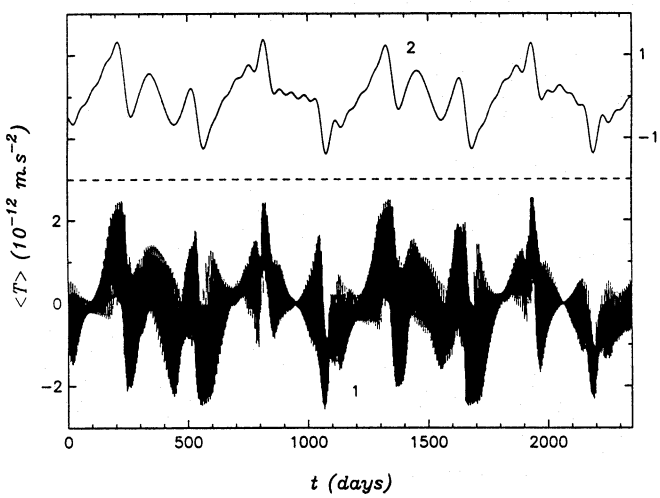
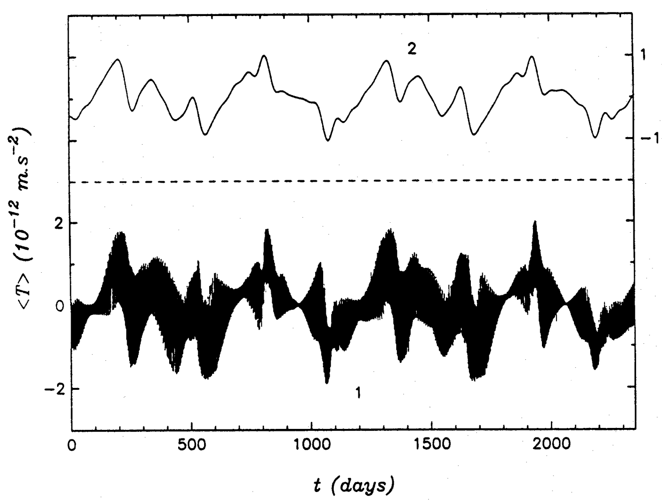


Fig. 1a–d. One–revolution averaged, transverse component ($\langle T \rangle$) of the radiative acceleration due to specularly reflected radiation vs. time, starting at LAGEOS' first orbit determination (May 15, 1976). The lower part of each plot (curve 1) shows the full results of the averaging procedure, including short–periodic components, while the upper part shows a filtered curve (2), obtained by keeping only the ten strongest Fourier components of the full $\langle T \rangle$ vs. time curve. Plot **a)** shows results from the simple mirror–like reflection mode using the ideal Fresnel coefficient $\mathscr{R}(\vartheta)$, as in Barlier et al. (1986) and Vokrouhlický et al. (1993b). Plots **b), c)** and **d)** have been derived by using instead the effective Fresnel coefficient $\langle \mathscr{R} \rangle (\vartheta; \alpha_\star)$, averaged over all the orientations of the reflecting surface elements, which were assumed to be distributed in different ways: a uniform distribution up to a cutoff $\alpha_\star = 5°$ [plot **b)**] or $\alpha_\star = 20°$ [plot **c)**]; a distribution consistent with data by Cox & Munk (1954) [plot **d)**]

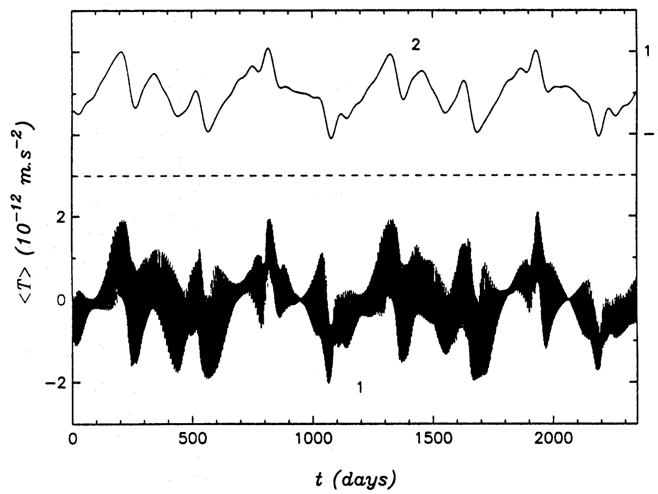
long–term perturbation changes at most by $\approx 20\%$, in agreement with the conclusions of paper I, and as expected is smaller when the average sea surface "roughness" is larger, that is in the cases of the uniform distribution with $\alpha_\star = 20°$ (Fig. 1c) and of the Cox & Munk model (Fig. 1d). Note that while the filtered curves reach amplitudes of $\approx 1.5 \times 10^{-12}$ m/s^2, the overlapping short–periodic effects have a similar magnitude, so the orbit–averaged perturbation undergoes large, rapid variations. The dominant frequencies are associated with the orbital period and the Earth's rotation rate for the short–periodic effects, and with combinations of the solar mean motion and the nodal motion of LAGEOS for the long–periodic effects (in agreement with the results of Barlier et al. 1986 and Vokrouhlický et al. 1993b). These long–periodic frequencies are clearly due to the fact that the intensity of the albedo effect due to specular reflection is modulated by the angle between the orbital plane and the solar position. The main spikes in the $\langle T \rangle$ vs. time curve repeat with a periodicity of 560 days, associated to the argument $(\lambda_\odot - \Omega)$, i.e. the difference between the solar longitude and LAGEOS' nodal longitude. Unfortunately, the same spectral components



(a)



(b)



(c)

Fig. 2a–c. The same as in Figs. 1b–d, but using the more refined (and computationally demanding) approach of accounting for the geometry of the reflected rays from an extended reflecting region on the ocean surface. The assumed probability distributions $\hat{p}(\alpha)$ and $\alpha_\star$ cutoff values are the same as in Figs. 1b–d, respectively

are common to other non-gravitational effects, such as "thermal thrust" and penumbra effects, so their presence in the data cannot be used to discriminate between different perturbation mechanisms. Anyway, it is clear that the albedo effect contributes in a significant way to the observed residuals, but cannot explain them alone. For a more detailed discussion of these spectral lines, their phases and amplitudes and the comparison with the data, we refer to Barlier et al. (1986) and Vokrouhlický et al. (1993b). We shall add some further comments on this in Sect. 4.

In the second series of figures (Figs. 2a–c), we present the results of the same models as in the previous ones (Figs. 1b–d), but using the more refined technique based on the geometry of reflected light rays, which accounts for the finite–lobe reflection pattern and the extended reflecting region "seen" on the Earth from the satellite. Of course, when performing the double integration yielding the radiative vector flux (Eq. (24) of paper I), one has to check for each ray whether the reflecting Earth element corresponds to a continent (and discard it) or to an ocean (and account for it). As a result, the shape of the reflecting spot on the Earth surface is a result not only of the reflection process (as discussed in the paper I, see Fig. 5 there), but also of the "continental mask". Some tests suggested to us that decreasing the number of grid points for the double integration below about 40×40 may degrade the accuracy of the results; as a consequence, a serious drawback of this approach consists in its comparatively large computing time requirements.

As shown in Figs. 2a–c, the behaviour of the $\langle T \rangle$ vs. time curves is very similar to that shown in Figs. 1, apart from some further decrease in the amplitude of the main long–periodic components, which is now reduced to about 1×10^{-12} m/s^2, namely $\approx 40\%$ less than it would be inferred assuming a simple mirror–like reflection mode (Vokrouhlický et al. 1993b).

In view of the computing time required by this kind of simulations, we did not test other possible models for the statistical distribution $\hat{p}(\alpha)$ – for instance the exponential *model 2* introduced in paper I. However, the results reported above and the one–revolution tests performed in paper I lead us to the conclusion that no really new feature is likely to be found. The quantitative factor for the decrease of the amplitude of the long–periodic effects may slightly change. However, we believe that the different approaches to compute the reflected light intensity, the different forms assumed for $\hat{p}(\alpha)$ and the different values chosen for the $\alpha_\star$ cutoff should provide reliable lower and upper limits for this decrease factor, of about 10% and 40% respectively. This is confirmed by inspecting Fig. 3, where we have collected all the filtered $\langle T \rangle$ vs. time curves resulting from the simulations described above.

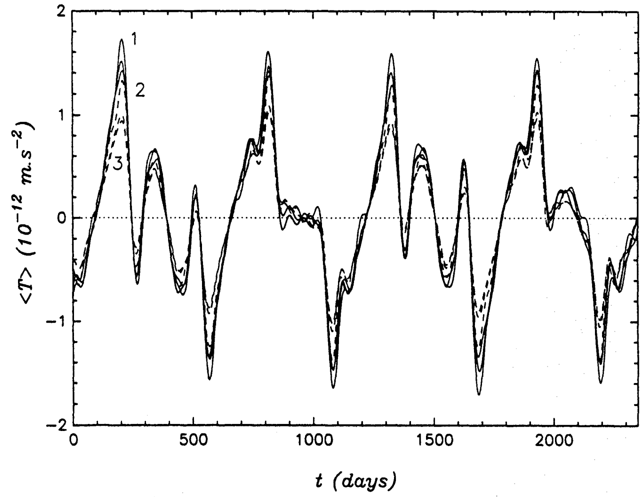


Fig. 3. The Figure collects in the same plot all the Fourier–filtered $\langle T \rangle$ vs. time curves shown in Figs. 1 and 2. The dashed and solid curves correspond to the two different approaches to model the reflection process, used to derive Figs. 1 and 2 respectively. Labels 1, 2 and 3 refer to the different models for the $\hat{p}(\alpha)$ probability distribution

So far, we have not taken into account the fact that for the real Earth the presence of large–scale cloud formations in the atmosphere affects in a very significant way the sunlight reflection process. It might be argued that since clouds shield the radiation reflected specularly from ocean surfaces, the perturbing accelerations obtained from the cloud–free Earth surface models should be decreased by a factor of the order of the average cloud filling fraction in the sky. This parameter, obtained by averaging over time (in particular, over the seasonal cycle) and location, is approximately 50% (Stowe et al. 1989). Were this true, we could conclude that the albedo effect contributes only a minor fraction of the observed LAGEOS along–track residuals. But the above argument is false for two distinct reasons.

First, large cloud formations are also likely to contribute, though in a different manner, to the revolution–averaged perturbing accelerations, because they do not diffuse sunlight in a Lambertian mode, but are characterized by anisotropic scattering processes (Vokrouhlický et al. 1993b).

Second and more important, the cloud coverage of part of the Earth surface does not remove nor weaken the essential mechanism for the appearance of the semimajor axis residuals, namely the anisotropic distribution of the specularly reflecting surface elements. Indeed, we have performed several test runs of our program with different assumed cloud distributions on the Earth surface, similar to the ones discussed in Vokrouhlický et al. (1993b): a full coverage on the Southern estimate, an equatorial cloud belt, two polar cloud caps, and so on. The resulting amplitudes of the $\langle T \rangle$ vs. time curves did not change much with respect to the no–cloud simulations for any of the assumed configurations. On the other hand, the spectra and the phases of the dominant Fourier components depend sensitively on the cloud distribution.

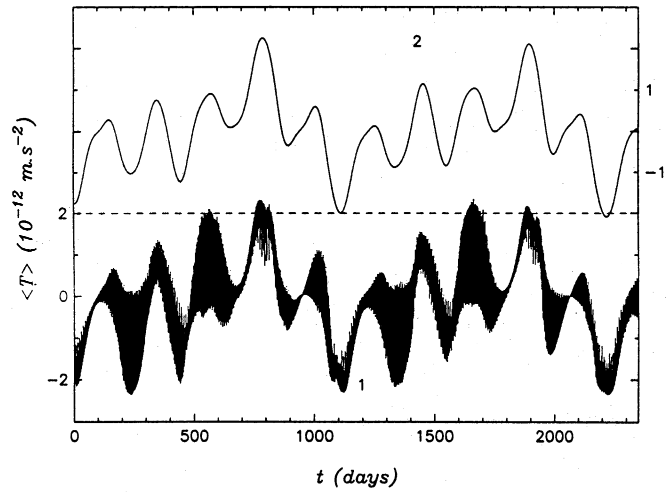
As an example, we have plotted in Figs. 4 the results from a model assuming a seasonally–variable cloud distribution, in agreement with the "probabilistic model" of Vokrouhlický et al. (1993b). In practice, we have introduced a probability $P(\lambda_\odot, \varphi_g) = (1 - \sin\lambda_\odot \cos\vartheta_g)/2$ of cloud coverage at the geographical colatitude ϑ_g and at a time of year specified by the solar longitude $\lambda_\odot$ (measured from the vernal equinox; obviously, 'the correct' form of this probability function is in reality much more complicated and hardly modeled in a deterministic way, see e.g. Mokhov & Schlesinger 1993,1994). Then, we have multiplied the intensity of reflected radiation from each of the Earth surface elements times the factor $P(\lambda_\odot, \varphi_g)$.‡ Fig. 4a shows the resulting long–term behaviour of the transverse acceleration component, using the same reflection model adopted to derive Fig. 1b (calculation of the effective Fresnel coefficient, $\hat{p}(\alpha)$ =constant for $\alpha < \alpha_\star = 5°$).

Comparing Figs. 4a and 1b, one can observe that the maximum amplitude of the $\langle T \rangle$ vs. time curve is little changed, whereas its shape is quite different. The reason is that the seasonal and latitudinal dependence of the cloud coverage factor introduce an additional modulation in the long–term pattern of the perturbation, in agreement with the suggestion of Anselmo et al. (1983) and the results of earlier studies (Barlier et al. 1986; Vokrouhlický et al. 1993b). As a consequence, any quantitative model of the albedo effect for LAGEOS must be based on the use of extensive data on the cloud coverage and its variability, in addition to data on the oceanic reflection modes.

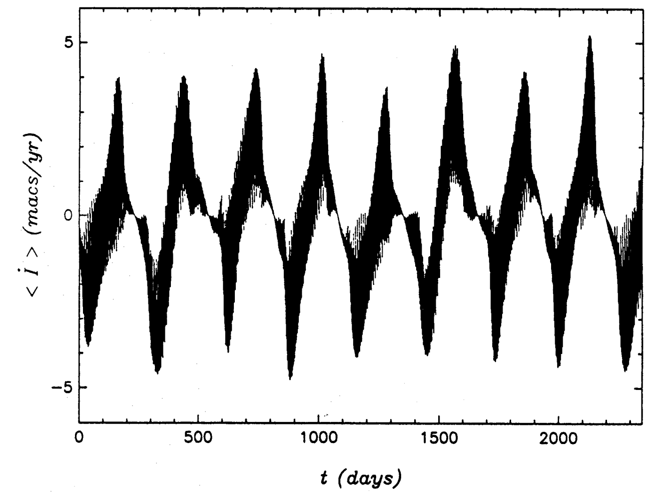
In parts b) and c) of the Fig. 4 we also show the long–term node and inclination effects resulting from this model. The node results are consistent with those of our previous study (Vokrouhlický et al. 1993) and of Lucchesi (1994, personal communication), who used the reflection models discussed in Lucchesi & Farinella (1992). Essentially, we have peak values of $\dot{\Omega}$ of the order of 10 mas/yr, but only a much smaller "secular" effect (about 0.5 mas/yr) survives when the periodic components are averaged out, over a time span of $\approx$ 6 years. This remaining long–term nodal drift is only a few percent of the general–relativistic Lense–Thirring precession, which may become detectable by analysing the orbital evolution of LAGEOS–type satellites (see Ciufolini 1989). However, such experiments should be based on the analysis of very long orbital arcs, lest the periodic components of the albedo effect would strongly degrade their accuracy.

Inclination effects are also present. As shown in Fig. 4c, they appear to be dominated by a $2(\lambda_\odot - \Omega)$ 280–days periodic component (half the synodic period of the orbit's node with respect to the Sun, namely the periodicity of eclipses), which has an amplitude of $\approx$ 3 mas/yr. As discussed by Farinella et al. (1990), such albedo effects on the inclination may well contribute in a significant way to the observed residuals on this orbital element, though other mechanisms (in particular the so–

‡ Actually, the probabilistic model of Vokrouhlický et al. (1993b) was based on the idea of performing a kind of Monte Carlo simulation, that is comparing $P(\lambda_\odot, \varphi_g)$ with a suitably chosen random number to assess whether the corresponding surface element was covered by clouds or not. We have checked that this procedure gives results very similar to those shown in Figs. 4, just with a slightly higher dispersion.

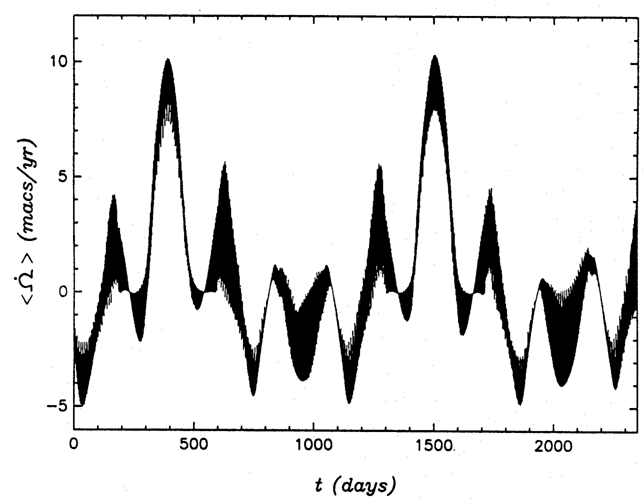


(a)



(c)

Fig. 4a–c. Orbit–averaged transverse acceleration component (a), node (b) and inclination (c) drift rates vs. time for LAGEOS, resulting from the same ocean reflection model adopted to derive Fig. 1b plus the cloud distribution model described in the text



(b)

called "thermal thrust" force) probably provide contributions of a similar magnitude – but with a very different time dependence.

4. Conclusions

The main results of this paper can be summarized as follows:

1. The amplitude of the long–periodic terms in LAGEOS' semimajor axis perturbations caused by specularly reflected sunlight from the oceans can be estimated to be of the order of 1 to 1.5×10^{-12} m/s^2, on the base of our models accounting for sea surface "roughness" and finite–lobe reflection patterns. This represents a reduction by $\approx 10 - 40\%$, depending on the details of the adopted models, with respect to our previous results derived by assuming a mirror–like reflection mode (Vokrouhlický et al. 1993b), in agreement with the preliminary conclusions drawn from an analysis of the dependence of the perturbing force upon mean anomaly (see paper I).
2. Taking into account that the amplitude of the observed along–track residuals of LAGEOS' orbit is about 3×10^{-12} m/s^2 (Barlier et al. 1986; Rubincam 1990), we conclude that specularly reflected solar radiation – and, more in general, the albedo effect – probably accounts for some 30–50% of the observed residuals. Unfortunately, an accurate quantitative model of this perturbation would require a much better knowledge of the properties of ocean surfaces (and possibly of their variability), together with extensive data on the distribution of large–scale cloud formations in the atmosphere.
3. The results summarized above support the conclusion that a number of other non–gravitational mechanisms of similar magnitude are simultaneously at work in generating the unmodelled LAGEOS along–track residuals (see also Vokrouhlický & Farinella 1994b). These are: "thermal thrust" effects due to the differential heating of the satellite body (Rubincam 1987, 1988; Afonso et al. 1989); neutral and charged particle drag (Barlier et al. 1986); solar radiation pressure during the Earth penumbra transitions (e.g. Vokrouhlický et al. 1994a; Inversi & Vespe 1994). All of these mechanisms – as well as that analyzed in this paper – have similar characteristic periodicities and depend on many input data and phenomenological parameters, which in most cases are poorly known. In our opinion, therefore, any quantitative *a posteriori* fit of the long–term LAGEOS orbit residuals runs the danger of giving rise to unwanted *aliasing* effects, namely of obtaining wrong parameter estimates by mixing up different perturbation mechanisms

and/or overestimating the role of some of them at the expense of others. An example of this may be the work of Scharroo et al. (1991), who tried to fit the LAGEOS semimajor axis residuals by a mixture of thermal thrust effects and radiation pressure perturbations due to an assumedly significant optical anisotropy of the LAGEOS surface (which had not been previously known by independent means), and at the same time neglected completely the albedo effect. Their best–fit value (55 K) for the maximum temperature difference between opposite LAGEOS hemispheres was thus much larger than the independent estimates of Rubincam (1988) and Afonso et al. (1989). We believe that a more promising way to make progress on this issue is that of applying the same models for non–gravitational forces used so far to analyse LAGEOS' semimajor axis perturbations to independent data sets – e.g., to long–term residuals for other orbital elements of LAGEOS, and/or to tracking data for satellites on different orbits, such as LAGEOS II.

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