

# Emission-Line Spectroscopy and Analysis

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November 2017

## Various topics to be addressed:

- Processes that govern ISM/nebular excitation and ionization
- Diagnostic information obtained from different kinds of transitions
- Why most ionized (nebular) regions have kinetic  $T_e \sim 10^4$  K
- The Balmer decrement (H recombination spectrum)
- Why forbidden lines are prominent in ionized gas regions
- e-databases useful for spectroscopy

## Reference Books:

- Osterbrock, D. E. & Ferland, G. J. 2006, *Astrophysics of Gaseous Nebulae and Active Galactic Nuclei* 2<sup>nd</sup> edn (Sausalito: Univ. Sci. Books)
- Pradhan, A.K. & Nahar, S.N. 2011, *Atomic Astrophysics & Spectroscopy* (Cambridge: Cambridge Univ. Press)
- Bashkin, S. & Stoner, J. 1975, *Atomic Energy Levels & Grotrian Diagrams* (North-Holland: Amsterdam)
- v2.05 Atomic Line List website: <http://www.pa.uky.edu/~peter/newpage/>
- NIST Atomic Spectra Database website: <https://www.nist.gov/pml/atomic-spectra-database>

stanley bashkin and john o. stoner, jr

# atomic energy levels & grotrian diagrams 1

hydrogen I – phosphorus XV



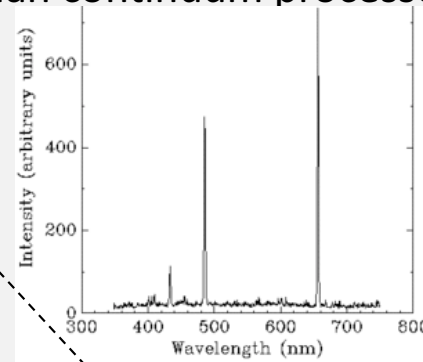
north-holland/american elsevier

# Lecture 1

- Emission spectra
- Spectroscopic notation
- Atomic processes
- Interaction rates
- Cross sections

Emission Spectrum: produced in **Statistical Equilibrium**:

- Mean free path of photons  $>$  size of object ('*opt thin*')
- Due to bound-bound (line) transitions have larger cross sections than continuum processes for emission



**Emission Spectrum**

- Originate in low density objects
- (Photon) mean free path  $= 1/(n\sigma)$

$$(n \propto R^{-3} \text{ for constant mass}) \propto R^3$$

**Therm Equilib**: applies whenever physical processes that determine thermodynamic properties of a gas ( $P, n, T$ ) occur over distance scales less than distances over which  $n$  &  $T$  change. (Mean free path small  $\rightarrow$  isolation)

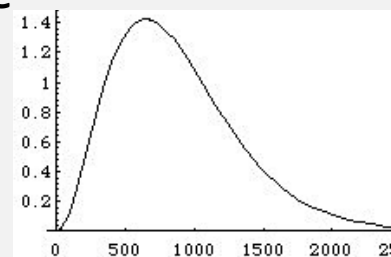
Continuous Spectrum: produced whenever .....

- Mean free path of photons  $\ll$  size of object ('*opt thick*')
- In high density, **Thermodynamic Equilibrium** applies:

**Isolation**  $\leftrightarrow$  'Detailed Balance'  $\leftrightarrow$  reaction rates = their inverse

- $>$  intensity,  $I_\nu = \text{Planck fn } B_\nu(T)$
- $>$  velocity distribution,  $f(v) = \text{Max-Boltzmann fn}$
- $>$  ionization & excitation = Saha and Boltzmann eqns

**Continuous Spectrum**





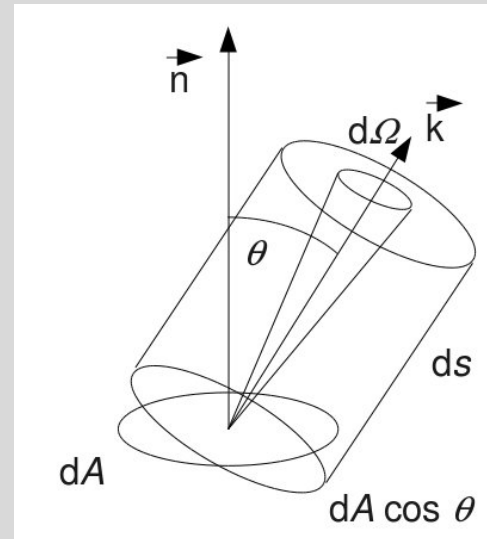
# Emission Spectrum

## Define:

*Intensity,  $I_{\nu}$*  = radiant energy crossing surface per unit (area•time•solid angle•freq) [erg/(cm<sup>2</sup> sec ster Hz)]

*Flux,  $F_{\nu}$*  = radiant energy crossing surface *in all directions* per unit (area•time•freq) [erg/(cm<sup>2</sup> sec Hz)]

$$= \int I_{\nu} \cos \Theta d\Omega$$



## Equation of Radiative Transfer:

$$dI_{\nu}/ds = j_{\nu} - \kappa_{\nu} I_{\nu} \longrightarrow I_{\nu} = \int j_{\nu} ds \quad \text{if no absorption}$$

*(Volume) Emission coefficient,  $j_{\nu}$*  = energy emitted per unit (volume•time•solid angle) [erg/(cm<sup>3</sup> sec ster)]  
for transition  $i \rightarrow j$

$$= n_i A_{ij} h\nu_{ij}/4\pi \quad \text{for emission *line* transitions } i \rightarrow j$$

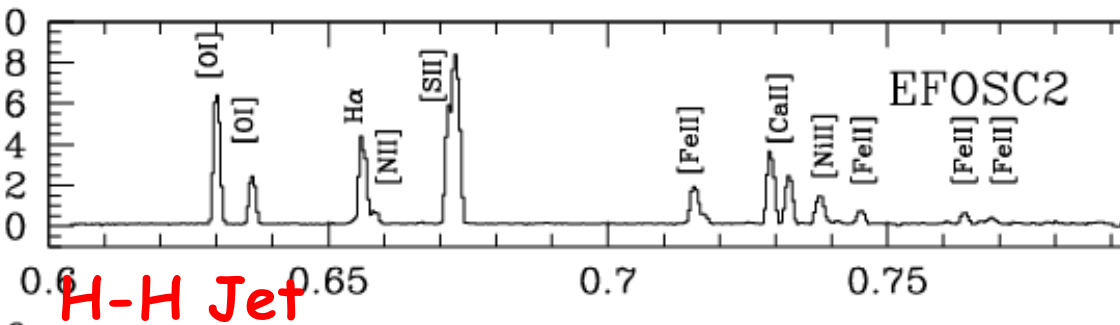
*(Volume) Absorption coefficient,  $\kappa_{\nu}$*  =  $n a_{\nu}$  = 1/(mean free photon path) [cm<sup>-1</sup>]

where  $n$  = density of absorbers [cm<sup>-3</sup>]

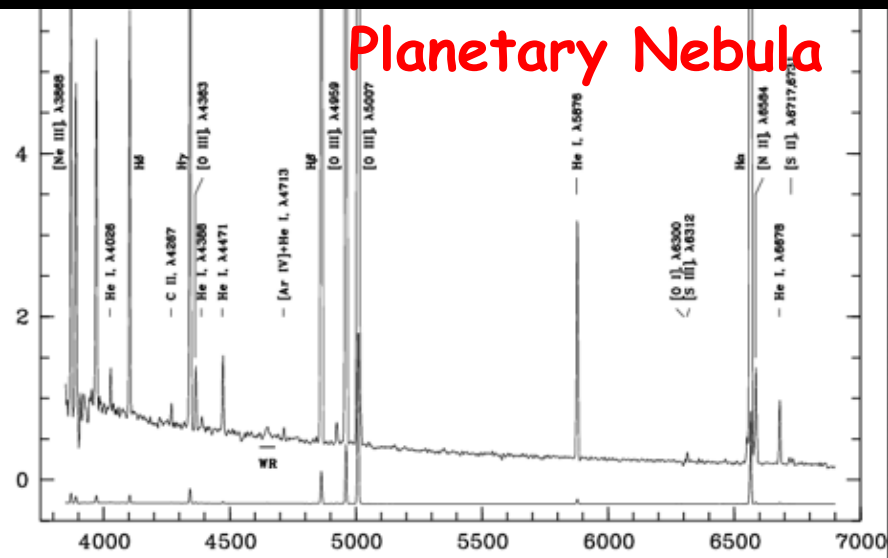
$a_{\nu}$  = absorption cross section [cm<sup>2</sup>]

# Emission-line Spectra

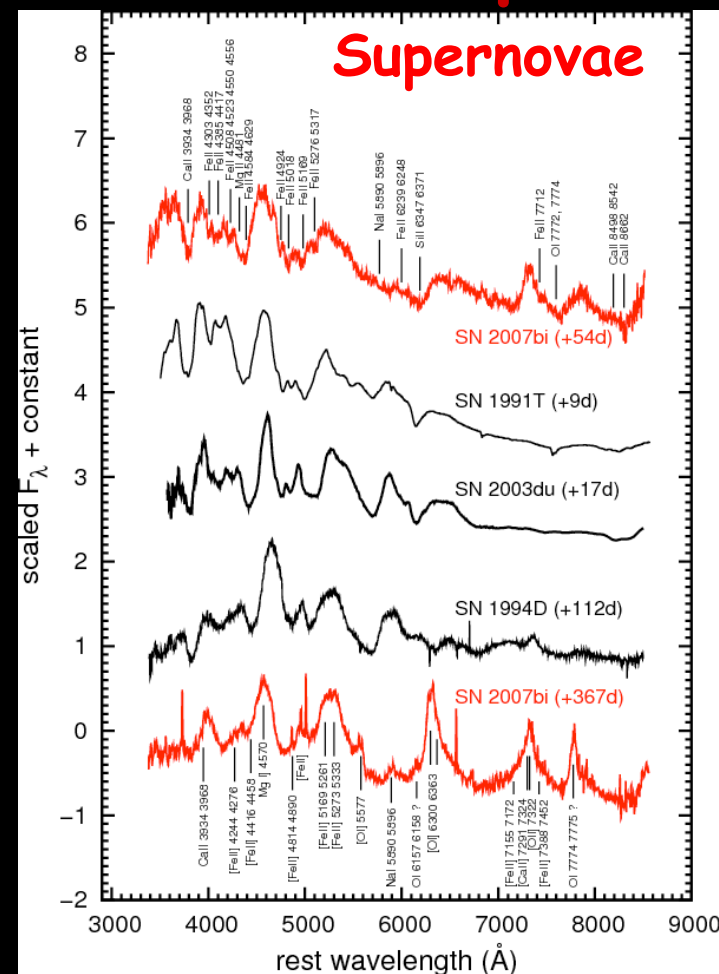
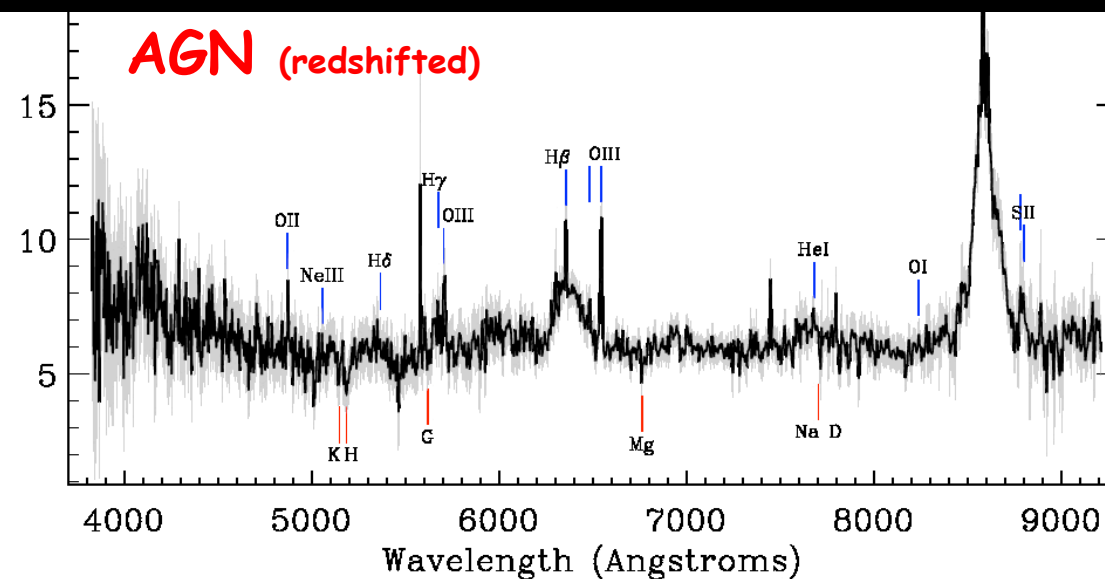
HH1 jet, knot G



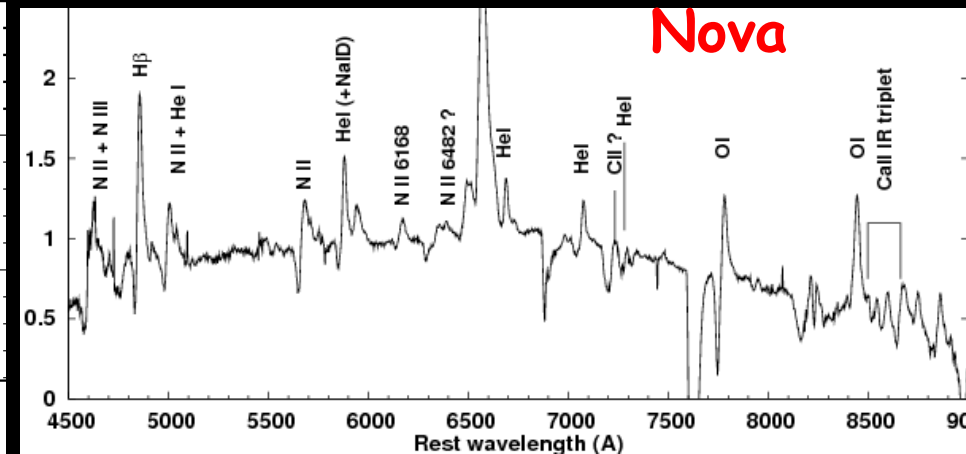
Planetary Nebula



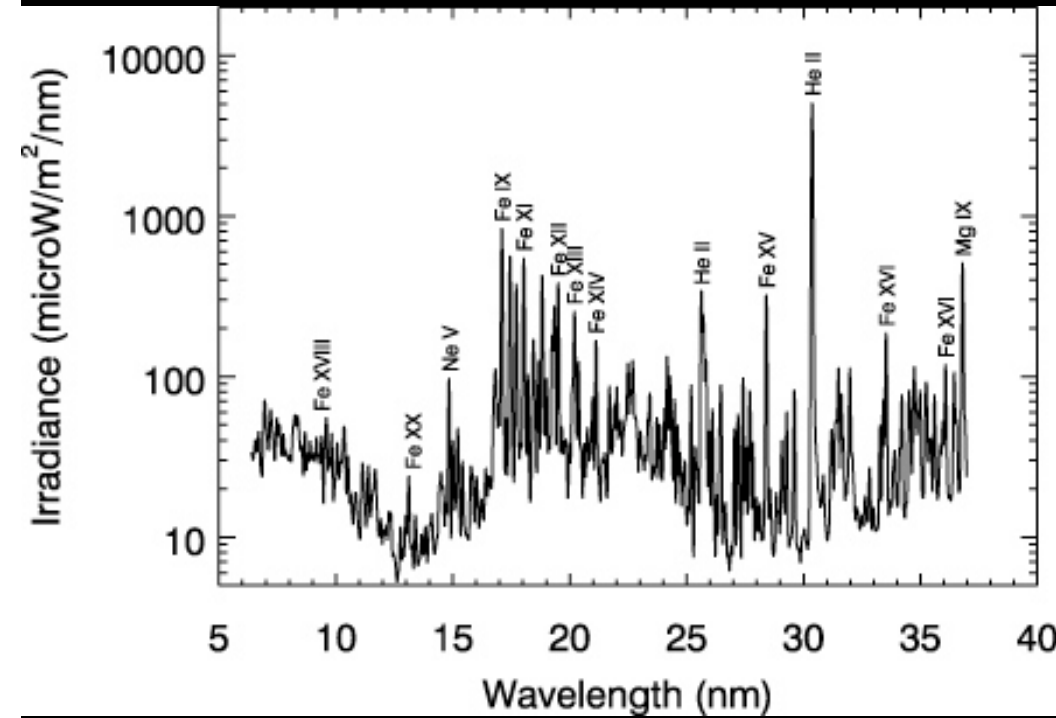
AGN (redshifted)



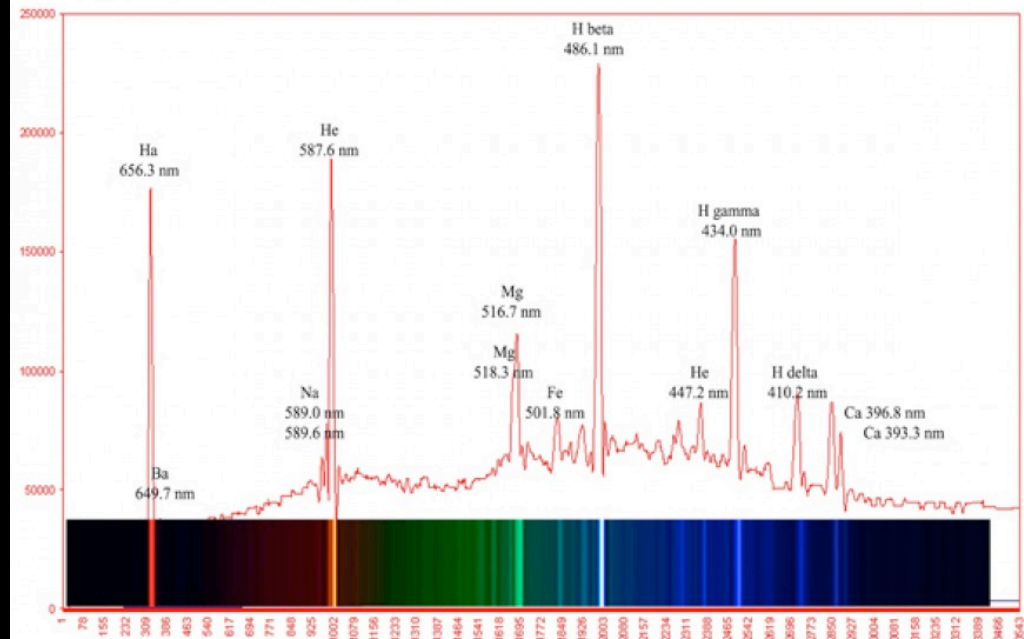
Nova



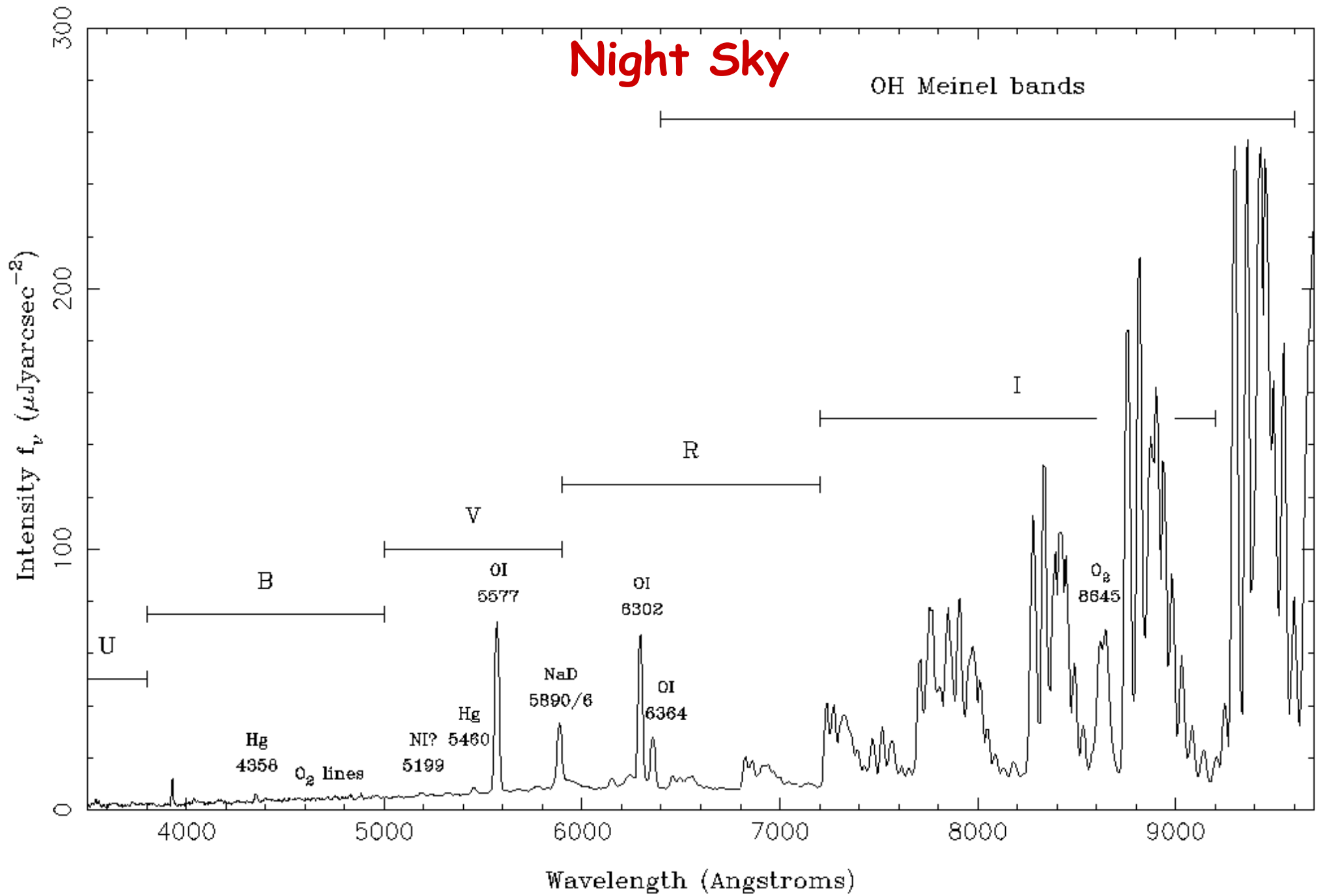
# Solar Corona & Chromosphere



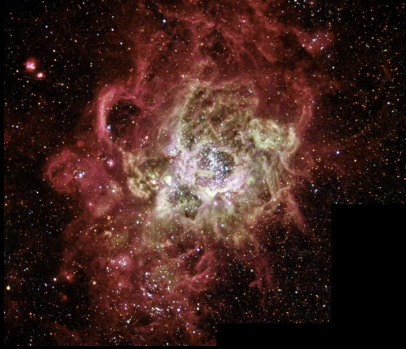
*The Solar Chromosphere Spectrum (Flash Spectrum)*







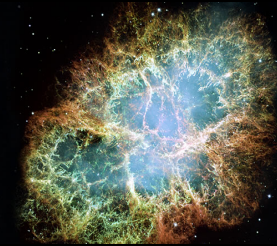
# Ionized ISM Constituents



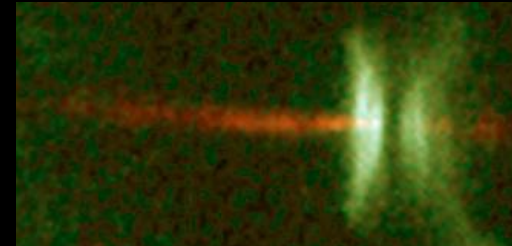
H II Regions



Planetary Nebulae



Nova & Supernova Remnants



Herbig-Haro Object Jet

**Size**

1-100 pc

0.2 pc

1 - 50 pc

1,000 au

**Mass**

$10^{4-6} M_{\odot}$

$0.3 M_{\odot}$

$1-10 M_{\odot}$

$10^{-6} M$

**Lifetime**

$10^{7-8}$  yr

$2 \times 10^4$  yr

$10^4$  yr

$10^6$  yr

**Abundances**

Solar

CNO~2 Solar

>10 Solar

~Solar

**Expansion velocities**

5-10 km/s

20-30 km/s

$10^{2-3}$  km/s

100 km/s

**Energy source**

O Association  
hot stars

RG→WD hot central star

Explosion of SN  
+ pulsar

Stellar grav  
Potential; mag field

# Atomic Energy Levels

- Energy levels are determined by the **Schroedinger Eqn**

$$H\psi = E\psi$$

where  $H$  is the Hamiltonian of a charged particle with spin in an electric field:

$$H = p^2/2m - Ze^2/4\pi\epsilon_0 r + (\hbar/2m) \underline{s} \cdot \underline{l} + (\hbar/2m) \underline{s} \cdot \underline{s}$$

In quantum mechanics the momentum operator  $p$  can be replaced by  $\hbar/i \partial/\partial x$ .

For a spherically symmetric system  $H\psi = E\psi \longrightarrow -\hbar^2/2m \nabla^2\psi - Ze^2/4\pi\epsilon_0 r = E\psi$

In spherical polar coordinates:

$$-\hbar^2/2mr^2 \left[ \partial/\partial r (r^2 \partial\psi/\partial r) + 1/\sin\theta \partial/\partial\theta (\sin\theta \partial\psi/\partial\theta) + 1/\sin^2\theta (\partial^2\psi/\partial\Phi^2) \right] - Ze^2\psi/4\pi\epsilon_0 r = E\psi$$

The solution of the Schroedinger eqn for the single  $e^-$  atom can be written generally in the form

$$\psi_{nlm} = A_{nlm} L_{nl}(r) P_{lm}(\cos\theta) e^{im\Phi}$$

with eigenstate energy:  $E_{nlms}$

  
Laguerre & Legendre polynomials

# Energy Levels & Spectroscopic Notation

- The energy levels of many-electron atoms/ions are specified by the quantum numbers of the electrons:

$n_i \rightarrow$  orbit of each electron  $i$

$\ell_i \rightarrow$  orbital angular momentum of each electron, with  $\ell_i < n_i$  ( $\ell_i=0$  [s],  $\ell_i=1$  [p])

$L \rightarrow$  orbital angular momentum of the state =  $\sum \ell_i$  (vector) ( $L=1$  [S],  $L=2$  [P])

$S \rightarrow$  spin angular momentum of the state =  $\sum s_i$  (vector) (where  $s_i = \frac{1}{2}$ )

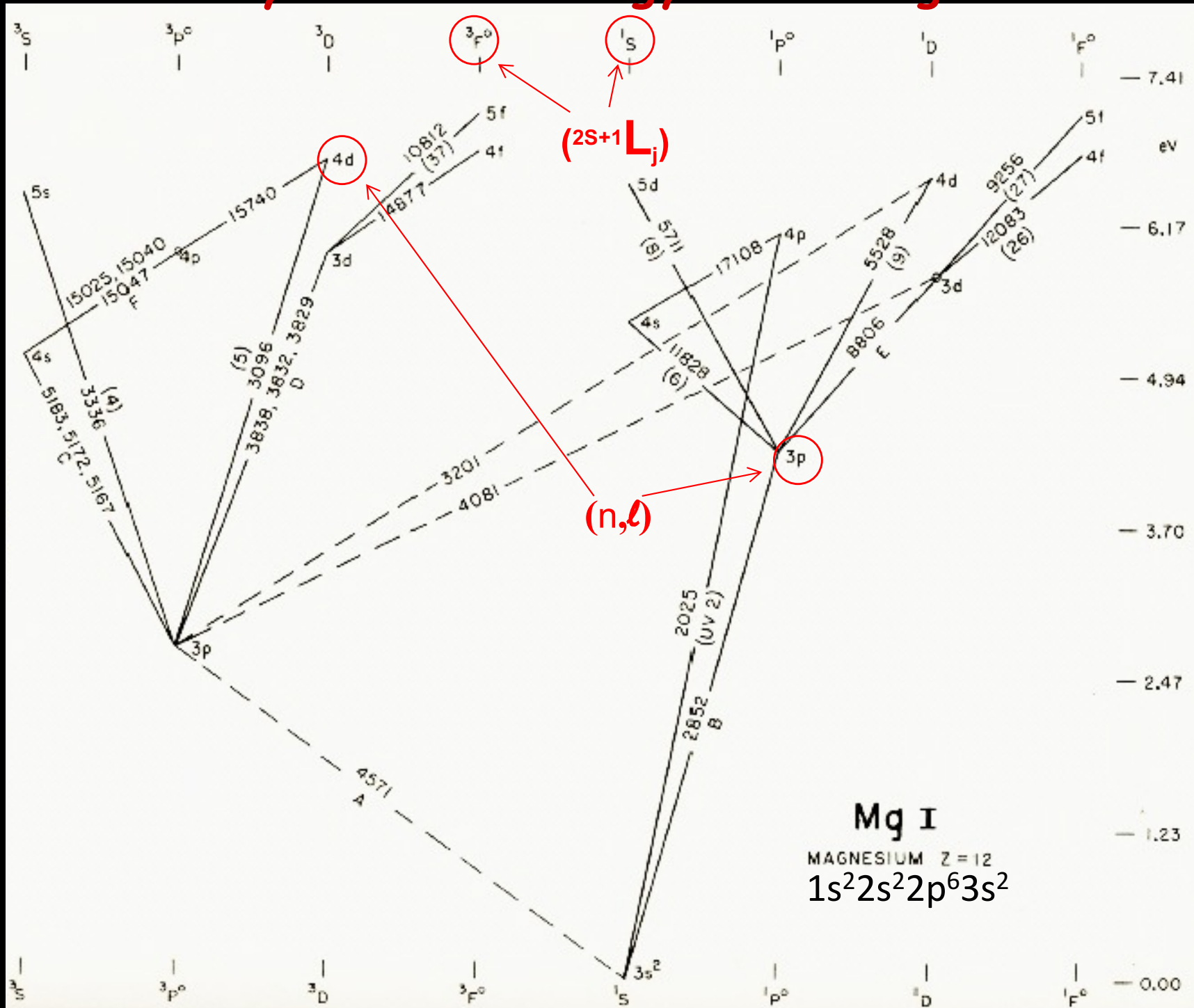
$J \rightarrow$  total angular momentum of the state =  $\underline{L} + \underline{S}$  (vector)

- Specification of these quantities represents a distinct state with a unique energy. For "L-S coupling", the energy levels are most sensitive to the quantum numbers  $n_i - \ell_i - L - S - J$  in that order.
- The spectroscopic notation of different energy states uses the convention:

$$(n_i, \ell_i) \text{ } ^{2S+1}L_J \quad \text{for example: } [1s^2 2s^2 2p^6] 3s^2 \text{ } ^1S_0$$

For example, the configurations  $1s^2 \text{ } ^1S_0$  &  $1s2p \text{ } ^3P_2$  represent two different energy states of helium, with the second state ( $^3P_2$ ) having the higher energy because one electron is in a higher orbit.

# Many-Electron Energy Level Diagram



# Ionized ISM Physical Processes

	Process	Description	Comments
<b>R A D I A T I V E</b>	Photoionization	$A^i + h\nu \rightarrow A^{i+1} + e^-$	Energy threshold: $h\nu \geq \chi_I$ . Main source of ISM ionization.
	Recombination	$A^{i+1} + e^- \rightarrow A^{i*} + h\nu$	Source of bound-free <b>continuum</b> radiation. Inverse of photoionization
	Spontaneous Radiative Decay	$A^{i*} \rightarrow A^i + h\nu_1$	No energy threshold. Most common form of de-excitation. Primary source of emission <b>lines</b>
	Photoexcitation/ Fluorescence Excitation	$A^i + h\nu \rightarrow A^{i*}$	$h\nu = \Delta E$ . Populates a few specific levels. Usually followed by spontaneous radiative decay
<b>C O L L I S I O N A L</b>	Collisional: Ionization Excitation De-excitation	$A^i + e^- \rightarrow A^{i+1} + e^- + e^-$ (ionization) $\quad \quad \quad \rightarrow A^{i*} + e^-$ (excitation) $A^{i*} + e^- \rightarrow A^i + e^-$ (de-excitation)	Threshold: $\frac{1}{2} m_e v^2 \geq \chi_I$ (i.e., $kT_e \geq \chi_I$ ) $\frac{1}{2} m_e v^2 \geq \chi_{exc}$ ( <b>e<sup>-</sup> coolant</b> ) No energy threshold
	Dielectronic Recombination	$A^{i+1} + e^- \rightarrow A^{i**}$	No radiation emitted. Large cross section, subject to condition:  $\frac{1}{2} m_e v^2 = \chi_{exc}^2 - \chi_{exc}^1$
	Autoionization	$A^{i**} \rightarrow A^{i+1} + e^-$	Often follows dielectronic recomb if there is no stabilizing emission: $A^{i**} \rightarrow A^{i*} + h\nu$
	Charge Exchange	$A^i + H^0 \leftrightarrow A^{i-1} + H^+ + \Delta E$	Large resonance ( $\Delta E \sim 0$ ) cross section. Controls $O^0 \leftrightarrow O^+$ ionization

$A^{i*}$  = excited state of ion i;  $A^{i**}$  = doubly excited state of ion i

## General 'Interaction Rate' Equation

(from Transport Theory)

\* Define  $\sigma = \pi r_0^2$  as the cross section for interactions between particles of types A & B, so that if an A & B particle come within distance  $r_0$  the interaction occurs (statistically). For most interactions  $\sigma(v) \propto v^{-n}$  ( $n > 0$ ) because higher velocity particles have greater self energy and spend less time influenced by the interaction field.

Let  $n_A, n_B$  = number density of particles A & B ( $\text{cm}^{-3}$ )

$f(v) dv$  = particle speed distribution function (usually Maxwell-Boltzmann)  $\propto v^2 \exp(-mv^2/2kT)$

$\sigma(v)$  = interaction cross section ( $\text{cm}^2$ ) (typically  $\sigma \sim 10^{-16} \text{ cm}^2$  for particle interactions)

$v_0$  = threshold velocity for interaction to occur (from energetics)

Then, the no. of interactions per unit volume and time is easily shown to be

$$\text{Number of A-B interactions}/(\text{cm}^3 \text{ sec}) = n_A n_B \langle \sigma(v) v \rangle \quad \text{cm}^{-3} \text{ s}^{-1}$$

$$\text{where } \langle \sigma(v) v \rangle \equiv q(T) \equiv \int_{v_0}^{\infty} \sigma(v) v f(v) dv$$

'collision coefficient'

$$\propto T^{-1/2} \exp(-mv_0^2/2kT) \quad [\text{for } \sigma(v) \propto v^{-2}]$$

\* For radiation the photon density  $n_\nu$  is related to the mean intensity of radiation  $J_\nu$  by

$$n_\nu = 4\pi J_\nu / (ch\nu)$$

Thus, the rate of radiative interactions (absorption, ionization, etc.) for an ion  $i$  is

$$\text{Number of absorptions by ion } i / (\text{cm}^3 \text{ sec}) = n_\nu n_i \langle \sigma_\nu c \rangle = n_i \int_{\nu_i}^{\infty} 4\pi J_\nu \sigma_\nu / h\nu d\nu$$

(typically  $\sigma_\nu \sim 10^{-18} \text{ cm}^2$  for continuum radiative interactions)

# Cross Sections

