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Icarus 179 (2005) 128-138

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**ICARUS** 

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# Yarkovsky detection opportunities II. Binary systems

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Received 14 December 2004; revised 7 June 2005

Available online 8 August 2005

#### Abstract

We consider the possibility of detecting the Yarkovsky orbital perturbation acting on binary systems among the near-Earth asteroids. This task is significantly more difficult than for solitary asteroids because the Yarkovsky force affects both the heliocentric orbit of the system's center of mass and the relative orbit of the two components. Nevertheless, we argue these are sufficiently well decoupled so that the major Yarkovsky perturbation is in the simpler heliocentric motion and is observable with the current means of radar astrometry. Over the long term, the Yarkovsky perturbation in the relative motion of the two components is also detectable for the best observed systems. However, here we consider a simplified version of the problem by ignoring mutual non-spherical gravitational perturbations between the two asteroids. With the orbital plane constant in space and the components' rotation poles fixed (and assumed perpendicular to the orbital plane), we do not examine the coupling between Yarkovsky and gravitational effects. While radar observations remain an essential element of Yarkovsky detections, lightcurve observations, with their ability to track occultation and eclipse phenomena, are also very important in the case of binaries. The nearest possible future detection of the Yarkovsky effect for a binary system occurs for (66063) 1998 RO<sub>1</sub> in September 2006. Farther out, even more statistically significant detections are possible for several other systems including 2000 DP<sub>107</sub>, (66391) 1999 KW<sub>4</sub> and 1996 FG<sub>3</sub>. Published by Elsevier Inc.

Keywords: Asteroids; Yarkovsky effect; Orbit determination

#### 1. Introduction

In a recent paper (Vokrouhlický et al., 2005; see also Vokrouhlický et al., 2000, 2001), we demonstrated that future radar and optical astrometric observations of near-Earth asteroids (NEAs) will likely provide many opportunities to detect the Yarkovsky effect (e.g., Bottke et al., 2002) in their orbital motion. We considered candidate asteroids with a variety of sizes, orbital parameters, shapes, rotation states, spectral types, etc., but we intentionally omitted discussion of binary systems in that work. The purpose of this paper is

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to extend the analysis in Vokrouhlický et al. (2005) to account for these cases.

The possibility of detecting the Yarkovsky effect for binary systems is both appealing and challenging. The attractive quality, as compared to solitary asteroids, is due to an independent constraint on the total mass of the system from analysis of the relative motion. Experience from the first successful detection of the Yarkovsky effect (Chesley et al., 2003) reveals an "unpleasant intrinsic correlation" of the surface thermal inertia and the asteroid's mass. When one of these parameters is independently known the Yarkovsky signal may be used to determine the other, or if both parameters are somehow independently constrained their uncertainties may be reduced. Binary systems naturally offer this possibility. The other face of the same reasoning, however, is the necessity of solving both the motion of the two asteroids relative each other and the motion of their center of mass

<sup>0019-1035/\$ –</sup> see front matter Published by Elsevier Inc. doi:10.1016/j.icarus.2005.06.003

(COM) about the Sun. This dramatically increases the complexity of the problem.

Although the methodology of our work remains basically the same as described in Vokrouhlický et al. (2005), and already in Vokrouhlický et al. (2000), we need to complement our discussion by an appropriate analysis of the relative motion of the two binary components. This is done in Section 2, and in Appendix A, using the simplest possible approach. In particular, we do not model the coupling between the gravitational perturbations (due to non-sphericity of the components) and the Yarkovsky perturbation, a problem marvelously difficult requiring numerical solution. Rather, we illustrate the principal Yarkovsky effect for the relative binary motion as if the mutual gravitational perturbations did not exist. This allows us to assume the orbital plane of the two components stays fixed in space, as well as their rotation axes (that we additionally assume perpendicular to their orbital plane). For that reason, however, our analysis is not sufficient to study long-term dynamics of the binary system and the role of the Yarkovsky forces for its stability, but we focus solely on a short-term scale. The theory is applied in Section 3, where we consider one binary system-2000 DP107-as an illustration, and some considerations concerning validity of our assumptions are in Section 4. Even though the Yarkovsky detection might be obtained sooner for other systems, we consider it premature to discuss them because their key physical and dynamical parameters have not yet been reported in the peer-reviewed literature. This paper thus sets the concepts, which could be applied to any binary system when enough data are available.

# 2. Theory

Determination of the Yarkovsky effect for close binary systems, which predominate among the NEA binary population, is difficult because it affects both parts of their motion in space: (i) heliocentric motion of the system's COM ("global motion"), and (ii) relative motion of the two asteroids about the COM ("local motion"). A combined analysis of global and local dynamics certainly relies on numerical simulation whose complexity goes beyond standard orbit determination programs. However, we show that the local and global motions are largely decoupled,<sup>1</sup> and the major Yarkovsky perturbation occurs for the global motion, for which the analysis is much simpler.

Let **R** denote the heliocentric position vector of the system COM and **r** the relative position of the secondary component with respect to the primary component. Then the global motion is described by

$$\frac{\mathrm{d}^2 \mathbf{R}}{\mathrm{d}t^2} = -\frac{GM}{R^3} \mathbf{R} + \text{(tidal quadrupole coupling)}$$

$$+X_1\mathbf{f}_1 + X_2\mathbf{f}_2,\tag{1}$$

where G is the gravitational constant and M is the solar mass. The term "tidal quadrupole coupling" denotes the leading tidal term due to interaction of the internal motion with the quadrupole part of the external gravity fields, which are, for the solar influence, smaller by a factor<sup>2</sup> of  $\simeq (r/R)^2 \simeq 10^{-10}$  than the solar monopole acceleration<sup>3</sup>  $GM/R^2$  and may be safely neglected. There are even smaller terms due to higher multipole interactions and the interaction of the solar monopole with quadrupole fields of both asteroids not shown in Eq. (1). In general, however, the point-masses model is a very good approximation of the global motion, except perhaps for very long timescales or deep encounters with planets. The Yarkovsky perturbation is represented by the terms in the second row;  $f_1$  and  $f_2$  are the Yarkovsky accelerations of both components with masses  $m_1$  and  $m_2$ , and  $X_1 = m_1/m$  and  $X_2 = m_2/m$  are the respective fractions with which they contribute to the total mass  $m = m_1 + m_2$  of the system. Note the relative magnitude of the Yarkovsky effective acceleration  $|X_1\mathbf{f}_1 + X_2\mathbf{f}_2|$  with respect to the solar monopole acceleration is  $\simeq 5 \times 10^{-10}$  for the 2000  $DP_{107}$  case studied below. But even more important than the absolute magnitude is a non-zero mean acceleration component transverse to the heliocentric position vector that produces a fast growing perturbation in the orbital longitude.

The local motion is given by

$$\frac{d^2 \mathbf{r}}{dt^2} = -\frac{Gm}{r^3} \mathbf{r} + (\text{quadrupole coupling}) + \mathbf{f}_2 - \mathbf{f}_1, \qquad (2)$$

where "quadrupole coupling" stands for quadrupole (and higher multipole) interactions of the two asteroids and for their monopole interactions with the solar and planetary (quadrupole) tidal fields. The Yarkovsky perturbation is again represented by the second row. Due to a typical proximity of the binary components in NEAs, the local motion solution is very complicated, also because the quadrupole interaction is intimately coupled with evolution of the spin state of both asteroids. *Representation of the relative motion with a point mass model is inadequate on any timescale of interest*.

While the Yarkovsky perturbation of the global motion is a weighted sum of both Yarkovsky forces, its effect on the local motion is given by their difference. Assuming that the rotation periods of both components are comparable (within an order of magnitude) and that their rotation axes collinear, and neglecting mutual shadowing effects, we may approximate  $\mathbf{f}_1 \simeq \xi \mathbf{f}_2$ , where  $\xi = D_2/D_1 \leqslant 1$ 

<sup>&</sup>lt;sup>1</sup> General discussion of a coupling between the global and local motions is in Damour (1987).

 $<sup>^2</sup>$  This interaction is generally very small and it does not result in a significant long-term perturbation of the orbit; a special topic that warrants a deeper analysis are close encounters with planets (Earth) that may temporarily surpass the solar tidal influence.

 $<sup>^{3}</sup>$  Note this is only slightly more than the uncertainty in *GM* or equivalently the astronomical unit (e.g., IERS, 2003).

is the ratio of characteristic sizes of the two components (here we arbitrarily assume  $D_1 \ge D_2$ ). With that, one easily shows<sup>4</sup>  $X_1 \mathbf{f}_1 + X_2 \mathbf{f}_2 \simeq (1 + \xi^2) \mathbf{f}_1 / (1 + \xi^3) \simeq \mathbf{f}_1$ . The effective Yarkovsky acceleration for global motion in (1) is dominated by the Yarkovsky effect on the *larger* component. On the contrary,  $\mathbf{f}_2 - \mathbf{f}_1 \simeq (1 - \xi)\mathbf{f}_2$  is dominated by the Yarkovsky effect on the *smaller* component (except for cases with binary components of comparable size). These rules likely maximize the total result; mutual shadowing, non-collinear rotation axes and other effects perhaps lessen the effective Yarkovsky force and should be investigated numerically.

### 2.1. Thermal solution for binaries

In the analyses shown below we develop a detailed thermal model for binary systems.<sup>5</sup> Our approach treats both components as irregular bodies whose shape is given by a polyhedral model (e.g., Simonelli et al., 1993). For each of the surface facets, we solve the heat diffusion problem (HDP) in a one-dimensional formulation where the temperature is considered a function of time and depth below the surface (see, e.g., Vokrouhlický and Farinella, 1998). The non-linear boundary condition at the surface, namely the energy balance between the absorbed solar radiation and the heat emitted into space and conducted into the body, is solved by a standard approximation scheme (e.g., Appendix of Spencer et al., 1989). We take into account the mutual shadowing of the surface elements on each of the components and also the effects of eclipses between the two asteroids<sup>6</sup>; both determine the input of solar radiation energy on each of the surface elements and thus affect the HDP. The timestep of the numerical HDP solution is a small fraction of the primary rotation period, typically 10–50 s. The numerical step in depth is a small fraction of the estimated penetration depth of the diurnal thermal wave. At large depths, in practice  $\simeq 10$  times the penetration depth of the seasonal thermal wave, we impose an isothermal core as a second boundary condition. The last boundary condition of the HDP is a periodicity in time after one revolution of the COM about the Sun. The solution of the HDP is iterated until the surface temperature has a fractional variation smaller than  $10^{-4}$ .

With the surface temperature determined for each of the surface facets at any time instant during one revolution about the Sun, we can compute the recoil force due to the thermally emitted radiation (e.g., Vokrouhlický and Farinella, 1998; Bottke et al., 2002). Summing up over the whole surface of the asteroid, one gets the total Yarkovsky accelerations  $f_1$  and  $f_2$  on each of the two components. These results are stored in a computer file<sup>8</sup> and used for further analysis, such as the prediction of global and local Yarkovsky displacements.

Fig. 1 illustrates several important features of the thermal solution in the case of the system 2000  $DP_{107}$ . Most importantly, we note the effective COM perturbation is indeed close to the effect on the primary component. Moreover, the smaller secondary causes only insignificant eclipse phenomena on the primary, so that a first glimpse of the effect may be obtained by considering the Yarkovsky force on the "solitary" primary component (see Fig. 2). On the contrary, the Yarkovsky perturbation of the local (relative) motion of the two components is fundamentally affected by the mutual eclipsing phenomena (see Appendix A) and only adds to the complexity of the interpretation of their relative motion. The principal Yarkovsky perturbations of the global and local motions are of the same nature, namely a secular change in semimajor axis of the respective orbits. The key difference, however, is due to the relative position of the Sun in the two cases. For the COM motion about the Sun only the COM (and not the Sun) is affected by thermal forces that are internal to the system, whereas for the binary relative motion, where the radiation is external to the system, the thermal effects act on both components of the binary.9 This circumstance produces two variants of the thermal-force perturbations (both previously studied in satellite geodesy):

- The heliocentric motion is steadily perturbed by the offradial force component due to the time lag between sunlight absorption and thermal re-emission in exactly the same way as the Yarkovsky effect acts on single asteroids.
- The relative motion of the two asteroids, for which the Sun is an external rather than internal source of radiation, is on a long term affected by the uneven thermal cooling and heating during and after the eclipse phases (see Appendix A and Fig. 1).

From the latter item it follows that eclipses are a necessary condition for Yarkovsky to secularly affect the internal motion. Compactness of the NEA binary systems implies such eclipses are the rule rather than the exception, and thus

<sup>&</sup>lt;sup>4</sup> Comparable densities of the two components are also assumed.

<sup>&</sup>lt;sup>5</sup> A zero level estimate of the Yarkovsky strength in the COM's motion may be obtained with a simpler model that contains only the primary's motion (Fig. 2). However, analysis of the Yarkovsky effect for the local motion necessarily requires a detailed thermal description of both the primary and secondary components because the main, long-term Yarkovsky perturbation of local motion stems from a sequence of shadowing effects.

<sup>&</sup>lt;sup>6</sup> We assume some zero approximation description of the relative motion for the two binary components, usually a circular orbit. This procedure may be, however, iteratively improved as the analysis becomes more constrained.

<sup>&</sup>lt;sup>7</sup> In fact we make sure the timestep is several times smaller than the timelag of the diurnal thermal effect (e.g., Vokrouhlický, 1998).

<sup>&</sup>lt;sup>8</sup> Our results are publicly available through http://sirrah.troja.mff.cuni. cz/~davok/.

<sup>&</sup>lt;sup>9</sup> At this stage we disregard mutual thermal irradiation of the two binary components.



Fig. 1. Time history of the magnitude of the Yarkovsky acceleration for the 2000 DP107 system. Bottom: The abscissa covers one revolution of the COM around the Sun with time equal zero at the pericenter. The larger-amplitude curve is the acceleration of the secondary  $|\mathbf{f}_2|$ , and the smaller-amplitude curve is the acceleration of the primary |f<sub>1</sub>|. The large oscillation of the secondary's acceleration is due to eclipse phenomena, mainly near the pericenter and apocenter where the eclipses are total; around quadrature (e.g., time  $\simeq$  80 days) the secondary eclipses are only partial and the relative oscillation is reduced. Top: Details of the Yarkovsky acceleration during one revolution of the binary components about their common COM. The gray solid curves (labeled "1" for the primary and "2" for the secondary) are the individual effects on the two components. The solid black curve is the magnitude of the weighted sum  $|X_1\mathbf{f}_1 + X_2\mathbf{f}_2|$  from Eq. (1), which is the effective Yarkovsky acceleration of the system's COM, and the dashed black curve is the magnitude of  $|\mathbf{f}_2 - \mathbf{f}_1|$  from Eq. (2), which is the effective Yarkovsky acceleration for the relative motion. The left part (A) is the situation near the pericenter of the heliocentric orbit (at  $\simeq 0.85$  AU), and the right part (B) is the situation near apocenter (at  $\simeq 1.88$  AU). In both cases mutual eclipses occur. First the primary component totally eclipses the secondary component and causes a deep drop of the perturbation, and a half-revolution later the secondary component partially eclipses the primary component causing a smaller effect. Since the primary component is larger by a factor  $\simeq 8/3$  than the secondary component, the Yarkovsky perturbation of the secondary is significantly larger. As discussed in the text, the net perturbation of the heliocentric motion is roughly the effect on the primary, while the effective perturbation of the relative motion is close to the effect on the secondary. The main reason for the larger (smaller) perturbation at the pericenter (apocenter) is the variation of solar radiation flux, which immediately explains a factor  $\simeq 5$  difference. The additional factor is due to the Yarkovsky force dependence on the thermal parameter  $\Theta$  (see, e.g., Vokrouhlický, 1998). For the dominating diurnal variant of the Yarkovsky effect, on the secondary component, for example,  $\Theta \simeq 0.74$ at the pericenter and  $\Theta \simeq 1.33$  at the apocenter. The larger value of  $\Theta$  near the apocenter means a longer relaxation time after the eclipse seen in part (B), whereas in part (A) the secondary acceleration returns much more quickly to the baseline. This effect is also seen for the primary component, where the partial eclipse is followed with a relaxation phase lasting a few rotation-cycles. Note that the details of the shadowing events are not important for the perturbation of the heliocentric motion, while they are essential for the perturbation of the relative motion.

we should readily expect a strong Yarkovsky influence on the local motion, as well as the global motion.

## 3. Candidate systems: an example

Currently, as of November 2004, we know of 23 binary systems in the NEA population (e.g., Merline et al., 2002, and updates on http://www.asu.cas.cz/~ppravec/). The amount of information about each of these systems is unequal and depends on their optical observation history and, especially, whether they have been observed by radar (Ostro et al., 2002). However, even for the best known systems today, their full orbital analysis is complicated and depends on many still unknown or poorly known parameters. For that reason we do not discuss all candidate systems,<sup>10</sup> but rather we summarize them in Table 1. We have selected one of them solely to illustrate the basic concepts that should be

 $<sup>^{10}</sup>$  A good candidate system is judged by estimating strength of the Yarkovsky effect together with a possibility to acquire high-quality astrometry data.

Table 1	
Selected Yarkovsky-detection candidates among the binary asteroid systems within the next t	two decades

Asteroid	$\begin{array}{c} \text{Spectral} & \rho^* \\ \text{class} & [g/ct] \end{array}$	ρ*	* $D_1/D_2^{**}$ g/cm <sup>3</sup> ] [m]	Year of Yarkovsky detectability	Required pre-detection observations	
		$[g/cm^3]$			Radar	Optical
1998 RO <sub>1</sub>			800/400	2006?	2005	2007
2000 DP <sub>107</sub>	С	1.7	800/300	2016	2008	2005, 2011, 2013
1999 KW4	Q	2.6	1200/400	2019	2017, 2018	2016
1996 FG3	C	1.4	1400/430	2022	2009, 2011	2010
1994 AW <sub>1</sub>			900/480	2023	2015, 2022	2008, 2016
2003 YT <sub>1</sub>	V		1000/200	2023	2009, 2011, 2016	
1998 ST <sub>27</sub>	С		800/100	2024	2021	2012, 2015, 2018

*Note.* Objects sorted according to the estimated year of Yarkovsky detection. None of the listed systems requires future astrometric recovery, and the indicated "optical observations" stand for putative photometry that should constrain solution of the asteroids' relative motion (those are typically also possible during the radar apparitions). Additional candidate objects will be posted on http://sirrah.troja.mff.cuni.cz/~davok/.

 $\rho^*$  is the bulk density from analysis of the relative motion of the two asteroids (typically with a large error bar).

\*\* Sizes are usually estimated from absolute magnitude and geometric albedo, when known, and from the lightcurve eclipses; in a few cases radar ranging allowed to better estimate the characteristic size of the components (none of these is however known better than to  $\simeq 10-20\%$ ).

kept in mind when analyzing binaries for which a prolific dataset is available.

# 3.1. 2000 DP107

Most information about 2000 DP107 comes from an extensive radar campaign in September/October 2000 (Margot et al., 2002). The system consists of two components, a primary of diameter  $D_1 \simeq 800$  m and a secondary of diameter  $D_2 \simeq 300$  m, each revolving about the system center of mass with a period of about 1.755 day. The primary rotates at a near critical rate with period of  $\simeq 2.775$  h (Pravec et al., 2000), while the secondary likely has a synchronous rotation rate. Separation between the two components is  $\simeq 2620$  m and the relative orbit is near circular. The primary component was found to be a C-type object,<sup>11</sup> and a preliminary solution of the relative motion from the radar imaging suggests a bulk density of  $1.7 \pm 1.1$  g/cm<sup>3</sup>, appropriate for that spectral type (Margot et al., 2002). There is not enough information to resolve poles of rotation for the two components. For sake of simplicity we assume here that the synchronized secondary component has rotation pole normal to the plane of mutual motion, which would be compatible with a spin-orbit synchronization history. Without observational constraints, we assume the same pole orientation for the fast rotating primary. The preliminary solution of the mutual motion of asteroids in this system from Margot et al. (2002) would give these poles at ecliptic longitude  $\ell = 280^{\circ}$  and ecliptic latitude  $b = 73^{\circ}$ , consistent with lightcurve-detected eclipses (P. Pravec, personal communication).

In our simulation we consider the following surface physical parameters for both components: thermal conductivity K = 0.01 W/m/K, specific heat capacity C = 800 J/kg/K, surface and bulk densities  $\rho_s = \rho_b = 1.7$  g/cm<sup>3</sup>. We note that, a priori, there is no strict reason why the components should have exactly the same physical parameters, and here this is only one of many simplifying assumptions. Both components are considered spherical and represented with 1000facet polyhedra.

or so

With these assumptions, we have determined the orbitaveraged value of the semimajor axis drift due to the Yarkovsky effect (Fig. 2). The effect of non-linearity is a 10–20% reduction of the Yarkovsky acceleration (see also Vokrouhlický and Farinella, 1998, 1999), while the contribution of the secondary component increases the Yarkovsky strength for larger values of the surface thermal inertia (this because of its slow rotation). Generally, though, the linearized theory yields fairly satisfactory results.

The nearest radar observation opportunities for this system are two close encounters in September 2008 and August 2016; there is also good reason to take precise optical astrometry during periods when the sky-plane uncertainty exceeds an arcsecond (e.g., during January–March 2005, November 2005–February 2006, and the early months of 2008). Photometry in December 2005, January 2011, and December 2013 would be also useful to track the relative motion of the components if mutual eclipses are recorded.

#### 3.1.1. Heliocentric motion of the center of mass

We primarily focus on the Yarkovsky perturbation in the heliocentric COM motion for which our force model is adequate and we can produce a reliable estimate. Currently, the available observations constrain the orbit too weakly to allow statistically significant detection of the Yarkovsky displacement during the next close approach in 2008. We thus adopt the same method as in Vokrouhlický et al. (2000, 2005), simulating the 2008 Arecibo astrometry to make the solutions with and without Yarkovsky accelerations statistically distinct during the close approach in 2016. The 2008 apparition is close enough (Arecibo peak SNR  $\simeq$  4000) to allow a high-quality determination of the system's COM; we assume one range measurement with formal accuracy of  $\simeq$ 50 m.

With this simulated data point, and with all past optical and radar astrometry, we propagated the two orbital solu-

<sup>&</sup>lt;sup>11</sup> Alternately X-type, e.g., Yang et al. (2003).



Fig. 2. The orbit-averaged value of the COM heliocentric motion semimajor axis change  $\langle da/dt \rangle$  (in 10<sup>-4</sup> AU/My) due to the Yarkovsky effect for 2000 DP<sub>107</sub> shown as a function of surface conductivity *K* (in W/m/K; other surface thermal parameters as described in the text). The thick solid line is the result from complete numerical model accounting for non-linear analysis of HDP on both binary components and their mutual shadowing effects (Fig. 1). The thin solid line is the non-linear HDP numerical model but only the primary asteroid is considered (as if the system did not contain the secondary). The dashed line shows the result from a linearized approximation of heat conduction and a perturbation of the primary component only (note the non-linearity makes the Yarkovsky signal smaller; see, e.g., Vokrouhlický and Farinella, 1998, 1998). Since the secondary component rotates slowly, its own Yarkovsky drift peaks for higher value of the conductivity and thus contributes more importantly to the total signal. This is the reason for the divergence of the solid curves at larger *K*.



Fig. 3. Range and range-rate projected  $3\sigma$ -uncertainty ellipses of the no-Yarkovsky (dashed) and Yarkovsky (solid) COM heliocentric orbits of 2000 DP<sub>107</sub> on August 24.9, 2016. Arecibo radar offers a  $\simeq$ 50 SNR ranging opportunity at that epoch. The origin is the no-Yarkovsky orbital solution. These solutions assume all past optical and radar astrometry and a simulated Arecibo radar astrometry in September 2008 as described in the text. Different displaced ellipses of the Yarkovsky-solution are for different values of the surface thermal conductivity *K*, whose values (in W/m/K) are indicated in the figure. Note a degeneracy—one ellipse may correspond to two different values of conductivity *K* (Fig. 2).

tions, with and without Yarkovsky, by numerically integrating (1) with all necessary planetary and other perturbations to 2016. Nominal predictions with their formal  $3\sigma$  confidence ellipses were projected onto the plane of radar observables, delay and Doppler, or equivalently range and rangerate. Fig. 3 confirms that during the 2016 close approach, when Arecibo can acquire astrometric data at SNR  $\simeq$  50, the Yarkovsky effect could be comfortably detected as a large COM displacement of the two solutions beyond their formal  $3\sigma$  uncertainty regions.

#### 3.1.2. Relative motion of the components

Our solution may also be used to get a preliminary estimate of the observability of the Yarkovsky perturbation in the relative motion of the two asteroids. However, we emphasize that these results are suggestive rather than predictive since the force model in our simulation is not entirely adequate to describe the details of the internal dynamics of the 2000 DP<sub>107</sub> system, and at present several key parameters, such rotation poles, are only weakly constrained.

With our solution of the Yarkovsky effect for both binary components we constructed the radial, transverse and normal projections of the effective perturbation  $\mathbf{f}_2 - \mathbf{f}_1$ , and from those quantities we determined the corresponding displacements about the zero order circular orbit (see (A.6)–(A.8) in Appendix A). The result is shown in Fig. 4.

First, we confirm the principal orbital perturbation of the local motion of the two components is a quadratic advance of the transverse displacement  $\eta$  (related to the linear radial displacement  $\xi$ ; upper panels) due to a non-zero mean transverse Yarkovsky acceleration (for a broader context see Appendix A). This effect is specific to the dissipative thermal effects and cannot, on a long-term, correlate with gravitational perturbations that are conservative. In spite of in-



Fig. 4. The Yarkovsky displacement of the relative motion of the two components of 2000 DP<sub>107</sub>, according to an approximate (linearized) solution described in Appendix A. Top: Time history of radial  $\xi$  (left) and transverse  $\eta$  (right) perturbations, relative to the circular orbit approximation, with origin at the midpoint of radar observations in 2000. The transverse displacement is folded into an interval equal to the circumference of the unperturbed circular orbit. The principal secular effect, due to the non-zero average value of the transverse Yarkovsky acceleration (discussed in Appendix A), produces a linear drift in the radial component (masked by periodic terms at this short timespan) and a quadratic drift in the transverse component. The latter effect is very distinct from (neglected) multipole interactions of the gravity fields of the two asteroids and it should be in principle observable. Obstacles are (i) the large observational uncertainty (that from the 2000 observations is shown as a shaded area; note this prevents an unambiguous link of the 2000 and 2008 observations), and (ii) its weakness (the total transverse displacement amounts to only  $\simeq$ 240 m in 19 yr shown in this figure). When sparse observational data are only available, a correlation between this quadratic and a linear terms in  $\eta$  may be high and hinder the Yarkovsky signal. Bottom: Same as top part, but here the average values of the radial and transverse perturbing accelerations have been removed. The remaining sub- to few meter displacements are well below the model uncertainties associated with the absence of multipole interactions of the gravity fields of the two asteroids.

completeness of our force model we may thus conclude that the principal Yarkovsky perturbation could be observable. Mismodeling of the system mass, which would produce a linear advance in the transverse perturbation, could mask the effect when data from only a few apparitions are available. In general, however, more observations constrain the solution better and thus sooner or later should reveal the Yarkovsky signal in the local motion.

However, Fig. 4 also indicates two obstacles of an early detection of the Yarkovsky effect in the relative motion of binaries. First, a large orbital phase uncertainty is currently the primary hindrance to detection of the Yarkovsky perturbation of the relative motion. By the end of 2002 the formal uncertainty of the solution by Margot et al. (2002) had already spread over the entire range of values from 0° to 360°. That situation does not allow proper linking of the 2008 phase observations with those from 2000 and thus prevents determination of the Yarkovsky effect. When more data are obtained from short-enough interval of time, such as possible photometry and radar observations in between March

2008 and January 2011 (Table 1), the uncertainty in phase might be reduced. Then a tie to phases of more distant observations in time, namely those in September/October 2000, and later December 2013, could be possible.

Even if successful, Fig. 4 suggests the Yarkovsky signal in the relative motion is very weak. The linear component in the radial displacement  $\xi$  (upper left panel) is not actually seen at this scale and the related quadratic effect in the transverse displacement  $\eta$  (upper right panel) amounts to only ~240 m in 19 years. This stretches to circumference of the unperturbed circular orbit in more then a century. A very long-term data about the system might in principle reveal the Yarkovsky signal, but a more involved study of the relative motion would be needed to see these prospects realistically.

Fig. 5 shows the mean along-track component  $f_{\tau}$  of the Yarkovsky acceleration  $\mathbf{f}_2 - \mathbf{f}_1$  as a function of the assumed surface conductivity *K* of both asteroids. Like the mean semimajor axis drift  $\langle da/dt \rangle$  for the global motion (Fig. 2), the mean along-track acceleration is the principal "tuning parameter" of the secular Yarkovsky effect for the local mo-



Fig. 5. Mean along-track Yarkovsky acceleration  $f_{\tau}$  (in  $10^{-3}$  pm/s<sup>2</sup>) as a function of the surface conductivity *K* (in W/m/K) for 2000 DP<sub>107</sub> system. Geometry of spin axes, and other surface physical parameters as in the text.

tion, since the rate/speed of the semimajor axis drift (Fig. 4, upper left) directly depends on its value. We find it natural that  $f_{\tau}$  depends on *K* is a similar way as  $\langle da/dt \rangle$ .

# 4. Conclusions

The formation and evolution of near-Earth binary systems is becoming a particular focus of planetary science. From the perspective of the present work, a key characteristic of binaries is the fact that one can solve for the total mass of the system from tracking the relative motion of the two asteroids about their COM. With principles of gravitational physics, which set a body's acceleration independent of its mass, this seems to be a singular opportunity. Moreover, non-gravitational perturbations are mass-selective and their detection opens a second possibility of estimating asteroids' mass (e.g., Chesley et al., 2003). This applies both to solitary asteroids (Vokrouhlický et al., 2000, 2005) and also to the binary systems.

In this paper we demonstrated that the prime nongravitational perturbation in the asteroid's motion—the Yarkovsky effect—can be detected in the global motion of the system's COM about the Sun and, in the best cases, also in the local motion of the two asteroids relative to each other. The double detection of the Yarkovsky effect for binaries if achieved in remote future or for more suitable systems than known today—would allow a more profound investigation of the systems physical parameters. Ideally one could characterize parameters of each of the two asteroids separately, but the degree of correlation with other perturbations and uncertainties should be studied for each of the binary systems individually.

The most restrictive assumption in this study, to be removed in a detailed analysis of particular systems, is that of fixed rotation poles of the two asteroids, and a related assumption of the fixed orbital plane of the relative motion of the two asteroids. Tidal evolution (whose timescale is uncertain but perhaps fast for close binaries; Margot et al., 2002; Merline et al., 2002) generally leads to spin synchronization of the satellite (lighter component) and tilts its axis toward the Cassini states 1 or 2, locked to the orbital plane motion (e.g., Peale, 1977, 1999; Gladman et al., 1996). The latter generally depends on dynamical flattening of the satellite and precession rate of its orbital plane; as in the lunar case, rapid precession of the orbital plane likely excludes Cassini state 1 and the only terminal occupancy of the satellite spin is perpendicular to the orbital plane. Radar observations of the 2000 DP<sub>107</sub> are consistent with this situation (Margot et al., 2002).

Long-term evolution of the primary's spin state is more uncertain. In compact systems with fast rotating primary, typical for NEA binaries (e.g., Merline et al., 2002), the total angular momentum of the system is weakly dominated by the rotational angular momentum of the primary.<sup>12</sup> This implies a possibly complicated interplay between the rotational and orbital motion, affecting stability of the system (a problem not considered in detail so far). Since the secondary components of typical NEA systems bear a non-negligible fraction of mass, gravitational torque due to the secondary dominates solar gravitational torque, causing the primary's axis to evolve toward a state similar to that occupied by the secondary, but timescales depend on many uncertain parameters. Nevertheless, dominance of the primary's angular momentum in the system makes us think its pole is the most stable direction. As a result, we think the COM Yarkovsky displacement (see Fig. 3), related to the thermal effects on the primary, is the most justified result above. On the other hand, as the orbit of relative motion precesses, or the secondary's pole moves, the linear drift in the relative distance of the two asteroids (see Fig. 4A) becomes periodic. The results given above are likely an overestimate of the internal Yarkovsky effect, though they are hopefully appropriate within the assumed model and the  $\simeq 20$  yr timescale.

To verify these conclusions we constructed a toy model describing relative motion and spin evolution of two quadrupole-field axial bodies that retains some, though certainly not all, dynamical elements of NEA binaries. Initial orbital data and various parameters were chosen close to the 2000 DP<sub>107</sub> system and the initial rotation poles of both components were varied within  $\pm 15^{\circ}$  in the ecliptic longitude and latitude about the normal to the orbital plane of their relative motion. With those initial data we numerically integrated the Euler–Lagrange equations for 20 yr timespan<sup>13</sup> and we found small variations of the primary's pole (generally within 10° from the initial position), but a rather large

<sup>&</sup>lt;sup>12</sup> For instance, in the 2000 DP<sub>107</sub> case the ratio between the rotation angular momentum of the primary and orbital angular momentum of the pair is  $\simeq$ (2–3).

<sup>&</sup>lt;sup>13</sup> The primary and secondary components were modeled as homogeneous spheroids with ratio of equatorial and polar radii equal to 1.05 and 1.1, respectively, yielding thus only moderate dynamical ellipticity.

variation in the secondary's pole<sup>14</sup> (up to tens of degrees from its initial position). Clearly, more observational constraints are needed to fully address details of the Yarkovsky perturbation of the local motion.

#### Acknowledgments

The work of D.V. and D.Č. has been supported by the Grant Agency of the Czech Republic, under the grant No. 205/05/2737. The research of S.R.C. and S.J.O. was conducted at the Jet Propulsion Laboratory, California Institute of Technology under a contract with NASA. We are grateful to Matija Ćuk who, as a referee, helped us to correct an error in the earlier version of this paper.

# Appendix A. Linearized perturbation of the local motion

In Section 2 we noted that the local motion of the binary system components relative each other is far more complicated than the global (heliocentric) motion of its COM. This is because proximity of the two components implies that couplings of many multipole harmonics in gravity fields of the two asteroids are important and contribute to complexity of their local motion. Moreover, these couplings intimately depend, and in the same time affect, rotation state of both components. Detailed analysis of these effects is well beyond the scope of this paper, both because they should be studied in context of the individual systems [it is hard to imagine that quantitative features, except from general constraints such as in Scheeres (2002a, 2002b), can be derived] and also because our primary focus here is to determine the major signatures of the Yarkovsky effect. For that reason, we relegate the in-depth model of the internal motion when rich enough data are available to study some of the systems.

Our approach here is to examine general properties of the linear perturbation theory of the Keplerian circular motion in Cartesian variables as the simplest possible representation of the Yarkovsky displacement in the relative position of the components in the binary system. We assume an (Yarkovsky) acceleration  $\delta \mathbf{f}$  affects a circular orbit  $\mathbf{r}_0$  (often a good zero approximation of the binary motion) to produce a small perturbation  $\delta \mathbf{r}$  ( $|\delta \mathbf{r}| \ll r_0$ ). With *m* the total mass of the system we have

$$\frac{\mathrm{d}^2 \delta \mathbf{r}}{\mathrm{d}t^2} + \frac{Gm}{r_0^3} \left[ \delta \mathbf{r} - 3\mathbf{r}_0 \frac{(\mathbf{r}_0 \cdot \delta \mathbf{r})}{r_0^2} \right] = \delta \mathbf{f}.$$
 (A.1)

It is common to split the displacement vector  $\delta \mathbf{r}$  into radial  $\xi = \delta \mathbf{r} \cdot \boldsymbol{\rho}$ , transverse  $\eta = \delta \mathbf{r} \cdot \boldsymbol{\tau}$  and normal  $\zeta = \delta \mathbf{r} \cdot \mathbf{n}$  components. Here  $\boldsymbol{\rho} = \mathbf{r}_0/r_0$  is unit vector in the radial direction,

**n** unit vector normal to orbital plane along direction of the orbital angular momentum, and  $\boldsymbol{\tau} = \mathbf{n} \times \boldsymbol{\rho}$  the unit vector in the transverse direction. We assume **n** is fixed, while  $\boldsymbol{\rho}$  and  $\boldsymbol{\tau}$  uniformly rotate in space with angular velocity equal to the mean motion<sup>15</sup>  $n_0$ :  $d\boldsymbol{\rho}/dt = n_0\boldsymbol{\tau}$  and  $d\boldsymbol{\tau}/dt = -n_0\boldsymbol{\rho}$ . It appears useful to introduce scaled quantities  $\delta \mathbf{f} \rightarrow (\alpha_{\rho}, \alpha_{\tau}, \alpha_{\zeta})$  so that  $\alpha_{\rho} = (\delta \mathbf{f} \cdot \boldsymbol{\rho})/n_0^2$ ,  $\alpha_{\tau} = (\delta \mathbf{f} \cdot \boldsymbol{\tau})/n_0^2$ ,  $\alpha_{\zeta} = (\delta \mathbf{f} \cdot \mathbf{n})/n_0^2$ , and replace time *t* with a phase angle of the rotating (sidereal) system  $t \rightarrow \tau = n_0 t$ . With these new variables, (A.1) reads<sup>16</sup> (e.g., Nordtvedt, 1991)

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}\tau^2} + \xi = 2\Phi + \alpha_\rho = \Sigma(\tau), \tag{A.2}$$

$$\frac{\mathrm{d}^2\zeta}{\mathrm{d}\tau^2} + \zeta = \alpha_{\zeta} = \Gamma(\tau), \tag{A.3}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}\tau} = \Phi - 2\xi,\tag{A.4}$$

where

$$\Phi = \int_{0}^{\tau} \mathrm{d}\tau' \,\alpha_{\tau}(\tau'), \tag{A.5}$$

When  $\delta \mathbf{f} = 0$ , solutions of the system (A.2)–(A.4) represent change from one elliptic orbit to another due to variation of the initial conditions. This is characterized by periodic terms in all variables (with anomalistic frequency), a constant term in  $\xi_{\text{free}}$  [a slight change in the semimajor axis due to variation of the orbital angular momentum, a term eliminated from the right had side of (A.2)] and a related linear term in  $\eta_{\text{free}}$ . Strictly speaking coefficients of constant and linear terms in  $\xi_{\text{free}}$  and  $\eta_{\text{free}}$  are correlated, but when the total mass of the system is not known they independently couple to this additional parameter.

When  $\delta \mathbf{f} \neq 0$ , the system (A.2)–(A.4) admits the following particular solution:

$$\xi(\tau) = \int_{0}^{\tau} d\tau' \, \Sigma(\tau') \sin(\tau - \tau'), \qquad (A.6)$$

$$\eta(\tau) = \int_{0}^{\tau} \mathrm{d}\tau' \left[ \Phi(\tau') - 2\xi(\tau') \right], \tag{A.7}$$

$$\zeta(\tau) = \int_{0}^{\tau} d\tau' \, \Gamma(\tau') \sin(\tau - \tau'). \tag{A.8}$$

Special cases warrant investigation. For  $(\alpha_{\rho} = \alpha, \alpha_{\tau} = 0, \alpha_{\zeta} = 0)$  the proper mode of the perturbation is  $\eta_{\text{prop}} =$ 

<sup>&</sup>lt;sup>14</sup> When the initial pole positions were placed further from the perpendicular orientation to the mutual orbital plane, the system often evolved quickly toward satellite escape or collision.

<sup>&</sup>lt;sup>15</sup> We note that  $n_0$  here plays the role of sidereal frequency, which is in fact affected by both the solar tidal field and the multipole fields of the two asteroids. Such details are not studied in this paper.

 $<sup>^{16}</sup>$  We note the unitary frequency in the left hand sides of (A.2) and (A.3) should be affected, in a more detailed theory, by the solar tidal terms and multipole interactions of the asteroids' gravity fields, and should become anomalistic frequency (i.e., the natural pericenter frequency).

 $-\alpha \tau/2$ , i.e., a "stronger gravity" requires motion with higher angular speed at the circular orbit of a given radius. Obviously, this perturbation is fully correlated with the solution of a priori unknown total mass of the system. More important is the case of a permanent along-track acceleration  $(\alpha_{\rho} = 0, \alpha_{\tau} = \alpha, \alpha_{\zeta} = 0)$ , which results in the proper perturbation modes  $\xi_{\text{prop}} = 2\alpha\tau$  and  $\eta_{\text{prop}} = -3\alpha\tau^2/2$ , analogous to the heliocentric COM perturbation due to the Yarkovsky effect: a linear drift in semimajor axis produces a quadratic advance in the longitude in orbit. This is by far the most significant effect both in the global and local dynamics. Finally, linearity of the system (A.2)–(A.4) implies any periodic term in  $\delta \mathbf{f}$  produces perturbation  $\delta \mathbf{r}$  containing the same spectral components. As typical in linear resonant systems, forcing terms with periodicity close to the natural (anomalistic) period of the binary component revolution are amplified due to the presence of small denominators (e.g., Nordtvedt, 1991, 1995).

Out of these fundamental modes, the one due to the nonvanishing along-track acceleration is the most important on a long-term (since the related orbital displacement propagates quadratically in time). Whether we should expect this effect in the internal motion of the binary system or not arises, interestingly, from an analogy with motion of the geodynamics Earth satellites among which the case of LAGEOS has been the most thoroughly studied (e.g., Afonso et al., 1989; Métris et al., 1997; Slabinski, 1997). Thermal effects (the so called photon thrust or Yarkovsky-Schach effect in that context) are well known to produce secular along-track orbital acceleration when the spacecraft enters the Earth shadow. The Earth-spacecraft pair represents "an extreme binary system" where the secondary component (spacecraft) has a vanishing size, and only the primary (Earth) eclipses the secondary. In the case of NEA binaries both components are of the same size and produce a complex series of mutual eclipsing events, but the overall conclusion is the same: thermal relaxation in re-radiation of the absorbed sunlight during eclipses results in a non-vanishing along-track acceleration. The analogy with the spacecraft dynamics makes us also to think that precession of the orbital plane, defined by the relative motion of the two components, approximately averages out this principal effect on a timescale larger than the precession period (roughly a couple of centuries as for the solar gravitational torque). Certainly this compensation is not exact, but the purpose of this paper is not to study the longterm dynamics of the binary asteroids due to the Yarkovsky forces, but to determine its observability on a short-term scale. Our basic model of the fixed zero-approximation relative orbital plane of the two components is thus justified.

In practice we chose a zero approximation relative orbit of the two asteroids and solve the HDP problem (Section 2.1) to obtain the Yarkovsky accelerations  $\mathbf{f}_1$  and  $\mathbf{f}_2$  for both asteroids. These series are combined to get the effective Yarkovsky accelerations in the global and local dynamics, Eqs. (1) and (2). From the latter, a difference  $\mathbf{f}_2 - \mathbf{f}_1$ , we compute the radial ( $\alpha_\rho$ ), transverse ( $\alpha_\tau$ ) and normal  $(\alpha_{\zeta})$  perturbation components and numerically evaluate the quadratures in (A.6)–(A.8). From the resulting displacement vector ( $\xi$ ,  $\eta$ ,  $\zeta$ ) we subtract a part identical to the free mode ( $\xi_{\text{free}}$ ,  $\eta_{\text{free}}$ ,  $\zeta_{\text{free}}$ ) which goes in the solution of the initial orbital elements and the total mass of the system. Most importantly, we disregard a linear term in  $\eta$ ; however, any non-linearity in  $\eta$  should be observable and possibly interpreted as a proper Yarkovsky perturbation in the local motion of the system.

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