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Thermal force perturbations of the LAGEOS orbit: the albedo radiation part

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Abstract. A role of the Earth reflected radiation heating for the thermal effects perturbing the motion of the LAGEOS satellite is clarified. A small, but not negligible, part of the constant drag acting on the satellite and detected via the orbit analysis is attributed to this phenomenon. The contribution to the observed eccentricity excitations is very small in contrast to the hopes raised in the recent analysis by Métris *et al.* (1996). Copyright © 1996 Elsevier Science Ltd

1. Introduction

Thermal effects, consisting in the anisotropic emission of the thermal radiation by the satellite surface, have been recognized as very important constituents of the empirically observed long-term along-track residuals of the LAGEOS orbit (Rubincam, 1982, 1987, 1988; Afonso *et al.*, 1989; Scharroo *et al.*, 1991; Farinella *et al.*, 1996). More recently, Métris *et al.* (1996) reported that similar effects contribute significantly to analogous long-term eccentricity excitations of the LAGEOS orbit.

Modelling of the thermal effects is highly complicated for several reasons. Despite the simple geometrical form of the LAGEOS satellite, heat transfer and related surface temperature distribution is a rather intricate problem due to the contribution of diverse constituents of the satellite body and their differential sensitivity to the visible and infrared heating. First models have been developed by Rubincam (1987, 1988) and Afonso *et al.* (1989). To our knowledge, the recent work of Slabinski (1996) is the most refined attempt to treat this complex problem. Yet, this careful model still needs empirical corrections when applied to the LAGEOS data (Slabinski, 1996). As we do not feel able to improve on this subject, we shall follow in this paper a simpler, but qualitatively acceptable, formulation by Rubincam (1987). A second fundamental complexity of the modelling of thermal effects is due to different sources of the satellite surface heating. In principle, one should account for the direct solar radiation (resulting in the version of the thermal effect often called Yarkovsky–Schach effect in the literature related to LAGEOS; Afonso *et al.* (1989) and Scharroo *et al.* (1991)), the Earth infrared radiation (the corresponding thermal effect is called Yarkovsky–Rubincam effect; Rubincam (1987, 1988)) and the sunlight reflected on the Earth's surface (hereafter called the albedo radiation). Up to now the third heating source — albedo radiation — has been neglected in the literature.

In this paper, we focus on understanding the essential nature of the orbit perturbations due to the satellite thermal effects related to the albedo radiation heating. Notice, that one can hardly reject the albedo heating of the satellite surface a priori, because the radiative flux due to the Earth reflected sunlight can instantaneously exceed the Earth infrared flux by a factor greater than 2. However, the albedo radiative field as a heating source is rapidly changing over the satellite revolution around the Earth. The average albedo radiative flux is thus about one half of the nearly constant infrared radiative flux (Afonso *et al.*, 1989).

Motivation to pursue this study is due to the surprising finding of Métris *et al.* (1995) that an empirical amplitude of the Yarkovsky–Schach thermal effect should be about twice as large as previously suggested values in order to explain anomalous excitations of the LAGEOS eccentricity together with the along-track residuals. Although the revised thermal model of the LAGEOS satellite by Slabinski (1996) does suggest an increase of the total amplitude of the effect, it generally fails to explain the necessary factor 2 (improving the factor of theoretical mismodelling to about 1.7; Métris *et al.* (1996)). It was thus hoped that the missing albedo heating mechanism, both in previous studies by Rubincam (1987, 1988) and

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Afonso *et al.* (1989) and a more recent study by Slabinski (1996), would, at least partly, fill the gap between the empirically needed and the theoretically expected values of the amplitude of the thermal effects. The weight of the empirical arguments for increasing the discussed amplitude of the thermal effects lies on the side of the eccentricity excitations. Thus, in the first instant we focus on the determination of the eccentricity excitations. However, as a side product of our analysis we shall also consider contributions of the studied effects to the long-term along-track force and inclination excitation.

Our method is fully analytical, which makes it appealing for long-term studies. This is a somewhat surprising fact, because the albedo radiation is generally thought to be a very complex phenomenon requiring either a complicated analytical formulation (e.g. Rubincam and Weiss, 1986; Borderies and Longaretti, 1990; Borderies, 1990) or a completely numerical setting. It has been, however, recently shown by Métris *et al.* (1996) that for the purpose of the long-term eccentricity excitations, the direct albedo force can be significantly simplified with only minor loss of precision. We employ the same scheme for the thermal effects studied in this work. The validity of our results is then checked by comparison of the analytical results with the completely numerical approach.

Finally, we note that this paper rests on the assumption of LAGEOS fast rotation (see for discussion Farinella *et al.* (1996)). In terms of physical assumptions it means that the thermal inertia time-scale for all satellite components is significantly greater than its rotation period. As a consequence, the satellite's fast rotation averages out thermal force components not aligned with the spin axis, leaving the sole non-vanishing component directed along the spin axis. The LAGEOS I rotation model suggests that this essential assumption holds well for the first 25 years of the mission (Farinella *et al.*, 1996), the time interval on which we currently dispose with orbital data.

The paper is organized as follows: in the next section, we derive an estimation of the LAGEOS I surface temperature variations due to the albedo radiation heating and compute the corresponding thermal force together with long-term orbit perturbations. Next, we apply the theory to the LAGEOS I satellite and discuss several implications for interpretation of its observed orbital excitations.

2. The albedo radiation heating of LAGEOS: formulation and dynamical consequences

Mutual configuration of the satellite spin axis and the direction towards the radiation source leads to differential heating of the satellite surface and corresponding distribution of the surface temperature. Once the temperature distribution is determined one can easily compute the thermal force, related to differences in the momentum carried away from the satellite by the thermal photons, acting on the satellite (e.g. Afonso *et al.*, 1989). Section 2.1 is devoted to the physical considerations yielding a simple law for the temperature distribution on the satellite surface and the resulting thermal force. In Section 2.2 we give the long-term perturbations of the satellite orbit.

2.1. *Force modelling*

Principal simplifications adopted in our study can be listed as follows:

- we neglect the finite width of the albedo radiation field near the satellite, referring to the radiation flux vector **F** as its full characteristics (for terminology see Vokrouhlický *et al.* (1993));
- we neglect "latitudinal heat conduction" (here the satellite's spherical coordinates are defined with respect to its spin axis) relating the heat budget of a particular surface element (retroreflector, mounting rings, etc.) to the equilibrium between absorbed/emitted radiation and the radial heat conduction in the thin surface shell.

We also recall implications of the satellite's fast rotation discussed in the Introduction (the thermal force is oriented along the spin axis). Our formulation is thus close to Rubincam's treatment (Rubincam, 1987) and we shall also follow his notation and terminology. We note that this approach seems appropriate for the treated albedo radiation thermal source because the effect is smaller than the principal thermal effects.

Following Rubincam's (1987; Section 2) analysis one easily shows that the temperature excess ΔT over the mean surface value T_0 satisfies in the linearized theory

$$\Delta T(t;\theta) = \frac{3\varepsilon}{\rho C_p R_R} \int_{t' \leq t} \mathrm{d}t' \Phi(t',\theta) \mathrm{e}^{-\nu_R(t-t')} \tag{1}$$

where $\Phi(t',\theta)$ is the radiation flux due to an external source applied to the outer surface at satellite colatitude θ and instant t'. In addition, formula (1) contains several material constants of the particular satellite surface element (in agreement with Rubincam (1987) we shall speak about the retroreflector): its emissivity ε, its density ρ, its specific heat C_p and its radius R_R. The result (1) corresponds well to physical intuition which suggests: (i) a linear response to the heating Φ provided the effects are sufficiently weak (nonlinear effects being neglected), and (ii) exponential damping of the excited temperature difference ΔT. Theory suggests

$$\nu_R = \tau_R^{-1} = \frac{12\varepsilon\sigma T_0^3}{\rho C_p R_R} \tag{2}$$

for the inverse of the characteristic damping time (also called time-lag in context of the satellite thermal effects). Note that we allow for a generally non-periodic (or at least not simply periodic) heating signal $\Phi(t,\theta)$. On the contrary, Rubincam (1987) assuming a sinusoidal function $\Phi_{Rub}(t,\theta) \simeq \exp(i\nu_{Rub}t)$, obtained a simpler result consisting merely in a constant time shift between the heating of a given retroreflector and its response expressed by the temperature increase. Our formulation (1) is more general, and more complicated, and reduces to Rubincam's formulae provided a simpler heating signal $\Phi_{Rub}(t,\theta)$ is assumed and particular conditions on the frequency ν_{Rub} are satisfied.

Due to simplifying conditions listed at the beginning of this chapter we relate the heating flux $\Phi(t,\theta)$ to the albedo radiation flux vector $\mathbf{F}$ by

$$\Phi(t',\theta) = (\mathbf{F}.\mathbf{s})\cos\theta \tag{3}$$

where $\mathbf{s}$ is the unit vector directed along the satellite spin axis (θ is the satellite's colatitude as before). Armed with the previous results we can easily compute the spin axis directed component of the thermal acceleration $\mathbf{a}$ (see equation (4) of Afonso *et al.* (1989))

$$\mathbf{a} = \frac{16}{3}\frac{\varepsilon\sigma T_0^3}{mc}A_{\perp}\int_{-1}^{1}\mathrm{d}(\cos\theta)\Delta T(t;\theta)\cos\theta\,\mathbf{s} \tag{4}$$

where $A_{\perp} \equiv \pi R_s^2$ is the satellite's geometrical cross-section, m its mass and c the velocity of light.

The essential step thus consists in approximating the albedo radiation flux vector $\mathbf{F}$ from (3). Here, we shall rely on the successful formulation by Métris *et al.* (1996) who used

$$\mathbf{F}(\sigma) = \frac{1}{4}I_0\bar{\mathscr{A}}\alpha^2\left[\cos\psi(\sigma)\left(\frac{8}{3}+3\alpha\right)\mathbf{r}-\alpha\mathbf{n}\right]\Gamma_{\mathrm{alb}}(\sigma) \tag{5}$$

where I_0 is the solar constant, $\bar{\mathscr{A}}$ the averaged Earth albedo, $\alpha \equiv R_e/a$ the satellite parallax (R_e the Earth radius and a the satellite's semimajor axis) and ψ the geocentric Sun–satellite angular distance. Unit vectors $\mathbf{r}$, resp. $\mathbf{n}$, denote the geocentric positions of the satellite, resp. the Sun. Notice that, for further use, we prefer to parametrize the flux vector $\mathbf{F}$ by the angle σ defined as the difference between the satellite's longitude in orbit λ (measured from the orbital node) and the solar longitude in orbit λ_s, i.e. $\sigma \equiv \lambda - \lambda_s$ (see Fig. 1 for definitions). Finally, "the albedo shadow function" $\Gamma_{\mathrm{alb}}(\sigma)$ is simply defined as the usual step function

$$\Gamma_{\mathrm{alb}}(\sigma) = 1 \text{ for } \sigma\in(-\pi/2,\pi/2)$$
$$= 0 \text{ for } \sigma\in(\pi/2,3\pi/2)$$

(there is the obviously trivial relation $\sigma = 2\pi\nu t+\sigma_0$ between the angle σ and the satellite's orbital frequency ν). Defining the instantaneous heating function $H_a(\sigma)$ and the thermally relaxed heating function $h_a(\sigma)$ by the following relations

$$F_S \equiv (\mathbf{F}.\mathbf{s}) = \frac{1}{4}I_0\bar{\mathscr{A}}\alpha^2 H_a(\sigma) \tag{6}$$

and

$$h_a(\sigma) \equiv \frac{1}{\sigma_R}\int_{-\infty}^{\sigma}\mathrm{d}\sigma' H_a(\sigma')\mathrm{e}^{-(\sigma-\sigma')/\sigma_R} \tag{7}$$

(here $\sigma_R \equiv 2\pi\nu/\nu_R$). The investigated thermal force due to the albedo radiation heating of the satellite reads

$$\mathbf{a}(\sigma) = \gamma h_a(\sigma)\mathbf{s} \tag{8}$$

with

$$\gamma \simeq \frac{2}{9}\varepsilon\bar{\mathscr{A}}\alpha^2\frac{A_{\perp}I_0}{mc}. \tag{9}$$

Employing estimated values for the quantities in (9) (see Rubincam, 1987, 1988; Slabinski, 1996), we expect γ to lie in the interval 1–5 pm s^{-2}. Similarly, a reasonable range of the phase-lag σ_R is 1.5–2.5 rad.

An essential step towards the fully analytical formulation of our problem consists in computing the relaxed heating function $h_a(\sigma)$. Interestingly, it can be worked out with only one minor restriction of the exact 2π-periodicity of the heating function $H_a(\sigma)$, thus neglecting changes of the mutual configuration of the satellite orbit and the Sun. More precisely, the following results hold if the characteristic time scale on which the Sun–orbit configuration changes is much greater than the thermal time-lag τ_R, a property which is well satisfied for LAGEOS. The results then read

$$h_a(\sigma) = C_1 + D_1\cos 2\sigma + D_2\sin 2\sigma$$

$$-(C_1-D_1)\frac{\mathrm{e}^{-(\pi/2+\sigma)/\sigma_R}}{1+\mathrm{e}^{-\pi/\sigma_R}} \text{ for } \sigma\in(-\pi/2,\pi/2) \tag{10}$$

$$= (C_1-D_1)\frac{\mathrm{e}^{(\pi/2-\sigma)/\sigma_R}}{1+\mathrm{e}^{-\pi/\sigma_R}} \text{ for } \sigma\in(\pi/2,3\pi/2) \tag{11}$$

provided the following parameters are introduced

$$C_1 = \frac{1}{2}\left(\frac{8}{3}+3\alpha\right)\left(n_x s_x + n_y s_y\right) - \alpha\cos\theta_s \tag{12}$$

$$C_2 = \frac{1}{2}\left(\frac{8}{3}+3\alpha\right)\left(n_x s_x - n_y s_y\right) \tag{13}$$

$$C_3 = \frac{1}{2}\left(\frac{8}{3}+3\alpha\right)\left(n_y s_x + n_x s_y\right) \tag{14}$$

$$D_1 = \frac{C_2 - 2\sigma_R C_3}{1+4\sigma_R^2} \tag{15}$$

$$D_2 = \frac{C_3 + 2\sigma_R C_2}{1+4\sigma_R^2} \tag{16}$$

Here, $\cos\theta_s \equiv \mathbf{n}.\mathbf{s}$ in agreement with the terminology of Scharroo *et al.* (1991) and Métris *et al.* (1996). Indices (x, y, z) correspond to the projection of the given vector onto a direct triad of unit vectors ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$): $\mathbf{e}_x$ is defined by

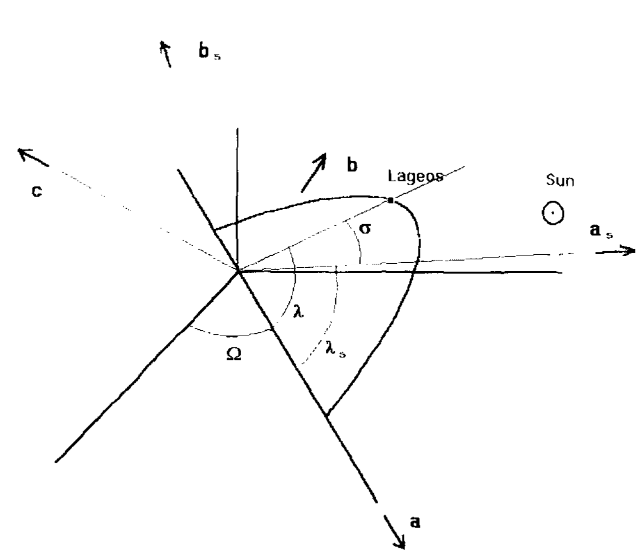

Fig. 1. Orbital parameters and related quantities introduced in the text

the projection of solar direction $\mathbf{n}$ onto the satellite orbit plane and $\mathbf{e}_z$ is in the direction of the satellite angular momentum vector. Note also, that the equality $\int d\sigma H_a(\sigma) = \int d\sigma h_a(\sigma) (= \pi C_1)$ assures equilibrium between the absorbed radiative energy and thermally emitted energy.

Formulas (10) and (11) (together with definitions (12)–(16)) complete our analytical approximation for the thermal force related to the albedo radiation heating of the LAGEOS satellite. As mentioned in the Introduction, we shall check the validity of our approximate results by employing a fully numerical model and comparing its results with those obtained by the simple analytical formulation given before. Practically it means that we shall compute components of the albedo radiation flux vector $\mathbf{F}$ exactly (without assuming simplification (5)) by employing

$$F_R = \frac{I_0}{\pi} \bar{\mathcal{A}} \alpha^2 J_x(\psi,\alpha)$$

$$F_T = -\frac{I_0}{\pi} \frac{(\mathbf{n}.\mathbf{t})}{\sin\psi} \bar{\mathcal{A}} \alpha^3 J_y(\psi,\alpha)$$

$$F_W = -\frac{I_0}{\pi} \frac{(\mathbf{n}.\mathbf{w})}{\sin\psi} \bar{\mathcal{A}} \alpha^3 J_y(\psi,\alpha) \tag{17}$$

for the radial, transverse and normal components of $\mathbf{F}$ in the orbital unit vector frame $(\mathbf{r}, \mathbf{t}, \mathbf{w})$. Formulation (17) has been given by Borderies and Longaretti (1990) for the case of the Earth with constant albedo and diffuse reflection on its surface (due to the small amplitude of the studied effects, we shall not go beyond the assumption of constant Earth albedo). Functions $J_x(\psi,\alpha)$ and $J_y(\psi,\alpha)$ are given in Borderies and Longaretti (1990) in the form of 1-D integrals. Once the components of the radiative flux vector are computed, we can follow formulas (6)–(9) to determine the thermal force acting on the satellite. Obviously, in the second case all quantities are computed numerically.

2.2. *Long-term orbital perturbations*

Analysis of the long-term orbital perturbations is the proper task of this study. We thus proceed to average the orbital perturbations corresponding to the thermal force formulated in the previous section over one satellite revolution around the Earth. Because the effects are very small we limit ourselves to direct computation of the mean values of the perturbations. We also neglect the eccentricity of the satellite orbit.

Starting with the averaged along-track acceleration T (related to the semi-major axis excitation) we obtain

$$T = \frac{\gamma}{3\pi} \left[(3C_1 + D_1)s_y - 2D_2 s_x - 3\sigma_R(C_1 - D_1)\frac{s_x + \sigma_R s_y}{1+\sigma_R^2} \right] + \mathcal{O}(e). \tag{18}$$

Similarly, excitations of the eccentricity vector components $(h,k) \equiv (e\cos\omega, e\sin\omega)$, given by the corresponding set of Gauss equations (e.g. Milani *et al.*, 1987), read

$$\dot{h} = \frac{\gamma}{8na} [6C_1 s_b + (D_1 s_y - D_2 s_x)\cos\lambda_s - (D_1 s_x + D_2 s_y)\sin\lambda_s] + \mathcal{O}(e) \tag{19}$$

$$\dot{k} = \frac{\gamma}{8na} [-6C_1 s_a + (D_1 s_x + D_2 s_y)\cos\lambda_s + (D_1 s_y - D_2 s_x)\sin\lambda_s] + \mathcal{O}(e) \tag{20}$$

(overdot means time derivative) with notation

$$s_a = s_x\cos\lambda_s - s_y\sin\lambda_s$$

$$s_b = s_x\sin\lambda_s + s_y\cos\lambda_s.$$

Finally, we compute the long-term excitation of the inclination I of the satellite orbit. After straightforward algebra we obtain

$$\frac{dI}{dt} = \frac{\gamma}{3\pi na}(\mathbf{s}.\mathbf{w}) \left[(3C_1 + D_1)\cos\lambda_s - 2D_2\sin\lambda_s - 3\sigma_R(C_1 - D_1)\frac{\sin\lambda_s + \sigma_R\cos\lambda_s}{1+\sigma_R^2} \right] + \mathcal{O}(e). \tag{21}$$

In the following, we apply the previous results to the case of the LAGEOS I satellite.

3. Application to LAGEOS I

For a decade or so, the orbital analysis of the LAGEOS I satellite has provided superb ground for geodynamics and physics of the Earth environment. Direct treatment of its orbit served for setting the top models of the Earth gravitational field together with its variations, Earth rotation monitoring, tectonic plate motion, etc. Yet, supplemental and highly interesting information is hidden in the series of empirical parameters necessary to be added to the basic dynamical model to reach laser tracking precision of about 1–2 cm. They are usually given in the form of time derivatives (called excitations) of the orbital elements. A detailed overview of the observed LAGEOS I orbit residuals and/or orbital element excitations is given in Tapley *et al.* (1993). It is generally believed that these excitations are due to tiny physical effects mostly of nongravitational origin. Their interpretation and link to particular physical phenomena offers interesting problems at the frontier between physics and dynamics.

The best understood among these orbit excitations are the empirical series of the along-track acceleration ("semi-major axis excitation"). Considerable effort has been paid to their modelling in the last ten years. Much less attention has been oriented towards understanding excitations of the other orbital elements. Yet, parallel understanding of several (or even all, if possible) excitations yields, apart from overall consistency, also the possibility of decorrelating some of the empirical parameters involved in the suggested models for the nongravitational effects (see Métris *et al.*, 1996). A recent attempt for a joint solution of the along-track series and the eccentricity excitations by Métris *et al.* (1996) resulted in the surprising conclusion that the empirically expected amplitude of the Yarkovsky–Schach thermal effect is about 1.7 times greater than its theoretical estimation. As mentioned in the Introduction, one possible suggestion to explain this discrepancy was the missing albedo heating mechanism in the models used for the thermal effects. We thus start by considering the eccentricity excitations. We also note that an important part of a correct determination of the thermal effects is a firm knowledge of the satellite spin axis orientation. In the case of LAGEOS I this has been for a long time a rather obscure issue. Recently, Farinella *et al.* (1996) succeeded in proving that the older theoretical model by Bertotti and Iess (1991) is well suited to recover the main features of the LAGEOS I spin axis evolution. We thus employ the LAGEOS I spin axis evolution as derived by Farinella *et al.* (1996).

Figure 2 shows the real (a) and the imaginary (b) parts of the eccentricity excitations corresponding to the albedo heated part of the thermal effects. The true LAGEOS I orbit is considered. We took $\gamma = 2.5$ pm s^{-2} for the amplitude of the effect and compared results corresponding to several values of the phase-lags σ_R (in rad): 2.5, solid line; 1.0, dashed line; 0.5, dashed-dotted line. Although there are small differences according to the value of the phase-lag, the essential information is given by the ordinate scale of both parts of Fig. 2. The eccentricity effect of the studied mechanism is much smaller than that needed for explaining the discrepancy between the theoretical and empirical values of the Yarkovsky–Schach effect discussed above. We have checked that, as in the case of the direct albedo radiation pressure (Métris *et al.*, 1996), the eccentricity excitations computed analytically, with use of the simplified formula (5) for the albedo radiative flux, fit with surprising precision the corresponding numerical solution with the complete computation (17) of the albedo radiative flux. Notice also, that the real part of the signal (in Fig. 2a) is in fact anticorrelated with the Yarkovsky–Schach effect contribution and thus acts rather negatively in respect of explaining the Yarkovsky–Schach thermal effect discrepancy mentioned above. It has been recently communicated to us by V. J. Slabinski (personal communication), that an intuitive explanation of this feature may be due to the fact that the Earth-albedo radiation preferentially illuminates the "dark" (anti-Sun) pole of the satellite.

Next, we turn to the possible contribution of the albedo part of the along-track thermal thrust. Figure 3 shows a comparison of the numerically determined thrust -- solid line -- and the analytically modelled thrust by formula (18) -- dashed line. As anticipated by analogy with the direct albedo radiation pressure, the coincidence of the two signals is weaker than in the case of the eccentricity excitations. Nevertheless, the analytical formulation is still not seriously corrupted, apart from occasional positive peaks. It is interesting to note that the expected averaged value of the drag is about $-(0.2$–$0.3)$ pm s^{-2}. This fact has

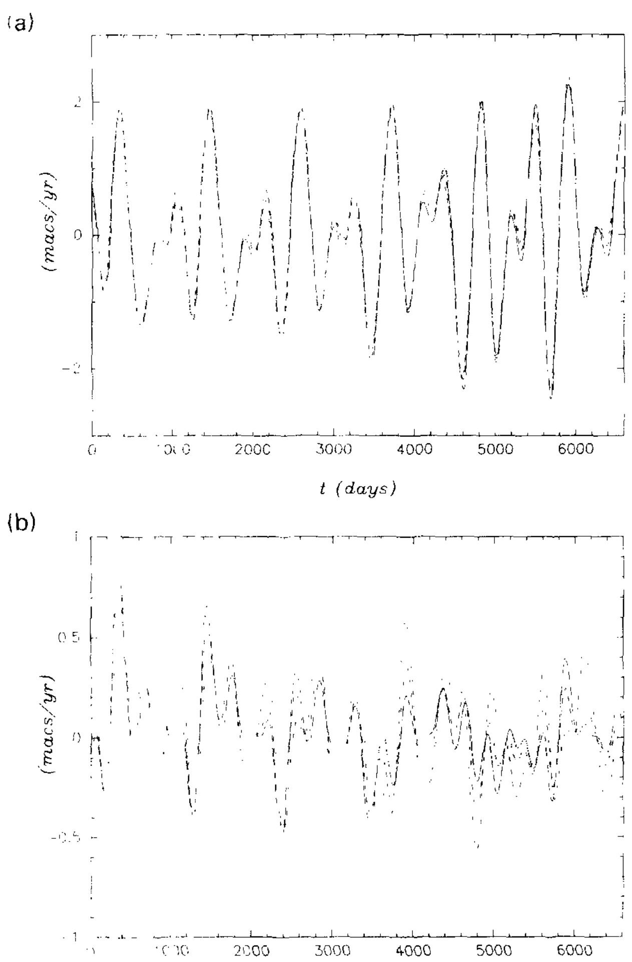

Fig. 2. LAGEOS eccentricity excitations due to the albedo heated thermal effect vs. time since launch: (a) the real part; (b) the imaginary part. Assumed value of the amplitude γ is 2.5 pm s^{-2}. Different curves correspond to different values of the phase-lag σ_R (in rad): (i) 2.5, solid curve; (ii) 1.0, dashed curve; (iii) 0.5, dashed-dotted curve

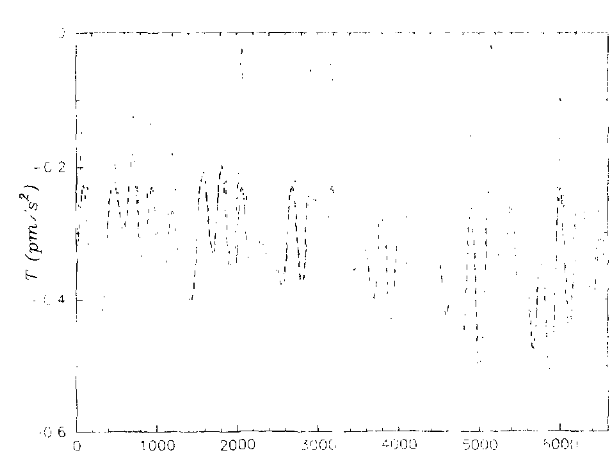

Fig. 3. Along-track thermal thrust due to the albedo heated part vs. time since launch. Solid curve corresponds to the completely numerical setting, while dashed curve to the fully analytical solution. Parameters: $\gamma = 2.5$ pm s^{-2} and $\sigma_R = 2.5$ rad

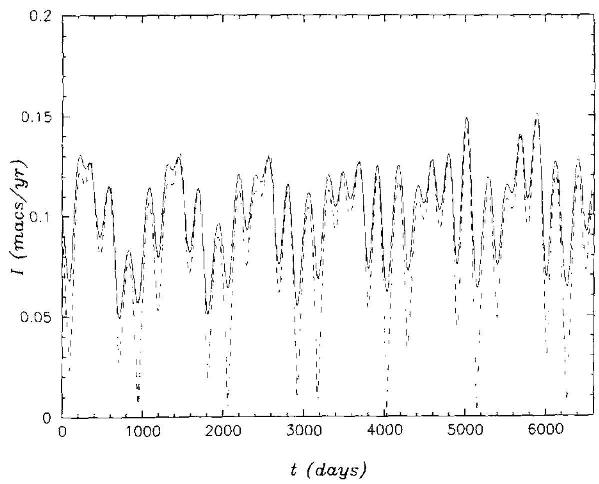

Fig. 4. Inclination excitation vs. time since launch. Plotted curves as in the previous figure: solid line for numerical solution, dashed line for analytical solution. Parameters: $\gamma = 2.5\,\mathrm{pm\,s^{-2}}$ and $\sigma_R = 2.5\,\mathrm{rad}$

a direct implication on the value of the adjusted constant along-track force which is to be attributed to the neutral and charged particle drag at the LAGEOS altitude (e.g. Afonso *et al.*, 1985; Barlier *et al.*, 1986). Interestingly, an analogous, but positive, additional effect of about $0.4\,\mathrm{pm\,s^{-2}}$ resulted from Slabinski's model (Slabinski, 1996) for LAGEOS I thermal thrust. Slabinski interpreted this effect in terms of the mutual coupling between the Yarkovsky–Schach and Yarkovsky–Rubincam effects related to the decrease of the satellite mean temperature during the shadow crossing periods. The albedo radiation as a heat source for the LAGEOS thermal effect thus seems to compensate for at least half of Slabinski's correction.

Finally, we present a similar solution for the inclination excitation in Fig. 4. A completely numerical solution (solid line) is again compared with the analytical solution (21) (dashed line). Apart from good coincidence of the two approaches, we note an average contribution to the inclination excitation of about $0.1\,\mathrm{mas\,yr^{-1}}$. This value is to be compared with the corresponding inclination effect of about 0.7–$1.0\,\mathrm{mas\,yr^{-1}}$ due to the Yarkovsky–Rubincam effect studied by Farinella *et al.* (1990). The observed value of the secular increase of the LAGEOS inclination is of about $1\,\mathrm{mas\,yr^{-1}}$ (P. Exertier, personal communication).

4. Conclusion

The main results of this study can be summarized in several items:

- the albedo radiation as a heating source for the LAGEOS thermal effects is unlikely to be responsible for the discrepancy between the theoretical and empirical estimated amplitude;
- it is expected that the albedo heated thermal thrust contributes by about $-(0.2$–$0.3)\,\mathrm{pm\,s^{-2}}$ to the secular part of the empirically observed along-track signal, partly absorbing Slabinski's recent correction due to coupling between the Yarkovsky–Schach and Yarkovsky–Rubincam effects;
- analogous secular contribution to the inclination excitation is of about $0.1\,\mathrm{mas\,yr^{-1}}$.

The first result points to a persisting discrepancy between the theoretical and empirical modelling of the LAGEOS I thermal effects. In the near future, some effort is still needed for its understanding.

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