Pergamon

Planet. Space Sci., Vol. 45, No. 4, pp. 419–425, 1997

0032–0633/97 $17.00+0.00

PII: S0032–0633(96)00147–X

Thermal force effects on slowly rotating, spherical artificial satellites—II. Earth infrared heating

David Vokrouhlický[1] **and Paolo Farinella**[2]

[1]Institute of Astronomy, Charles University, Švédska 8, 15000 Prague 5, Czech Republic
[2]Gruppo di Meccanica Spaziale, Dipartimento di Matematica, Università di Pisa, Via Buonarroti 2, 56127 Pisa, Italy

Received 16 August 1996; revised 5 November 1996; accepted 6 November 1996

Abstract. The theory developed in Farinella and Vokrouhlický (*Planet. Space Sci.* **44**, 1551–1561, 1996) is applied for the thermal force effects affecting the motion of LAGEOS-type artificial satellites to the specific case of the so-called Yarkovsky–Rubincam effect, due to anisotropic heating of the satellite's surface by the Earth's infrared radiation. This theory allows the usual assumption of a fast rotational motion of the satellite to be removed, so as to allow for a misalignment between the direction of the thermal force and the spin axis. When applied to the LAGEOS satellite, which has been strongly despun by magnetic torques since its launch in 1976, the theory shows that in the next decade the orbital perturbations associated with the Yarkovsky–Rubincam effect will become much different than those predicted by the simple rapid-rotation approach. This is the case both for the drag-like effect causing a semimajor axis decay, and for the inclination changes. © 1997 Elsevier Science Ltd

1. Introduction

Among the nongravitational forces affecting the motion of passive, geodynamical satellites such as LAGEOS, those due to thermal effects are very important, i.e. the asymmetrical heating of the satellite body by external radiation sources (the Sun and the Earth) and the corresponding temperature gradients on the surface, resulting in a net recoil force related to anisotropic emission of thermal radiation. A first generation of theoretical models for these effects appeared in the late 1980s and early 1990s (Rubincam, 1987, 1988, 1990; Slabinski, 1988; Afonso *et al.*, 1989; Scharroo *et al.*, 1991). However, because of the complexity of the physical processes under scrutiny, these models often adopted crude simplifications. In recent years, the need for more accurate and/or general models has been highlighted by the increasing accuracy and time span of the LAGEOS orbital data in particular the series of along-track acceleration residuals, of the order of a few pm s^{-2} and spinning two decades and by the improved understanding of the rotation rate and spin axis evolution of the satellite under magnetic and gravitational torques (for a brief recapitulation of the subject, we refer the reader to the introduction of Farinella and Vokrouhlický (1996), hereinafter referred to as paper I; the notations and terminology introduced in that paper will be kept here without significant changes).

In paper I we developed a new theory for the orbital perturbations related to thermal effects on LAGEOS-type satellites, modelled as spherical bodies with homogeneous physical and optical properties. The main improvement of this theory with respect to the previous ones is that it is not restricted to the case when the rotation of the satellite is fast with respect to its thermal inertia and orbital period, and that as a consequence the recoil force in general is not directed along the spin axis, but has nonzero "equatorial" components. In other words, if we consider the satellite as a small spinning planet, we account for "diurnal" effects as well as for "seasonal" ones. In paper I, this theory was applied to the so-called Yarkovsky–Schach effect, caused by the solar illumination of the satellite interrupted by passages through the Earth's shadow, and a complete solution was derived for LAGEOS's long-term orbital perturbations due to this effect. Here, we plan to perform a similar study but for the so-called Yarkovsky–Rubincam effect, in which the satellite's heating due to the Earth's infrared radiation is responsible for the surface temperature gradients and the related perturbing force. Of course, in reality both effects are present at the same time and cause orbital changes appearing in the observed residuals of the orbital elements (in particular, the semimajor axis).

Rubincam (1982) was the first to point out the importance of this particular thermal effect for the understanding of the observed along-track residuals of LAGEOS's orbit.

Correspondence to: D. Vokrouhlický

It was an important first step, because Rubincam clearly understood that a critical factor was the "seasonal" character of the effect in the case of LAGEOS, in contrast with the "diurnal" version of the Yarkovsky thermal force usually described in previous literature about non-gravitational forces on natural solar system bodies (see, i.e. Radzievski, 1952; Peterson, 1976; Burns *et al.*, 1979). This was justified because of the very fast rotational motion of LAGEOS at the time of launch (about 1.6 rotations per second), compared to the typical timescales for the satellite's thermal inertia and orbital motion (which are three to four orders of magnitude longer). A quantitative analysis of the along-track perturbations caused by this effect was given by Rubincam (1987, 1988), who showed that such perturbations could explain most of the previously unmodelled semimajor axis decay of LAGEOS ($\approx 1\,\mathrm{mm\,day}^{-1}$). Rubincam (1987) used a highly simplified thermal model, while the thermal behaviour of the real satellite is quite complex, as recently shown by Slabinski (1997). However, his simplifying assumptions did not lead to quantitatively important errors, and allowed him to introduce a fairly simple mathematical formulation of the problem. For these reasons, in paper I we have followed a similar approach.

The reason why a new theory accounting for the "diurnal" effects on a slowly-spinning satellite is needed is that LAGEOS's rotation is actually slowing down. As suggested by both theoretical models and observations (Bertotti and Iess, 1991), this despinning process—due to dissipation by eddy currents generated by the satellite's motion through the geomagnetic field—is an exponential one as a function of time, with an *e*-folding characteristic time of ≈ 3 years only. As a consequence, by the end of this century LAGEOS's rotational period will be of the order of 10^3 s, namely comparable to the typical thermal inertia timescale. In such a situation strong "diurnal" temperature gradients will arise on the satellite's surface, and thermal force will cease to be aligned with the spin axis. The new "equatorial" components of the force will give rise to qualitatively new perturbation patterns, and therefore it is clear that, the previously adequate fast-rotation approximation will be no longer useful. This is the main motivation of this work, in which we are going to reformulate the Yarkovsky–Rubincam effect for arbitrary values of the satellite's rotational period.

The remainder of this paper is organized as follows: in Section 2 we recall the theoretical setting of the problem as given in paper I and adapt it for the analysis of the thermal effects caused by the Earth's infrared heating. Next, in Section 3 we apply the theory to the case of the LAGEOS satellite and show that the effects due to the equatorial thermal force components appearing in the slow-rotation case will soon become detectable. In Section 4 we summarize our conclusions and sketch some future work on this subject.

2. Theory

2.1. *Recalling the basic formulae*

Because we are going to use the same mathematical and physical formulation of the problem as given in Section 2.1 of paper I, we will only give here the main formulae needed for understanding what follows. Given the constants specifying the material and geometric properties of the satellite (R its radius, m the mass, α and ϵ the absorption and emission coefficients of the surface, C_p and κ the specific heat and thermal conductivity, ρ the density, T_0 the average equilibrium temperature), the perturbing thermal force per unit mass of the satellite can be expressed as

$$\mathbf{a}(t) = -\frac{8}{mc}\frac{\alpha\epsilon\sigma}{\rho C_p R}T_0^3 \int_{t'<t} \mathrm{d}t'\, \mathrm{e}^{\nu_R(t-t')}\mathbf{h}(t,t';R) \qquad (1)$$

where σ and c are the Stefan–Boltzmann constant and the velocity of light, respectively, whereas the "kernel function" $\mathbf{h}(t,t';R)$ is defined by

$$\mathbf{h}(t,t';R) = \int_{\Omega^*(t)} \mathrm{d}A(\theta,\phi)\Delta\Phi(t';R,\mathbf{n}')\mathbf{n}(\theta,\phi). \qquad (2)$$

Here, $\Delta\Phi(t;R,\mathbf{n})$ is the scalar radiative flux due to the external source(s) through the satellite surface element defined by the unit vector $\mathbf{n}(\theta,\phi)$ at time t, (θ,ϕ) being the spherical coordinates defined on the satellite surface. The integration domain $\Omega_*(t)$ in the integral is implicitly defined by the illumination condition $\Delta\Phi(t;R,\mathbf{n})\geq 0$ (at time t). We recall that the primed variables in equations (1) and (2) are to be computed at the retarded time $t'<t$.

The fundamental parameters controlling the thermal effects in our model are: (i) $\sigma_R \equiv 2\pi\tau_R/T_{\mathrm{rev}}$, where τ_R is the thermal lag time characterizing the response of the satellite surface to external heating and T_{rev} the period of the satellite's orbit around the Earth; and (ii) $r \equiv T_{\mathrm{rev}}/T_{\mathrm{rot}}$, where T_{rot} is the satellite's rotational period. The basic set of timescales relevant for our problem is thus: $(T_{\mathrm{rot}},T_{\mathrm{rev}},\tau_R)$. We assume that all of these timescales are much shorter than those over which the satellite's spin axis direction and its orbit evolve.

The only (formal) constraint of our solution is $r\geq 1$, due to the fact that the predicted evolution of LAGEOS's spin axis direction has been derived from an averaged model (Farinella *et al.*, 1996), which cannot be applied in the transition through a spin-orbit resonance (Habib *et al.*, 1994).

2.2. *The Yarkovsky–Rubincam effect*

Let us start by introducing a suitable reference frame, which will be used throughout this paper: the Z-axis (with unit direction vector $\mathbf{e}_Z$) is directed along the satellite's spin axis, whereas the X-axis (unit direction vector $\mathbf{e}_X$) is chosen in such a way that the satellite's geocentric position vector always lies in the X–Z plane (with a *negative* X-component). This defines our system uniquely, with the exception of some singular configurations (spin axis in the orbital plane), which do not require a separate treatment. Note that in general, this frame rotates in a non-uniform way with respect to inertial space.

Contrary to the case treated in paper I (satellite heating caused by sunlight), the position unit vector of the heat source—the Earth in the current case—is not fixed in our reference system. Actually, for the colatitude $\theta_s(\lambda)$ of the heating source in the XYZ-frame we obtain

$$\cos\theta_s(\lambda) = -\sqrt{1-s_c^2}\sin\lambda \tag{3}$$

where $s_c \equiv \mathbf{s}\cdot\mathbf{c}$ is the cosine of the angle defined by the unit vectors directed along the satellite's spin axis ($\mathbf{s}$) and the perpendicular to the orbital plane ($\mathbf{c}$), while λ is the satellite's orbital longitude (like in paper I, we approximate the satellite's orbit as a circular one, so λ can be used instead of the time t as an independent variable). We have arbitrarily chosen the origin of λ so that it corresponds to $\theta_s = \pi/2$; otherwise one should have considered an additional constant λ_0 in the corresponding angular argument of equation (3). Since none of the final results depend on this arbitrary quantity λ_0, we have preferred to take directly $\lambda_0 = 0$. Note, also that, the parameter s_c is the only geometrical quantity which plays a role in the formulation and solution of our problem.

As a second step, we have to specify the "rotation law" of the satellite in the XYZ reference system. This means that we have to describe the motion of each of its surface elements. Because the Z-axis coincides with the spin axis, the rotational motion of any surface element changes its "longitude" ϕ and leaves unchanged its "colatitude" θ. If we consider a surface element having an initial longitude ϕ_0 at an arbitrary starting time, we obtain for its longitude ϕ as a function of λ:

$$\phi(\lambda) = \phi_0 + (r-1)\lambda - f(\lambda) \tag{4}$$

where we have introduced the auxiliary function

$$f(\lambda) = \arctan(s_c\tan\lambda) - \lambda. \tag{5}$$

Finally, we need to specify the external radiation field which heats the satellite's surface. In this respect, the Earth's infrared illumination is more complicated than the sunlight case, because the former is not unidirectional, but is better represented by a cone with an $\approx 60°$ aperture at LAGEOS's altitude. However, we shall neglect this geometrical complication, and approximate the Earth's infrared radiation field at the satellite's altitude as a homogeneous field of intensity Φ_0 ($\approx 65\ \mathrm{W\,m^{-2}}$ in the LAGEOS case). Then, if we take the satellite's surface element characterized by the spherical coordinates (θ,ϕ), we have

$$\Delta\Phi(\lambda;R,\mathbf{n}) = \Phi_0[\sin\theta_s(\lambda)\sin\theta\cos\phi + \cos\theta_s(\lambda)\cos\theta]. \tag{6}$$

The intermediate quantities that we need for computing the perturbations on the orbital semimajor axis and inclination of the satellite are the instantaneous values of the transverse $[T(\lambda)]$ and binormal $[W(\lambda)]$ components of the thermal force. Simple geometry shows that they can be written as

$$T(\lambda) = (1-s_c^2)\frac{\sin\lambda\cos\lambda}{\sin\theta_s(\lambda)}F_X - \frac{s_c}{\sin\theta_s(\lambda)}F_Y + \sqrt{1-s_c^2}\cos\lambda\,F_Z \tag{7}$$

and

$$W(\lambda) = \frac{\sqrt{1-s_c^2}}{\sin\theta_s(\lambda)}(s_c\sin\lambda\,F_X + \cos\lambda\,F_Y) + s_cF_Z. \tag{8}$$

where (F_X, F_Y, F_Z) are the projections or the thermal force vector onto the three unit vectors $(\mathbf{e}_X, \mathbf{e}_Y, \mathbf{e}_Z)$. Equation (3) yields

$$\sin\theta_s(\lambda) = \sqrt{\cos^2\lambda + s_c^2\sin^2\lambda} \tag{9}$$

for the same factor in equations (7) and (8).

Considering the rotation law (4), one can easily show that the vector $\mathbf{h}(\lambda,\lambda';R)$ appearing in equation (2) can be expressed as

$$\mathbf{h}(\lambda,\lambda';R) = \frac{2\pi}{3}R^2\Phi_0\begin{pmatrix}\sin\theta_s(\lambda')\cos\xi(\lambda,\lambda')\\ \sin\theta_s(\lambda')\sin\xi(\lambda,\lambda')\\ \cos\theta_s(\lambda')\end{pmatrix} \tag{10}$$

where we have defined

$$\xi(\lambda,\lambda') = (r-1)(\lambda-\lambda') + f(\lambda') - f(\lambda). \tag{11}$$

Combining this result with equations (1) and (2), we can see that only the Z component can be computed analytically by simple quadratures. Thus, for the component of the thermal force directed along the spin axis we obtain

$$F_Z(\lambda) = -\gamma\sqrt{1-s_c^2}\,\frac{\sin\lambda - \sigma_R\cos\lambda}{1+\sigma_R^2} \tag{12}$$

with

$$\gamma = -\frac{16}{3}\frac{\pi R^2\Phi_0}{mc}\tau_R T_0^3\frac{\chi\epsilon\sigma}{\rho C_p R}. \tag{13}$$

F_Z obviously vanishes for $s_c = 1$. Like in the case of the Yarkovsky–Schach effect, discussed in paper I, equation (13) for the normalizing factor γ provides just a good semi-quantitative estimate for the intensity of the thermal force, but since real satellites are not perfectly spherical and homogeneous, in practical applications the precise value of γ should be determined empirically by a fit to the available data rather than calculated a priori.

Introducing the previous result into equations (7) and (8), we obtain the contribution of the Z thermal force component to the along-track perturbing acceleration (which, for small orbital eccentricities, is just proportional to the time derivative of the semimajor axis):

$$T_Z = \frac{\gamma}{2}\frac{\sigma_R}{1+\sigma_R^2}(1-s_c^2). \tag{14}$$

Gauss' perturbation equations also give the corresponding excitation of the orbital inclination:

$$\left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_Z = \frac{\gamma}{2na}\frac{\sigma_R}{1+\sigma_R^2}s_c\sqrt{1-s_c^2}. \tag{15}$$

These results reproduce those of Rubincam (1987, 1990): in equation (12) the coefficient γ gives the amplitude of the perturbing effect, the factor containing the σ_R parameter corresponds to Rubincam's thermal lag factor [$(\sin\delta)$, in his notations], and the $(1-s_c^2)$ term is exactly the angular-dependent piece of Rubincam's formula. Whenever $s_c = \pm 1$ (spin axis normal to the orbital plane), the T_Z ("seasonal") component of the thermal drag-like effect vanishes. On the contrary, it is maximum for $s_c = 0$, as expected. Similarly, equation (15) corresponds to the results of Farinella *et al.* (1990), in particular their equation (18), provided one substitutes their "delay angle" θ

and amplitude factor K by the relationships $\sin\theta \equiv \sigma_R/\sqrt{1+\sigma_R^2}$ and $K \equiv \gamma/\sqrt{1+\sigma_R^2}$.

The contribution of the thermal force components perpendicular to the spin axis (F_X and F_Y) to the along-track and out-of-plane perturbations is more complicated. We did not succeed in integrating equations (1) and (2) analytically and our results can only be expressed as formal quadratures. However, an efficient numerical evaluation is always possible (some applications will be discussed in the next section). For this purpose, we define the auxiliary quantities

$$I_X(\lambda) = \int_{\lambda-2\pi}^{\lambda} \mathrm{d}\lambda' \sin\theta_s(\lambda')\cos\xi(\lambda',\lambda)\,\mathrm{e}^{-(\lambda-\lambda')/\sigma_R} \quad (16)$$

$$I_Y(\lambda) = \int_{\lambda-2\pi}^{\lambda} \mathrm{d}\lambda' \sin\theta_s(\lambda')\sin\xi(\lambda',\lambda)\,\mathrm{e}^{-(\lambda-\lambda')/\sigma_R}. \quad (17)$$

Using this definition and applying the same averaging method we explained in Section 2 of paper I, we obtain for the mean value of the along-track perturbing acceleration due to the X and Y thermal force components:

$$\begin{aligned} T_{XY} = &-\frac{\gamma}{2\pi\sigma}\hat{C}(r-1,\bar{e})\int_0^{2\pi} d\lambda \\ &\times\left[I_X(\lambda)\frac{\partial}{\partial\lambda}\sin\theta_s(\lambda)+I_Y(\lambda)\frac{s_c}{\sin\theta_s(\lambda)}\right] \\ &+\frac{\gamma}{2\pi\sigma}\hat{S}(r-1,\bar{e})\int_0^{2\pi}\mathrm{d}\lambda \\ &\times\left[I_Y(\lambda)\frac{\partial}{\partial\lambda}\sin\theta_s(\lambda)-I_X(\lambda)\frac{s_c}{\sin\theta_s(\lambda)}\right] \end{aligned} \quad (18)$$

with

$$\hat{C}(r,\bar{e}) \equiv \frac{1-\bar{e}\cos(2\pi r)}{1-2\bar{e}\cos(2\pi r)+\bar{e}^2} \quad (19)$$

$$\hat{S}(r,\bar{e}) \equiv \frac{\bar{e}\sin(2\pi r)}{1-2\bar{e}\cos(2\pi r)+\bar{e}^2}. \quad (20)$$

The total value of the thermal drag-like acceleration is of course $T \equiv T_Z + T_{XY}$.

In general the asymptotic behaviour of T_{XY} for large values of the parameter $r = T_{\mathrm{rev}}/T_{\mathrm{rot}}$ (the rapid-spin case) is obscured by the complicated structure of the previous formulae. We just mention that for large values of r the arguments of the trigonometric functions in the integrands of equations (16) and (17) change rapidly, leading to small mean values I_X and I_Y. In a few special cases, we can obtain an analytical result. When $s_c = \pm 1$, we have

$$T_{XY}(r; s_c = 1) = -\gamma\frac{(r-1)\sigma_R}{1+(r-1)^2\sigma_R^2} \quad (21)$$

and $T_{XY}(r; s_c = -1) = T_{XY}(r+2; s_c = 1)$. Both values of T_{XY} decay proportional to $1/r$ for large values of r. On the other hand, it can be shown by a straightforward but tedious computation that $T_{XY}(r; s_c = 0) \propto 1/r^2$. Therefore, we expect that for generic values of s_c the averaged T_{XY} component decays proportional to $1/r^{\alpha}$, where $\alpha \in (1,2)$.

In a similar way, the contribution of the X and Y thermal force components to the averaged excitation of the orbital inclination can be expressed as

$$\begin{aligned} \left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)_{XY} = &-\frac{\gamma\sqrt{1-s_c^2}}{2\pi na}\hat{C}(r-1,\bar{e})\int_0^{2\pi}\mathrm{d}\lambda \\ &\times\frac{\cos\lambda}{\sin\theta_s(\lambda)}[s_c I_X(\lambda)\sin\lambda + I_Y(\lambda)\cos\lambda] \\ &-\frac{\gamma\sqrt{1-s_c^2}}{2\pi na}\hat{S}(r-1,\bar{e})\int_0^{2\pi}\mathrm{d}\lambda \\ &\times\frac{\cos\lambda}{\sin\theta_s(\lambda)}[s_c I_Y(\lambda)\sin\lambda - I_X(\lambda)\cos\lambda]. \end{aligned} \quad (22)$$

Again, we could not carry out the integrations appearing in equation (22) analytically and, in the next section, we will raise a numerical method for their evaluation.

A brief comment is to be made about the eccentricity excitations, which we have not, mentioned so far. As discussed by Métris and Vokrouhlický (1996), a simple order-of-magnitude estimate shows that the corresponding Yarkovsky–Rubincam perturbing effects, in the rapid-spin approximation, do not contribute in a significant way to the observed eccentricity residuals for LAGEOS. One can easily check that, the same holds also for the more general case including the X and Y force components, with which we are dealing here. As a consequence, we shall not discuss the eccentricity effects in the next section.

3. Applications

The purpose of this section is to demonstrate that in the specific case of the LAGEOS satellite the components of the thermal force perpendicular to the spin axis, which are significant when the satellite's rotation is slow enough, give rise to excitations of the orbital semimajor axis (T_{XY}) and inclination [$(\mathrm{d}I/\mathrm{d}t)_{XY}$] which cannot be neglected within an accurate orbit analysis and/or determination. As we will see, for LAGEOS the slow-rotation condition will be met during the next decade.

In Fig. 1 we compare the results for T in the case of LAGEOS from the Z-component alone (rapid-spin approximation, curve Z) to the contribution of the equatorial components, as a function of the geometrical parameter $s_c \equiv \mathbf{s}\cdot\mathbf{c}$. Because the "diurnal" effect associated with the equatorial components depends on the ratio r between the orbital and rotational periods, we have plotted separately the curves corresponding to $r = 1$ (curve 1), $r = 2$ (curve 2), and so on.

The thermal inertia parameter σ_R has been set to 2.4 rad, in agreement with Métris *et al.* (1996), whereas the amplitude parameter γ has been chosen in such a way that: $\gamma\sigma_R/2(1+\sigma_R^2) \simeq 3.08\ \mathrm{pm\,s^{-2}}$, which is consistent with recent analyses of the LAGEOS along-track residuals (i.e. Scharroo *et al.*, 1991; Farinella *et al.*, 1996). This value is in good agreement with the approximate equation (13), once the constants appropriate for the LAGEOS satellite are inserted in it. This set of parameters is used throughout this section.

In Fig. 1 we can observe several important features of the Yarkovsky–Rubincam effect. When the spin axis is perpendicular to the orbital plane ($s_c \pm 1$), the "seasonal" contribution vanishes while the "diurnal" contribution is

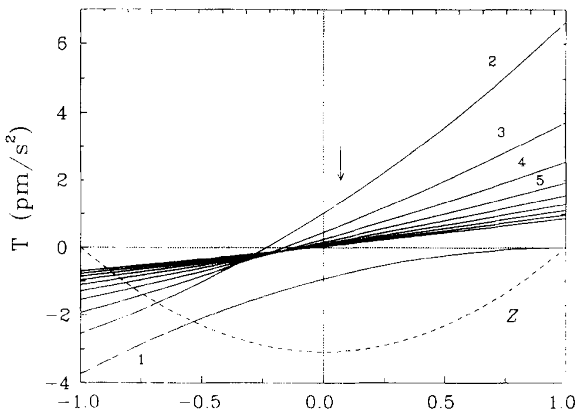

Fig. 1. The drag-like along-track component T of the Yarkovsky–Rubincam thermal force for LAGEOS vs. the s_c parameter (defined as the cosine of the angle between the satellite's spin axis and the perpendicular to its orbital plane). The dashed curve shows the contribution of the force component directed along the spin axis, whereas the solid curves show the contribution of the "equatorial" force components. The latter components depend on the ratio r between the orbital and rotational periods, so several cases have been plotted, corresponding to different values of r

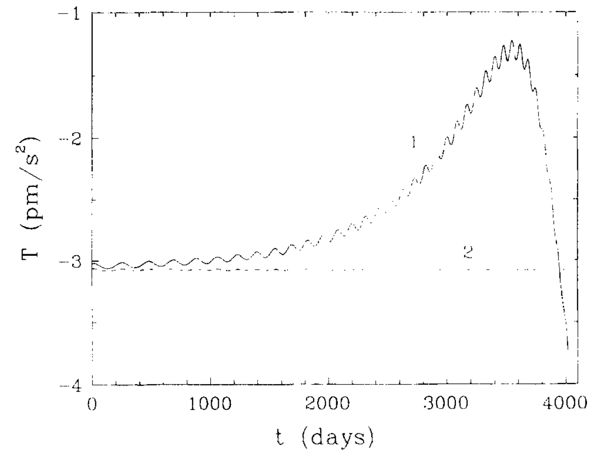

Fig. 3. Predictions of the averaged along-track thermal force component on LAGEOS due to the Yarkovsky–Rubincam effect vs. time (in days), starting on Jan. 1, 2000. The dashed curve 2 corresponds to the rapid-spin approximation, whereas the solid curve 1 shows the prediction of the theory developed here, including the equatorial force components as well. The evolution of LAGEOS's rotational state has been predicted according to the dynamical theory of Bertotti and Iess (1991), following Farinella *et al.* (1996)

maximum. In agreement with equation (21), the latter contribution decays as $1/r$ for increasing values of r. On the contrary, when the spin axis lies in the orbital plane, the "seasonal" contribution is maximum while the "diurnal" contribution is relatively small, and decays as $1/r^2$ for large values of r. The arrow in the figure points to the predicted value of s_c when the $r = 1$ spin-orbit resonance will be met by LAGEOS (according to Farinella *et al.* (1996)). This shows that the "diurnal" contribution will be a moderate one compared to its possible values for a high s_c, but as discussed below even this limited effect will be large enough to be observed.

A similar plot can be made for the Yarkovsky–Rubincam inclination effects. Figure 2 shows the time derivative (or excitation) of LAGEOS's mean inclination (dI/dt) as a function of s_c. When $s_c \pm 1$ it is easy to see that the thermal force lies in the orbital plane, and as a consequence the excitation of the inclination vanishes. The

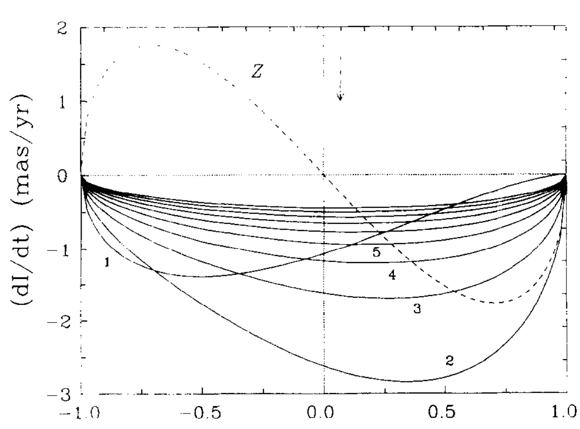

Fig. 2. The same as Fig. 1, but for LAGEOS's inclination excitation (dI/dt) vs. s_c

arrow again points to the value of the s_c parameter corresponding to the spin-orbit resonance for LAGEOS. Note that the contribution of the equatorial terms $(dI/dt)_{XY}$ is now quite important, as it will be confirmed below.

Figure 3 shows the Yarkovsky–Rubincam drag-like along-track perturbations predicted for LAGEOS after Jan. 1, 2000, assuming that its rotation rate and spin axis direction evolve as predicted by Farinella *et al.* (1996) on the basis of the theory developed by Bertotti and Iess (1991). The (dashed) curve obtained from the rapid-spin approximation is almost constant, while the (solid) curve corresponding to the total force, including the equatorial components, shows significant changes. We recall that the r.m.s. deviation of the empirical along-track residuals with respect to current dynamical models is of about $0.5\,\mathrm{pm\,s^{-2}}$, considerably smaller than the difference between the two curves shown in Fig. 3. Therefore a model accounting for the Yarkovsky–Rubincam slow-rotation effects will become essential to analyse the LAGEOS orbital residuals in the first decade of the next century.

Figure 4 shows the excitation of the mean orbit inclination of LAGEOS due to the Yarkovsky–Rubincam effect as a function of time since the launch epoch.

We can again conclude that in the near future the total effect (solid line) will become considerably different than the predictions based on the rapid-spin approximation (dashed line). Farinella *et al.* (1990) studied this inclination effect assuming a constant orientation of the satellite's spin axis, and concluded that a secular value of about $-0.84\,\mathrm{mas\,yr^{-1}}$ had to be expected (they also assumed a somewhat larger value of the phase-lag parameter). This result is in good agreement with the average value we find for the first 15– 20 years after launch, but later on the situation changes, due to the spin axis evolution. The influence of the equatorial thermal force components results in a fairly large negative value of the incli-

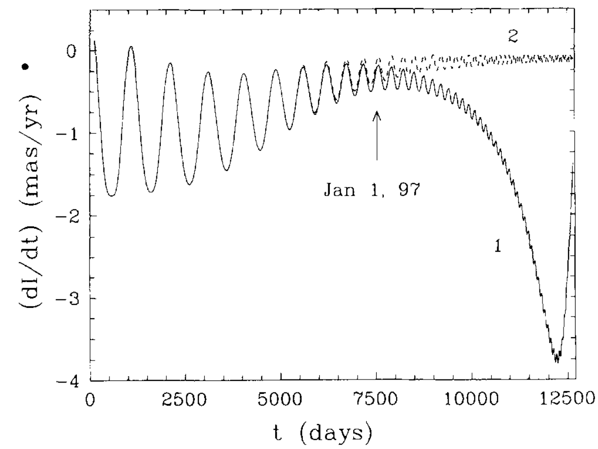

Fig. 4. The same as in Fig. 3 but for LAGEOS's inclination excitation (dI/dt) vs. time, starting at launch on May 4, 1976. The arrow points to the epoch Jan. 1, 1997, when the influence of the equatorial thermal force components will begin to be significant

nation excitation, occurring some 33 years after launch. This feature should be easily observable by accurate orbital determination. The arrow in the plot points to Jan. 1, 1997, an approximate date when the slow-rotation corrections will start to be significant.

4. Conclusions and future work

The main results obtained in this paper can be summarized as follows:

1. We have completed our theoretical treatment of the thermal effects acting on the motion of LAGEOS-type geodynamics satellites by taking into account the Earth's infrared heating (i.e. the so-called Yarkovsky–Rubincam effect) after the solar heating (Yarkovsky–Schach effect), which had been dealt with in paper I. In both cases our theory can be applied for arbitrary values of the satellite's rotation rate.
2. In the case of LAGEOS, the slower and slower rotation due to magnetic despinning and the corresponding "diurnal" temperature gradients associated with the Earth's infrared flux will soon give rise to significant components of the thermal force misaligned with the spin axis. As a consequence, within the next decade the long-term drag-like semimajor axis decay caused by the Yarkovsky–Rubincam effect will undergo significant modifications with respect to the rapid-spin approximation adopted in previous analyses. The secular part of the orbital inclination excitation is also going to show major changes.

We stress that the theory developed in paper I and applied again in this paper is based on several approximations and simplifying assumptions, and therefore further work should lead to significant improvements from the quantitative point of view. In particular, our spherical and homogeneous thermal model is a rather primitive one even for LAGEOS-type satellites. The complex structure of real satellites and the different interactions of their parts with the external radiation fields and with each other (including their mutual irradiation and shadowing effects) should be investigated and modelled in some detail. Slabinski (1997) performed a significant step forward in this direction, but limited his analysis to the rapid-spin case. Removing this assumption and generalizing Slabinski's thermal model to the situation investigated here appears as a promising area for future work.

Other issues which we have neglected are the possible coupling between the different thermal effects and the finite aperture of the Earth's infrared radiation field. As for the former problem, we have treated separately the case when the satellite surface is heated by the Sun's visible radiation (paper I) and by the Earth's infrared radiation (this paper). On the other hand, Slabinski (1997) correctly pointed out that in reality there is some interaction between the two thermal effects, because the average surface temperature T_0 depends on the occurrence of eclipses. The latter issue, arising from the finite angular size of the Earth as seen from the satellite, probably results in a slight decrease of the corresponding temperature gradients on the satellite's surface, and consequently of the thermal force perturbations. Quantitative studies of both these effects appear worth some effort.

Finally, we stress that in order to improve our capability of predicting the perturbations due to thermal effects for LAGEOS-type satellites we need to better understand and model the evolution of their rotation rate and spin axis direction. The theory of Bertotti and Iess (1991) should be refined by taking into account new perturbing effects (e.g. Vokrouhlický, 1996), and the complete system of Euler's equations should be studied numerically without the averaging assumption, in order to deal with the spin-orbit resonance crossings and the possible associated chaotic dynamics (see Habib *et al.* (1994) for some work in this direction).

Acknowledgements. We thank G. Métris for useful discussions and comments. D.V. is grateful to the University of Pisa (Italy) for its kind hospitality and to the Observatoire de la Côte d'Azur (Dept. CERGA, Grasse, France) where he worked on this project thanks to the "H. Poincaré" research fellowship. P.F. acknowledges support from the Italian Space Agency (ASI) and the Italian Ministry for University and Scientific Research (MURST).

References

Afonso, G., Barlier, F. and Carpino, M. (1989) Orbital effects of LAGEOS' seasons and eclipses. *Ann. Geophys.* **7,** 501–514.

Bertotti, B. and Iess, L. (1991) The rotation of LAGEOS. *J. Geophys. Res.* **96,** 2431–2440.

Burns, J. A., Lamy, P. L. and Sorer, S. (1979) Radiation forces on small particles in the Solar System. *Icarus* **40,** 1–48.

Farinella, P. and Vokrouhlický, D. (1996) Thermal force effects on slowly rotating, spherical artificial satellites—I. Solar heating. *Planet. Space Sci.* **44,** 1551–1561.

Farinella, P., Nobili, A. M., Barlier, F. and Mignard, F. (1990) Effects of thermal thrust on the node and inclination of LAGEOS. *Astron. Astrophys.* **234,** 546–554.

Farinella, P., Vokrouhlický, D. and Barlier, F. (1996) The rotation of LAGEOS and its long-term semimajor axis decay: a self-consistent solution. *J. Geophys. Res.* **101,** 17861–17872.

Habib, S., Holz, D. E. and Kheyfetz, A. (1994) Spin dynamics of the LAGEOS satellite in support of a measurement of the Earth's gravitomagnetism. *Phys. Rev.* **D50,** 6068–6079.

Métris, G. and Vokrouhlický, D. (1996) Thermal force perturbations of the LAGEOS orbit: the albedo radiation part. *Planet. Space Sci.* **44,** 611–617.

Métris, G., Vokrouhlický, D., Ries, J. C. and Eanes, R. J. (1997) Non-gravitational phenomena and the LAGEOS eccentricity excitations. *J. Geophys. Res.* (in press).

Peterson, C. (1976) A source mechanism for meteorites controlled by the Yarkovsky effect. *Icarus* **29,** 91–111.

Radzievski, V. V. (1952) The influence of the anisotropic reradiation of the sunlight on the orbital motion of asteroids and meteorites (in Russian). *Astron. Zh.* **29,** 162–170.

Rubincam, D. P. (1982) On the secular decrease in the semimajor axis of LAGEOS. *Celest. Mech.* **26,** 361–382.

Rubincam, D. P. (1987) LAGEOS orbit decay due to infrared radiation from Earth. *J. Geophys. Res.* **92,** 1287–1294.

Rubincam, D. P. (1988) Yarkovsky thermal drag on LAGEOS. *J. Geophys. Res.* **93,** 13805–13810.

Rubincam, D. P. (1990) Drag on the LAGEOS satellite. *J. Geophys. Res.* **95,** 4881–4886.

Scharroo, R., Wakker, K. F. and Ambrosius, B. A. C. (1991) On the along-track acceleration of the LAGEOS satellite. *J. Geophys. Res.* **96,** 729–740.

Slabinski, V. J. (1988) LAGEOS acceleration due to intermittent solar heating during eclipse periods (abstract). *Bull. Am. Astron. Soc.* **20,** 902.

Slabinski, V. J. (1997) A numerical solution for LAGEOS thermal thrust: the rapid-spin case. *Celest. Mech.* (in press).

Vokrouhlický, D. (1996) Radiation effects and LAGEOS' rotation. *Geophys. Res. Lett.* **23,** 3079–3082.